EC Workshop 2019 (March 25, 2019)

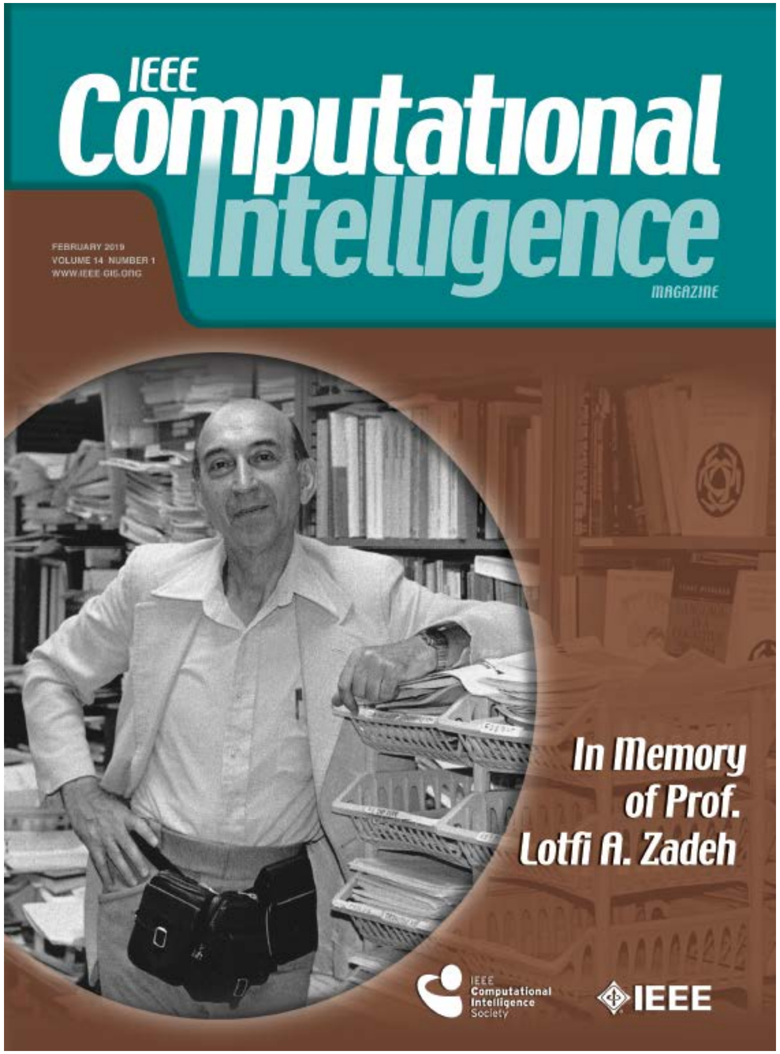
Evolutionary Many-Objective Optimization

Hisao Ishibuchi

Southern University of Science Technology (SUSTech)

Editor-in-Chief of IEEE Computational Intelligence Magazine

February 2019 Issue



May 2019 Issue

Top 4 Popular Articles in IEEE Computational Intelligence Magazine

(The number of downloads from IEEE Xplore)

Recent Trends in Deep Learning Based Natural Language Processing [Review Article]
Tom Young ; Devamanyu Hazarika ; Soujanya Poria ; Erik Cambria
7-20 2018
NLP & Deep Learning

Time Series Prediction Using Support Vector Machines: A Survey
Nicholas I. Sapankevych ; Ravi Sankar
4-24 2009
SVM

Evolutionary Fuzzy Systems for Explainable Artificial Intelligence: Why, When, What for, and Where to?
Alberto Fernandez ; Francisco Herrera ; Oscar Cordon ; Maria Jose del Jesus ; Francesco Marcelloni
1-11 2019
Explainable AI

Jumping NLP Curves: A Review of Natural Language Processing Research [Review Article]
Erik Cambria ; Bebo White
4-10 2014
NLP

Top 4 Popular Articles in IEEE Trans. on Evolutionary Computation

https://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=4235

(EiC: KC Tan)

Popular Articles

A fast and elitist multiobjective genetic algorithm: NSGA-II
K. Deb ; A. Pratap ; S. Agarwal ; T. Meyarivan
8-07 2002

NSGA-II

A Survey on Evolutionary Computation Approaches to Feature Selection
Bing Xue ; Mengjie Zhang ; Will N. Browne ; Xin Yao
11-30 2015

Feature Selection

An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems With Box Constraints
Kalyanmoy Deb ; Himanshu Jain
9-16 2013

NSGA-III

MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition
Qingfu Zhang ; Hui Li
11-27 2007

MOEA/D

Top 4 Popular Articles in Evolutionary Computation Journal

https://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=6720222

(EiC: Emma Hart)

Popular Articles

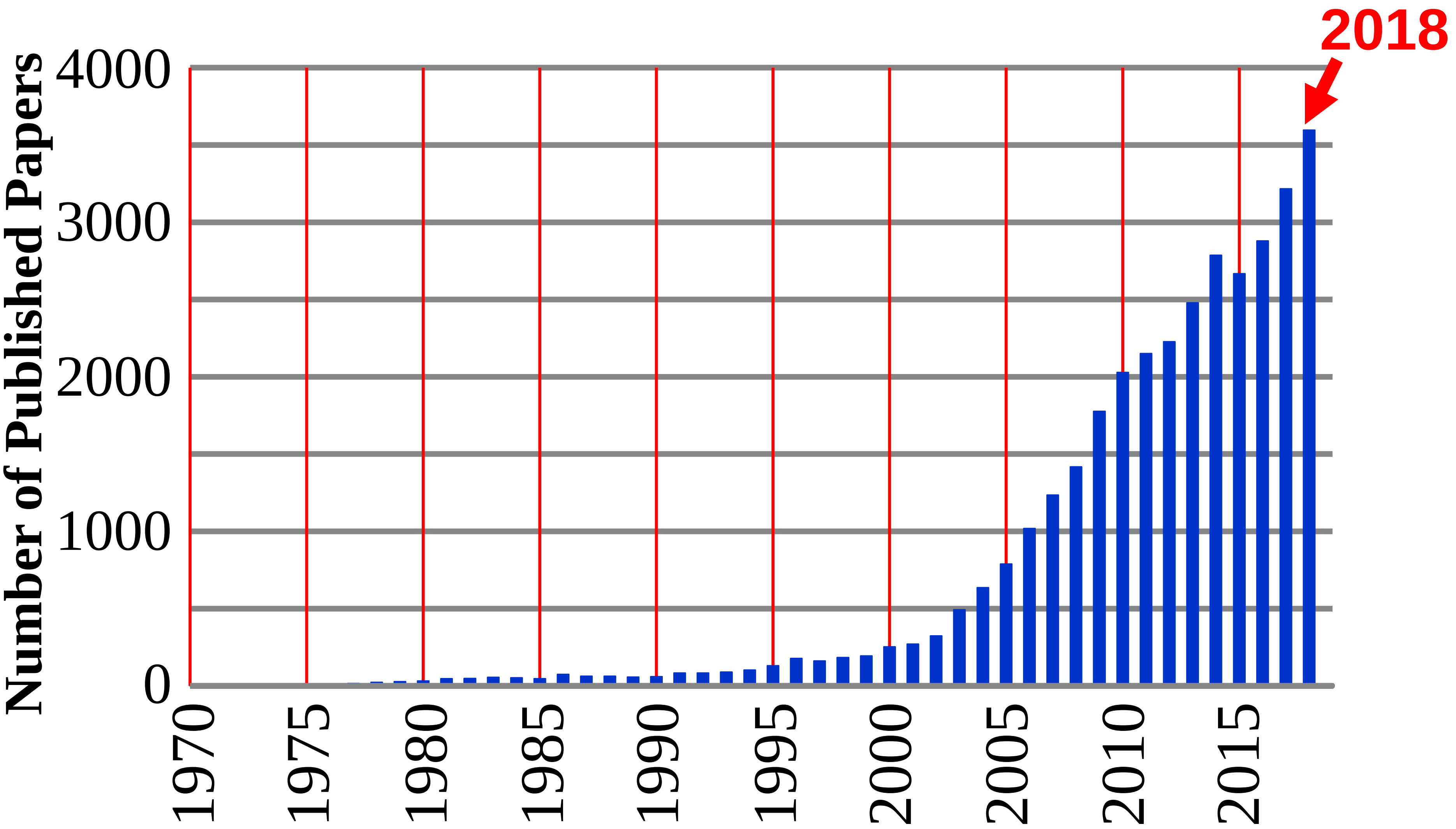
Emergent Solutions to High-Dimensional Multitask Reinforcement Learning
Stephen Kelly ; Malcolm I. Heywood
2-07 2019
Multitask RL

An Overview of Evolutionary Algorithms for Parameter Optimization
Thomas Bäck ; Hans-Paul Schwefel
5-19 2014
Parameter Optimization

Evolutionary Algorithms for Constrained Parameter Optimization Problems
Zbigniew Michalewicz ; Marc Schoenauer
5-19 2014
Constrained Optimization

Multimodal Optimization Using a Bi-Objective Evolutionary Algorithm
Kalyanmoy Deb ; Amit Saha
5-19 2014
EMO for Multimodal Optimization

Number of Papers with "Multi-objective" or "Multiobjective" in the Paper Titles (Source: Scopus Database)



Popularity of EMO Research

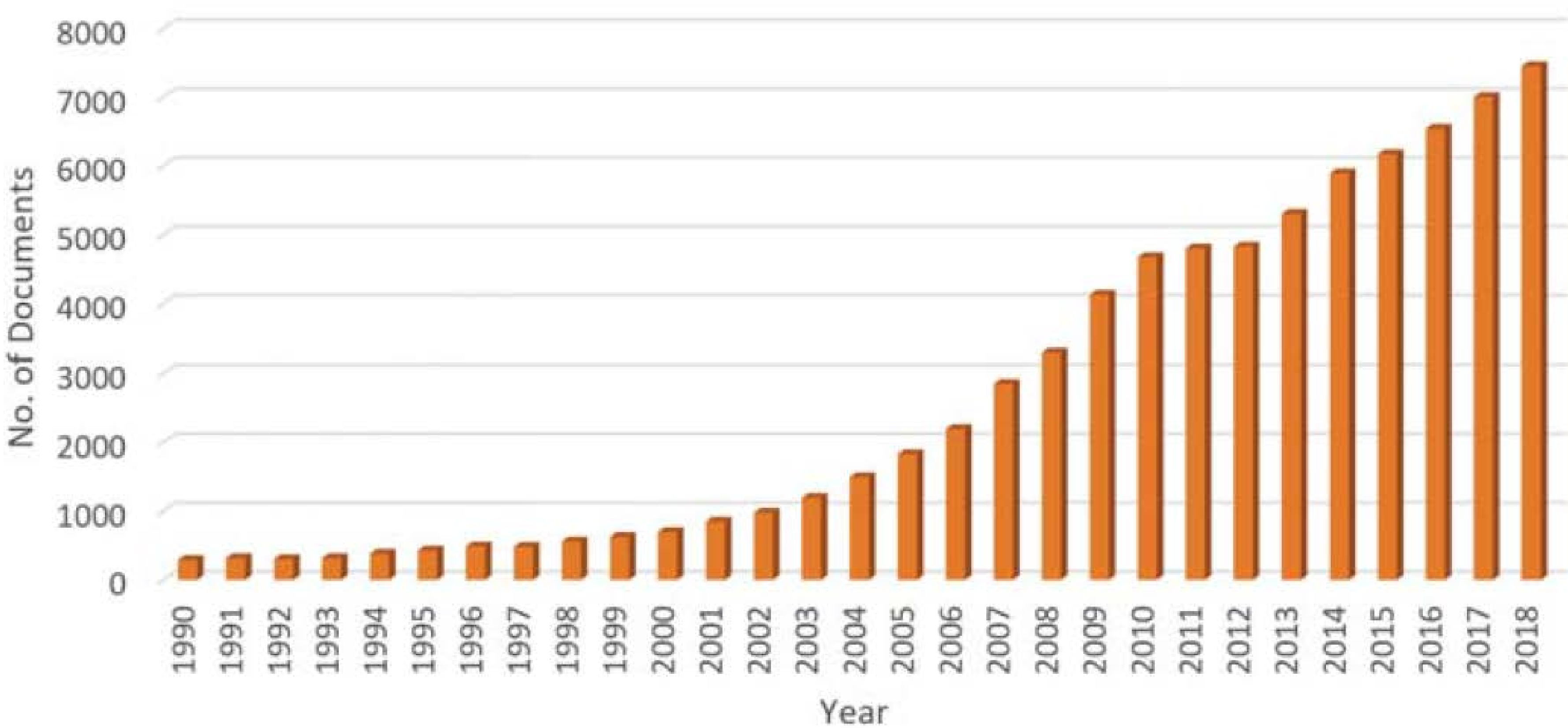


Fig. 1. Number of articles of EMO papers since 1990.

Copy from the preface of the EMO 2019 Proceedings

EMO 2019

March 10-13, 2021, Michigan State University

EMO
East Lansing '19

10th International Conference on
Evolutionary Multi-Criterion Optimization

In Partnership with HEEDS Design Space Exploration

EMO 2019

March 10-13, 2021, Michigan State University

EMO 2021 at Shenzhen, CHINA
March 28-31, 2021, SUSTech

General Chairs

Hisao Ishibuchi, Qingfu Zhang

Program Chairs

Hui Li, Handing Wang

Publication Chairs

Aimin Zhou, Ke Li

Organizing Chair

Ran Cheng

MCDM Chair

Kaisa Miettinen








Many-objective Optimization

Single-Objective Optimization: **Maximize** $f(\boldsymbol{x})$

Multi-Objective Optimization:

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})$

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x})$

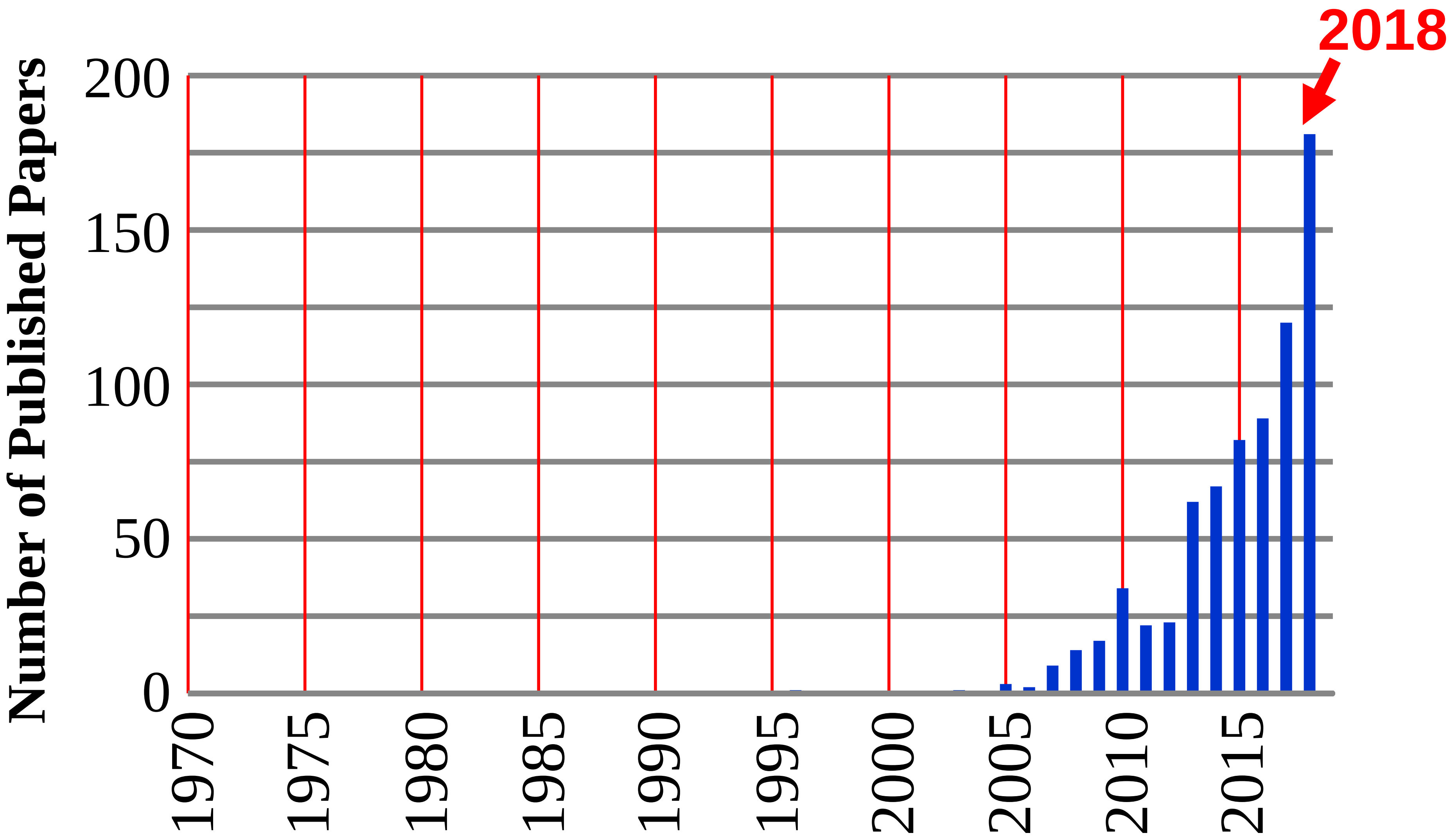
Many-Objective Optimization:

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x})$

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x}),\ f_5(\boldsymbol{x})$

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x}),\ f_5(\boldsymbol{x}),\ f_6(\boldsymbol{x})$

Number of Papers with "Many-Objective" in the Paper Titles (Source: Scopus Database)



Survey Paper on Evolutionary Many-Objective Optimization

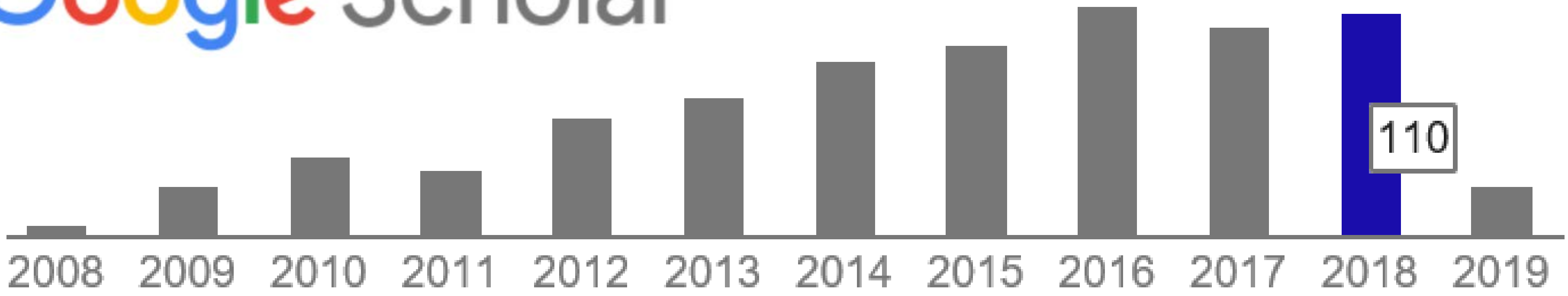
IEEE CEC 2008 (Based on Invited Talk at IEEE CEC 2007)

Evolutionary many-objective optimization: A short review

Authors	Hisao Ishibuchi, Noritaka Tsukamoto, Yusuke Nojima
Publication date	2008/6/1
Conference	2008 IEEE Congress on Evolutionary Computation

55 References (6 on EMO and 49 on many-objective)

Google Scholar



My Main Research in the Last 5 Years

Search Behavior Analysis of Many-Objective Algorithms

H. Ishibuchi et al., **Behavior of Multi-Objective Evolutionary Algorithms on Many-Objective Knapsack Problems**, *IEEE Trans. on Evolutionary Computation*, 2015. **156 Citations**

H. Ishibuchi et al., **Performance of decomposition-based many-objective algorithms strongly depends on Pareto front shapes**, *IEEE Trans. on Evolutionary Computation*, 2017. **120 Citations**

Analysis of Many-Objective Test Problems

H. Ishibuchi et al., Pareto fronts of many-objective degenerate test problems, *IEEE Trans. on Evolutionary Computation*, 2016. **38 Citations**

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Many-objective optimization is difficult: It is very difficult to search for a wide variety of Pareto optimal solutions.

My Main Research in the Last 5 Years

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Analysis of Many-Objective Test Problems

H. Ishibuchi et al., Pareto fronts of many-objective degenerate test problems, *IEEE Trans. on Evolutionary Computation*, 2016. **38 Citations**

Creation of many-objective test problems is difficult: We need a wide variety of test problems with various features.

My Main Research in the Last 5 Years

Search Behavior Analysis of Many-Objective Algorithms

H. Ishibuchi et al., **Behavior of Multi-Objective Evolutionary Algorithms on Many-Objective Knapsack Problems**, *IEEE Trans. on Evolutionary Computation*, 2015. **156 Citations**

H. Ishibuchi et al., **Performance of decomposition-based many-objective algorithms strongly depends on Pareto front shapes**, *IEEE Trans. on Evolutionary Computation*, 2017. **120 Citations**

Fare performance evaluation is difficult: It is very difficult to evaluate EMO algorithms on many-objective problems.

My Current Research: Indicator

Analysis of Performance Indicators

H. Ishibuchi et al., **How to Specify a Reference Point in Hypervolume Calculation for Fair Performance Comparison,** *Evolutionary Computation Journal*, 2018.

H. Ishibuchi et al., **Reference Point Specification in Inverted Generational Distance for Triangular Linear Pareto Front**, *IEEE Trans. on Evolutionary Computation*, 2018.

H. Ishibuchi et al., **Comparison of hypervolume, IGD and IGD$^+$ from the viewpoint of optimal distributions of solutions**, *EMO 2019*.

My Current Research: Test Problems

Multi-Objective Test Problems

H. Ishibuchi et al., **Regular Pareto Front Shape is not Realistic**, *IEEE CEC 2019.*

T. Matsumoto et al., **A Multiobjective Test Suite with Hexagon Pareto Fronts and Various Feasible Regions**, *IEEE CEC 2019.*

Y. Nojima et al., **Constrained Multiobjective Distance Minimization Problems**, *GECCO 2019.*

Multi-Modal Multi-Objective Test Problems

H. Ishibuchi et al., **A Scalable Multimodal Multiobjective Test Problem**, *IEEE CEC 2019.*

My Current Research: Algorithms

Performance Comparison of EMO Algorithms

R. Tanabe & H. Ishibuchi, **Non-elitist Evolutionary Multi-objective Optimizers Revisited**, *GECCO 2019.*

R. Tanabe & H. Ishibuchi, **An Analysis of Control Parameters of MOEA/D under Two Different Optimization Scenarios**, *Applied Soft Computing 2018.*

H. Ishibuchi et al., **Two-layered Weight Vector Specification in Decomposition-based Multi-objective Algorithms for Many-objective Optimization Problems**, *CEC 2019.*

Y. Liu et al., **Searching for Local Pareto Optimal Solutions: A Case Study on Polygon-based Problems**, *CEC 2019.*

Today's Plan

Difficulties in Evolutionary Many-Objective Optimization Studies

1. Difficulties related to many-objective search
2. Difficulties related to test problems
3. Difficulties related to performance evaluation

Today's Plan

Difficulties in Evolutionary Many-Objective Optimization Studies

1. Difficulties related to many-objective search
2. Difficulties related to test problems
3. Difficulties related to performance evaluation

Many-Objective Optimization
Frequently Discussed Difficulties

1. **Search for Pareto Optimal Solutions**
   Pareto dominance does not work well.
2. **Approximation of the Entire Pareto Front**
   A huge number of solutions are needed.
3. **Presentation of Obtained Solutions to DM**
   Visualization of high-dimensional solutions is difficult.
4. **Selection of a Single Final Solution**
   Choice of a single final solution is difficult for DM.
5. **Examination of Search Behavior**
   Visual observation of many-objective search is difficult.
6. **Large Diversity of Solutions in a Population**
   Usefulness of crossover may be degraded.

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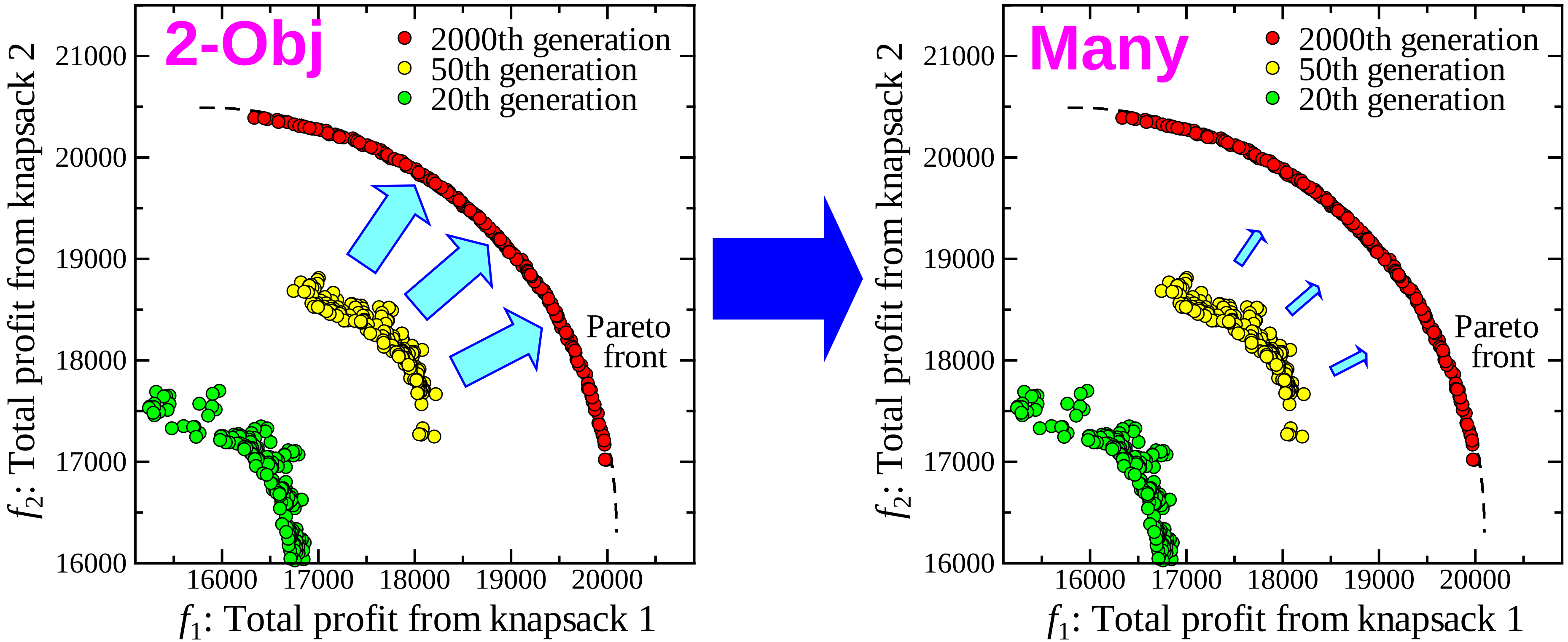
1. Search for Pareto Optimal Solutions
Pareto dominance does not work well

Q. Why are many-objective problems difficult for EMO ?

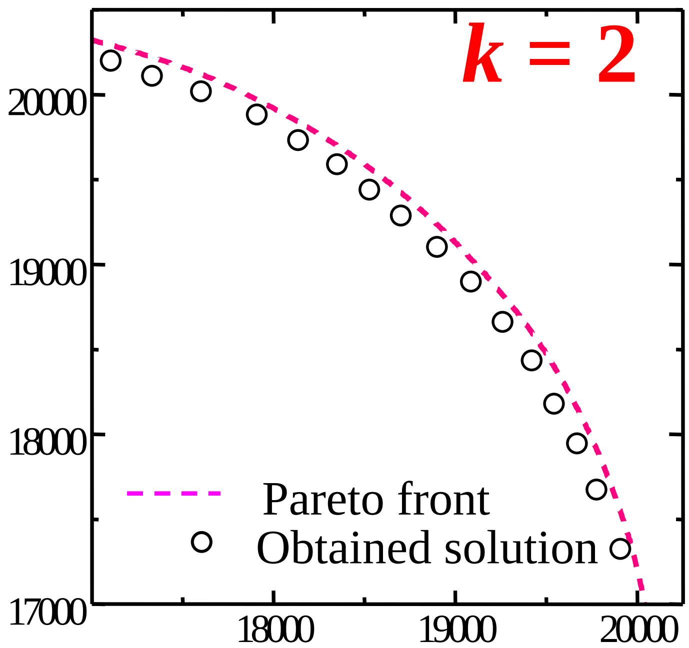
A. Solutions with many objectives are usually non-dominated with each other. Thus no strong selection pressure towards the Pareto front can be generated by Pareto dominance.

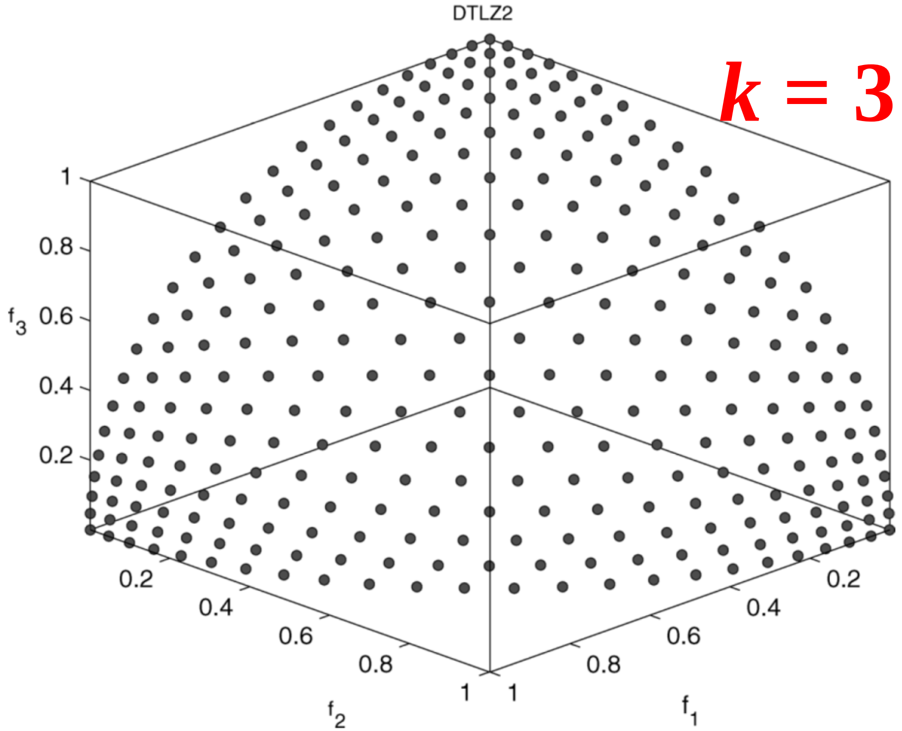


2. Approximation of the Entire Pareto Front

A huge number of solutions are needed

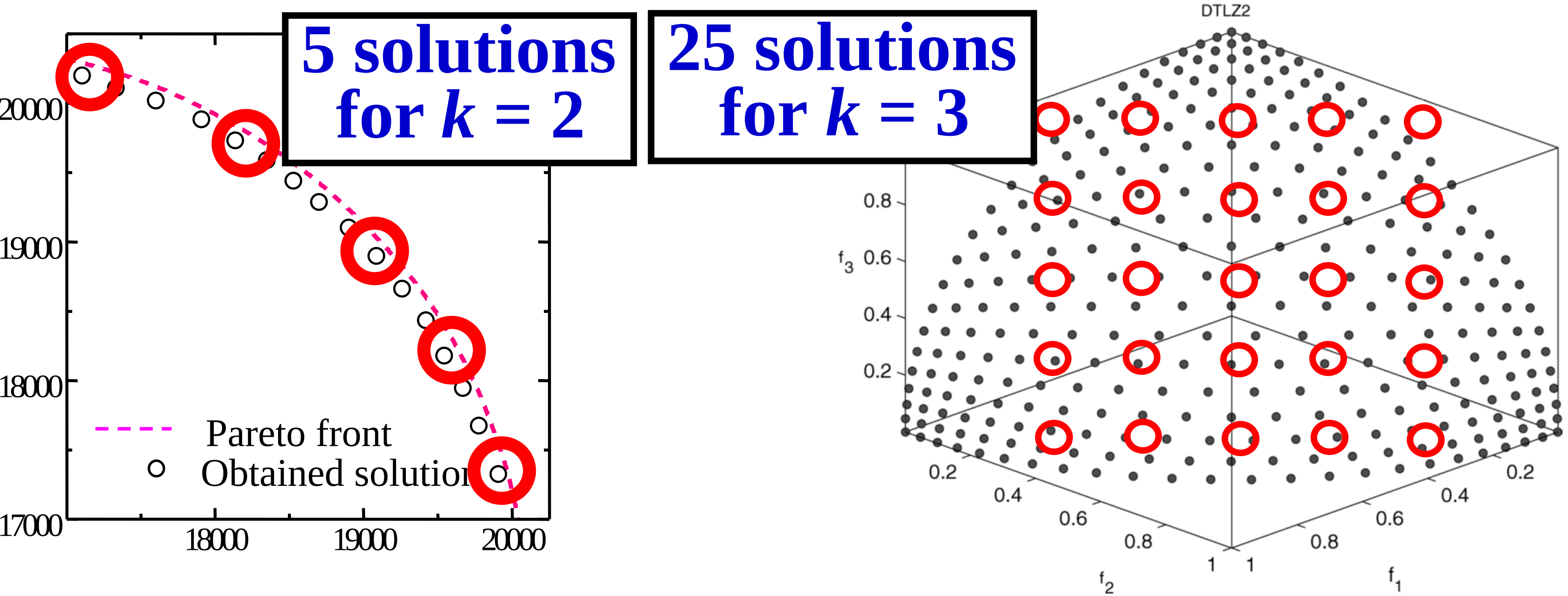
Q: How many non-dominated solutions are needed to approximate the entire Pareto-front of the *k*-objective problem? (*k* = 2, 3, 4, ...)





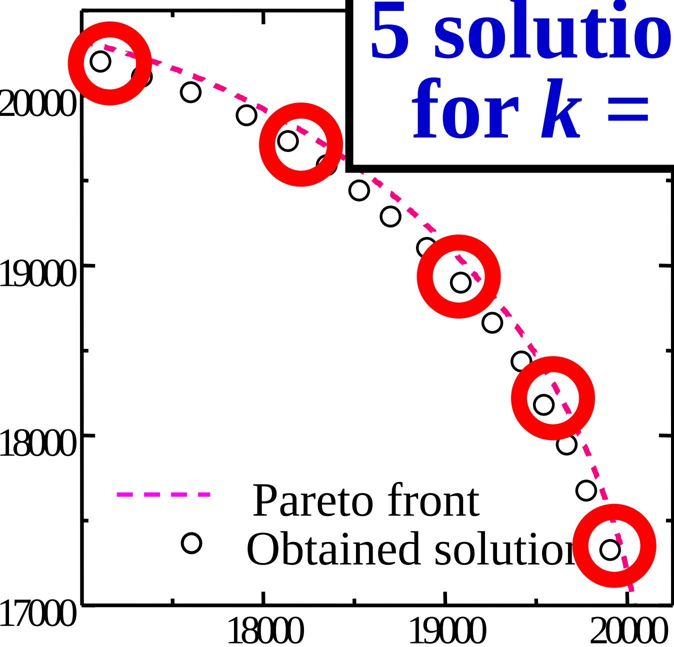
2. Approximation of the Entire Pareto Front

A huge number of solutions are needed



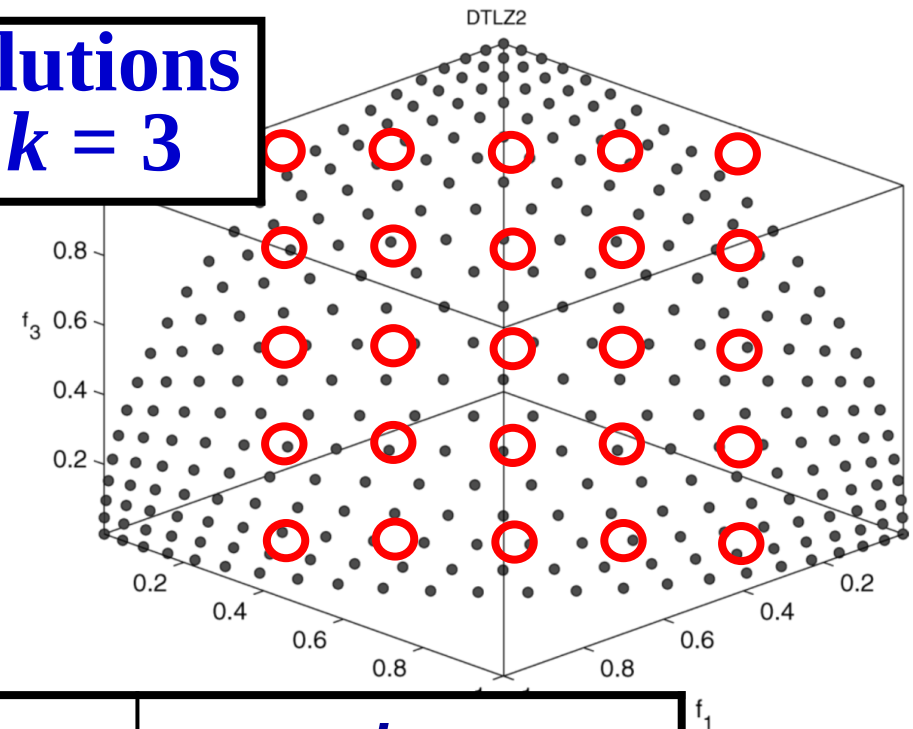
2. Approximation of the Entire Pareto Front

A huge number of solutions are needed



5 solutions for k = 2

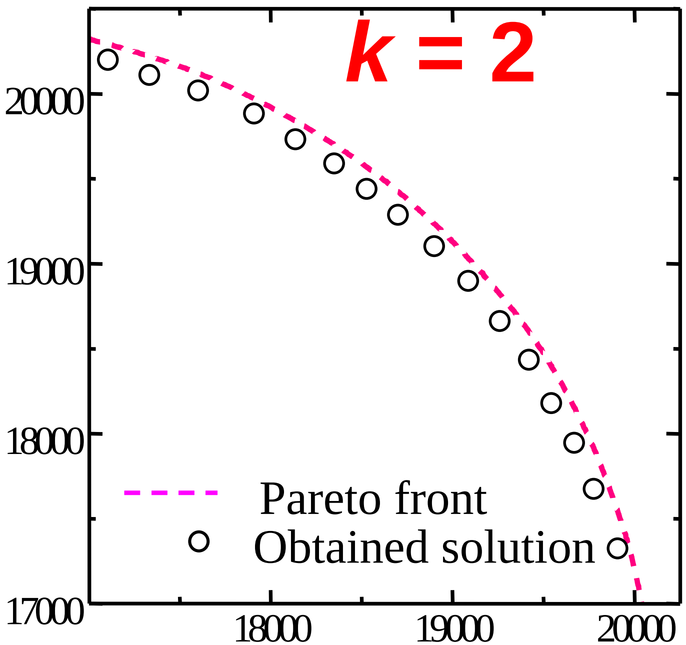
25 solutions for k = 3

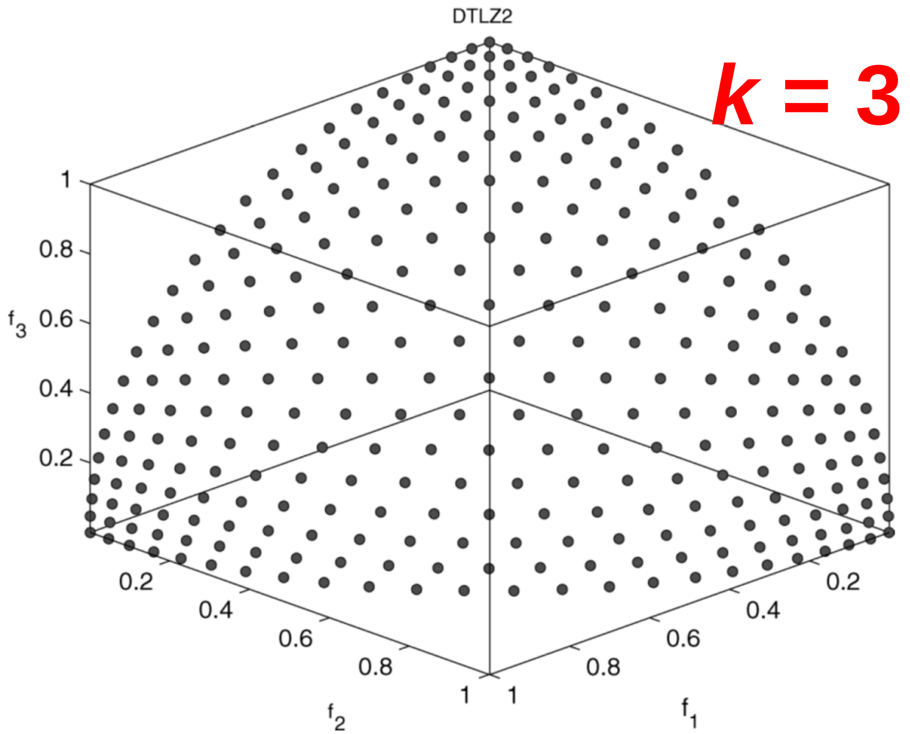


***k*-Objective Problem**	**$5^{(k-1)}$**
2-Objective Problem	**5**
3-Objective Problem	**25**
10-Objective Problem	**10 million**

3. Presentation of Obtained Solutions to DM

Visualization of high-dimensional solutions is difficult





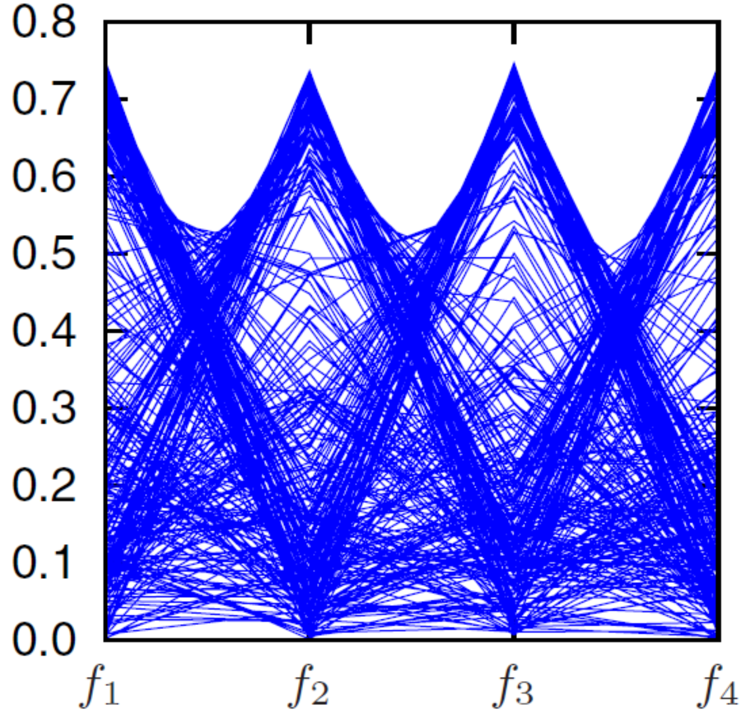
***k* = 4**

How can we show a number of four-dimensional vectors to the decision maker?

3. Presentation of Obtained Solutions to DM

Visualization of high-dimensional solutions is difficult

***k* = 4**

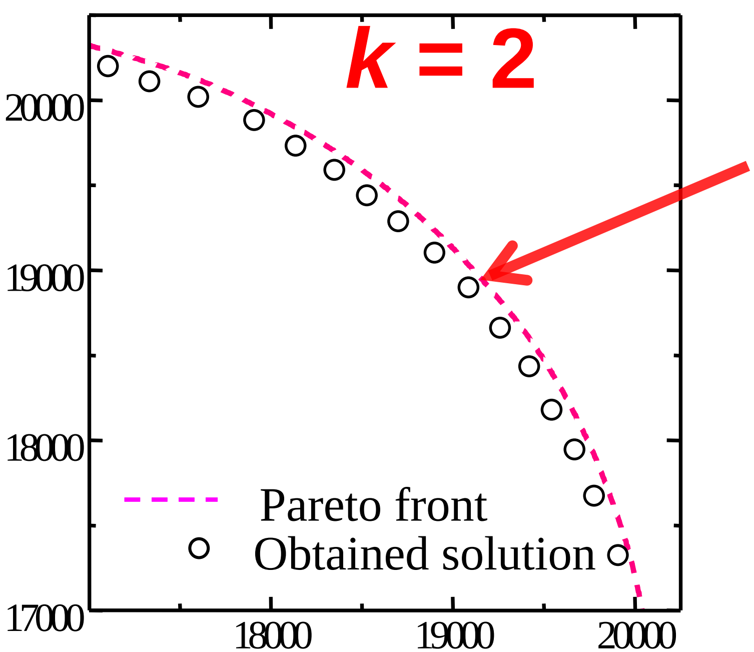


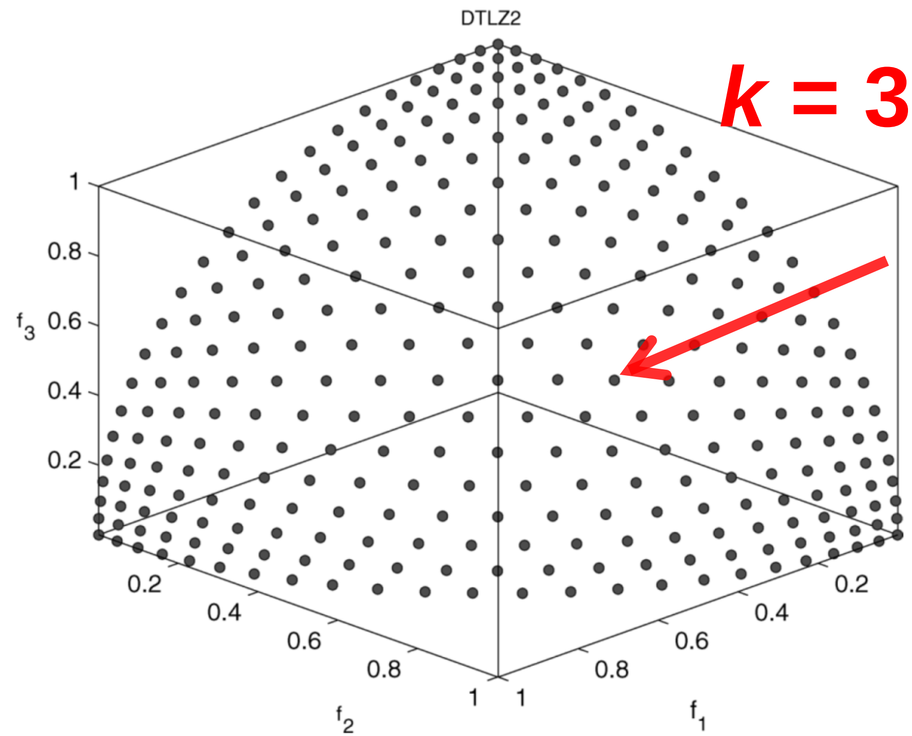
Obtained Solutions for a Four-Objective Problem

We can see that a wide variety of solutions are obtained. But, it is difficult to examine each solution.

4. Selection of a Single Final Solution

Choice of a single final solution is difficult for DM





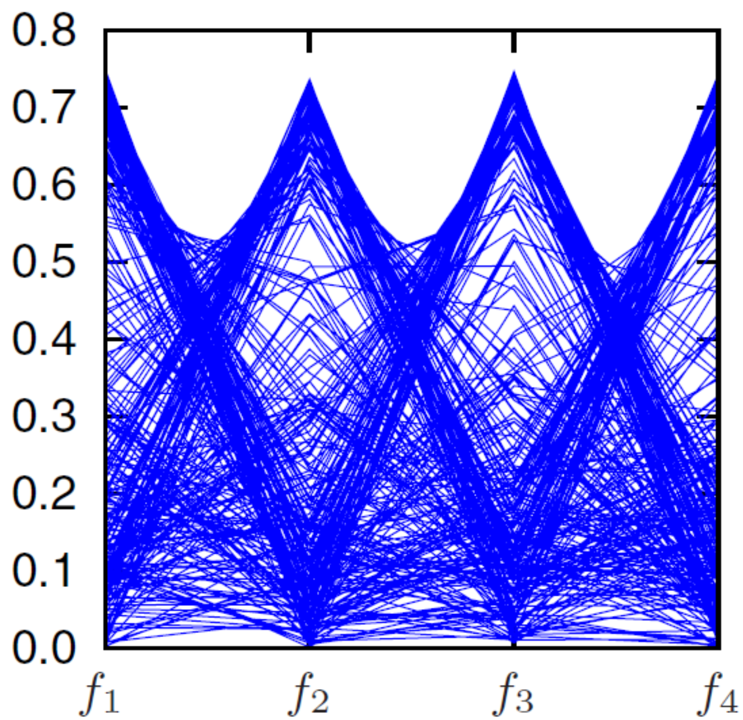
k = 4

How can we choose a single final solution from a large number of four-dimensional vectors?

4. Selection of a Single Final Solution

Choice of a single final solution is difficult for DM

***k* = 4**



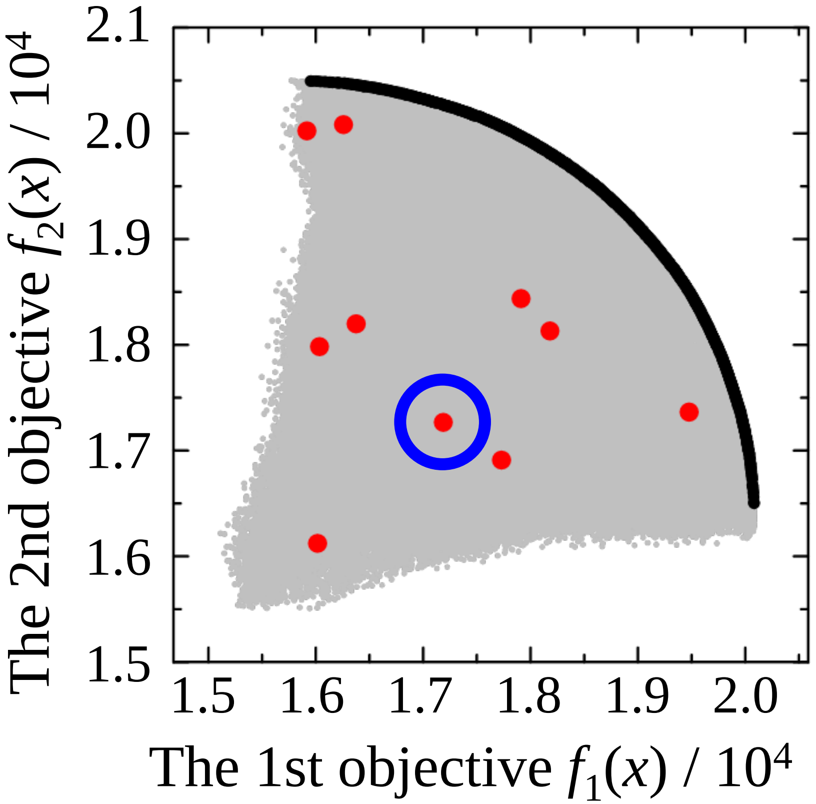
Obtained Solutions for a Four-Objective Problem

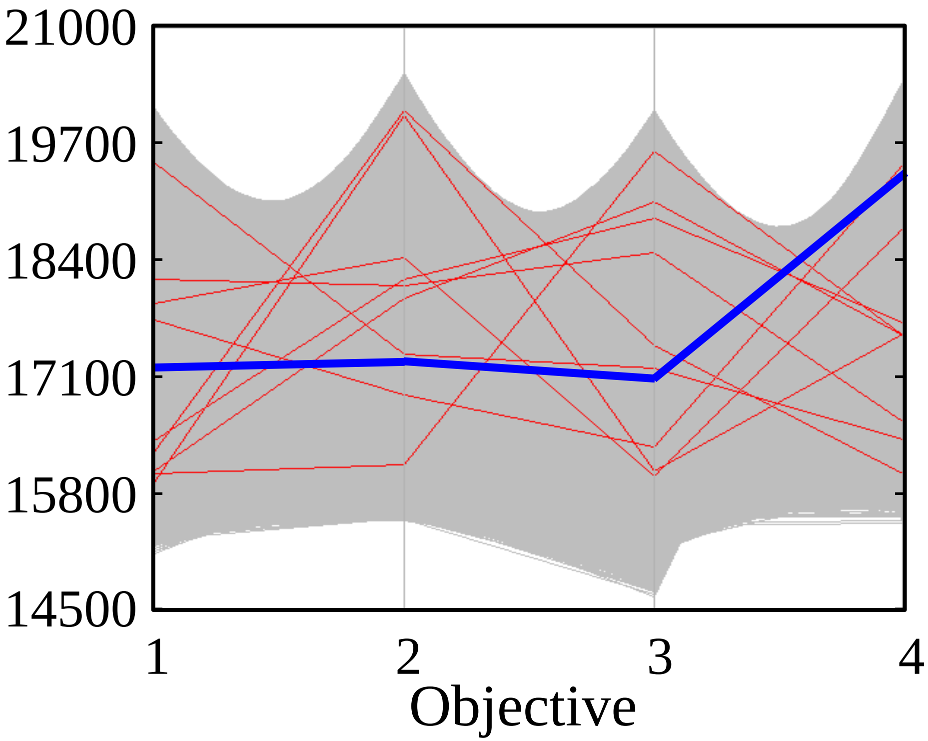
It may be very difficult for the decision maker to choose a single final solution from a large number of obtained non-dominate solutions.

4. Selection of a Single Final Solution

Choice of a single final solution is difficult for DM

Presentation of only a small number of solutions may help the decision maker. (Solution subset selection)



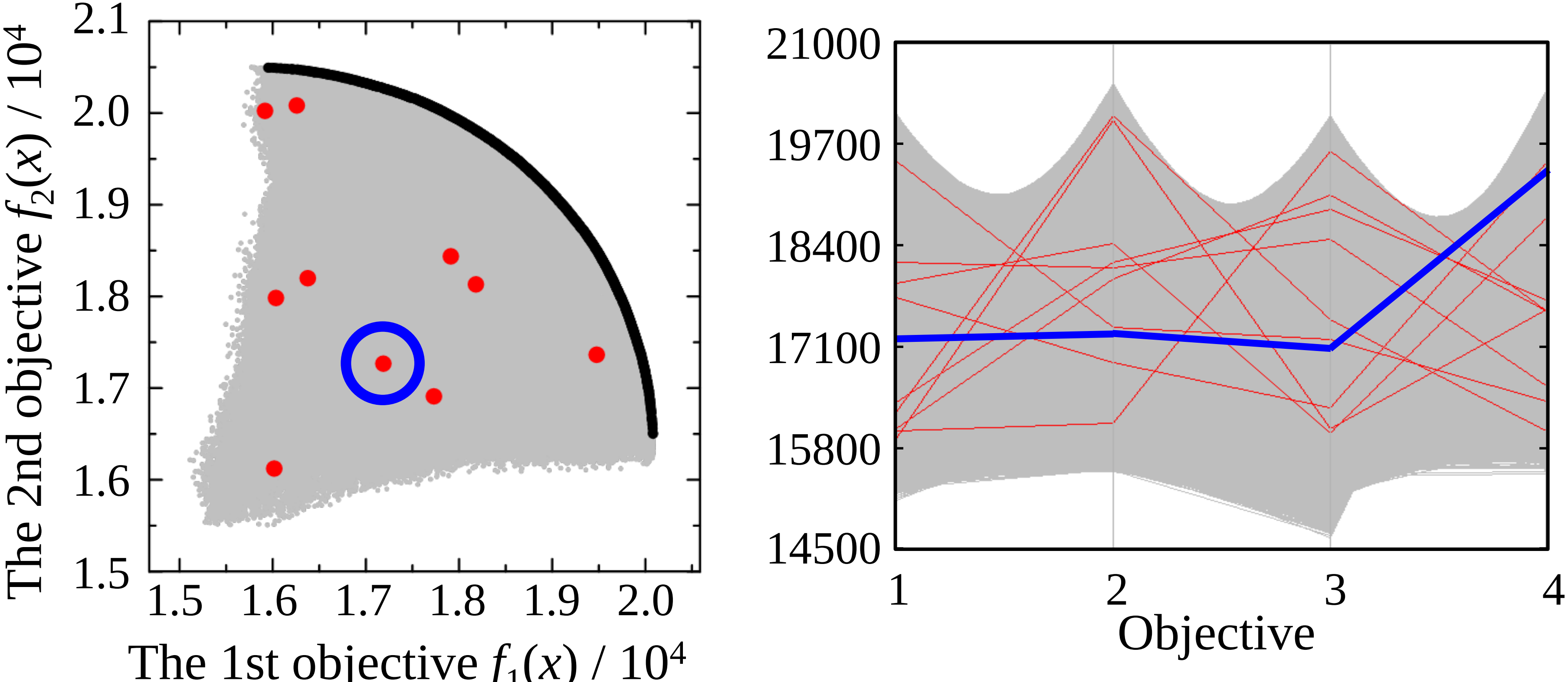


Ten solutions selected from 220,298 non-dominated solutions.

4. Selection of a Single Final Solution

Choice of a single final solution is difficult for DM

Presentation of only a small number of solutions may help the decision maker. How to select those solutions?



Ten solutions selected from 220,298 non-dominated solutions.

Many-Objective Optimization
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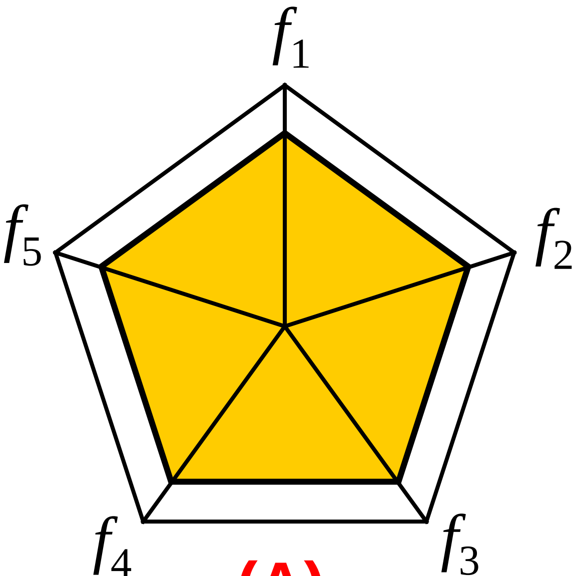
Ishibuchi et al., CEC 2008, IEEE TEVC 2015.

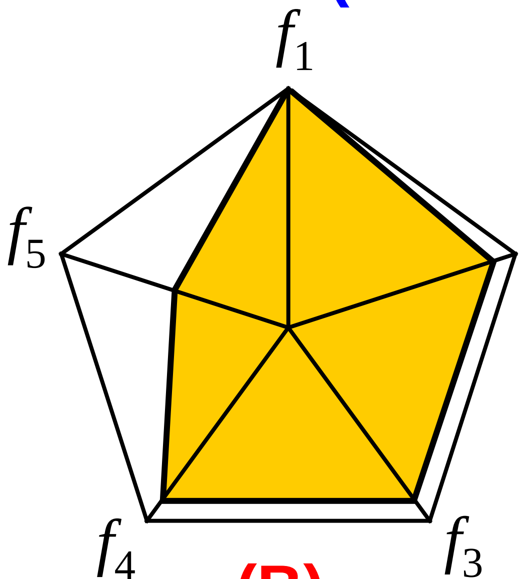
Difficulties of Many-Objective Problems

Three non-dominated solutions (Five-objective maximization)



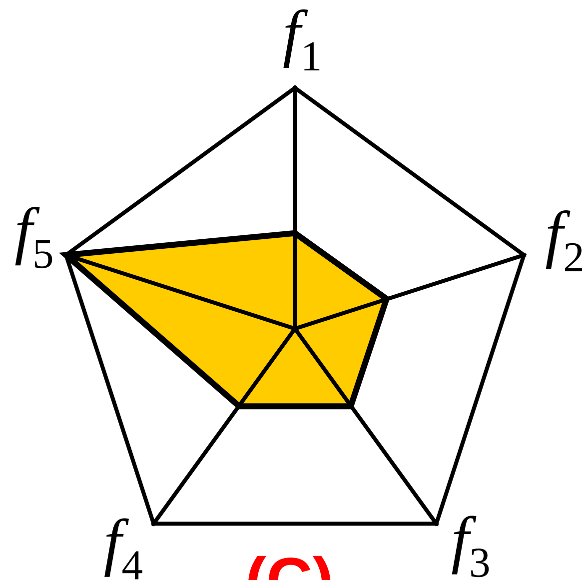
(A)

Good for all objectives.



(B)

Very good except for f_5.



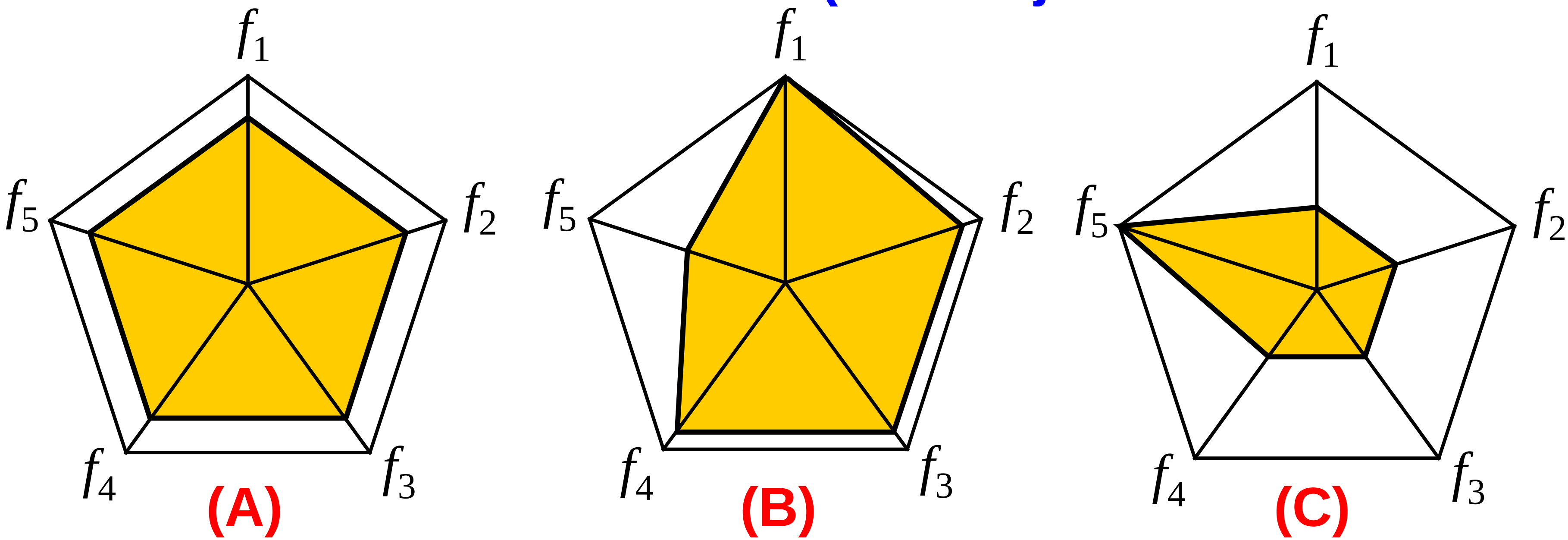
(C)

Only f_5 is good.

These three solutions are non-dominated.

Difficulties of Many-Objective Problems

Three non-dominated solutions (Five-objective maximization)



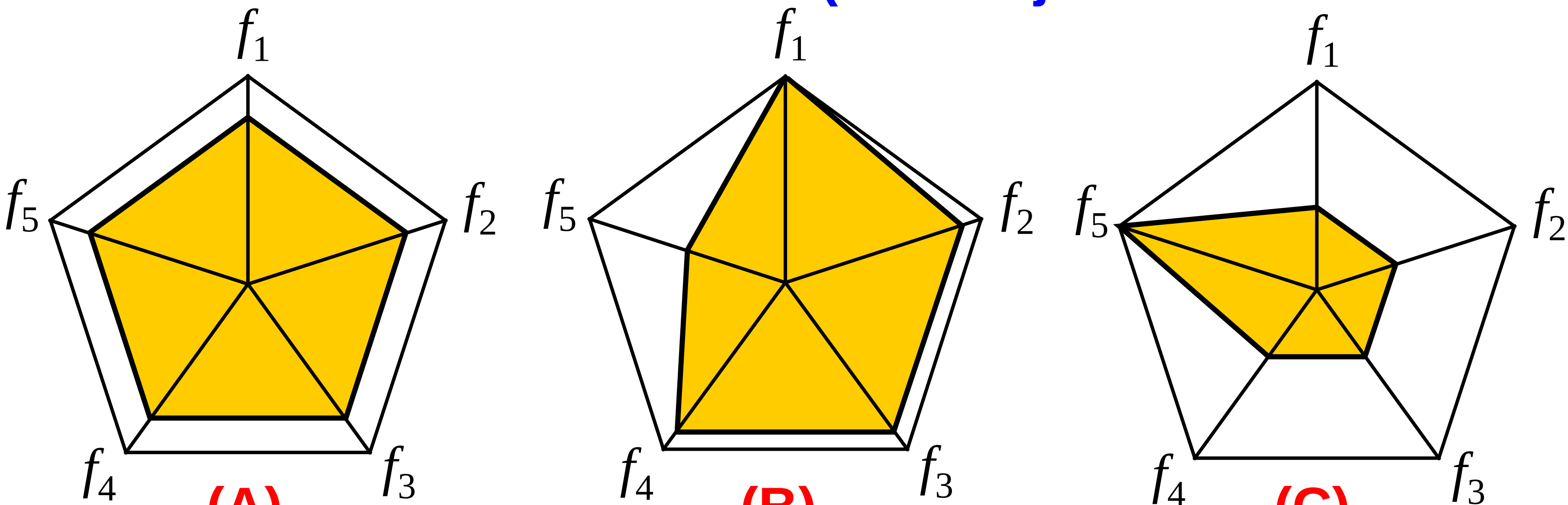
Good for all objectives. **Very good except for f_5.** **Only f_5 is good.**

These three solutions are non-dominated.

=> We need additional information about the decision maker's preference.

Difficulties for Many-Objective Problems

Three non-dominated solutions (Five-objective maximization)



(A) **(B)** **(C)**

Good for all objectives. **Very good except for f_5.** **Only f_5 is good.**

These three solutions are non-dominated.

By increasing the number of objectives, almost all solutions become non-dominated.

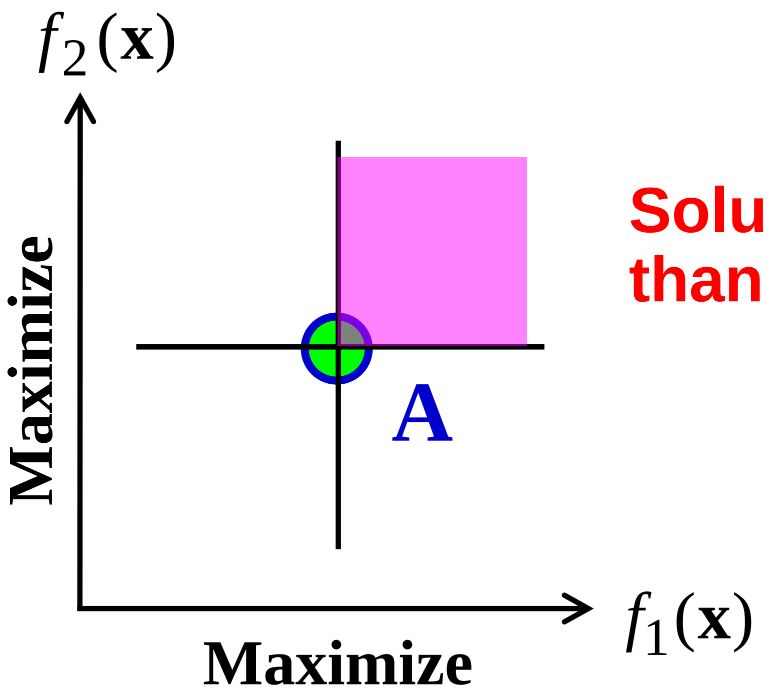
Many-Objective Optimization

Many-objective optimization is difficult.

- It is very difficult to find a better solution than the current one.

Better Solution: Two-Objective

Maximize $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}))$

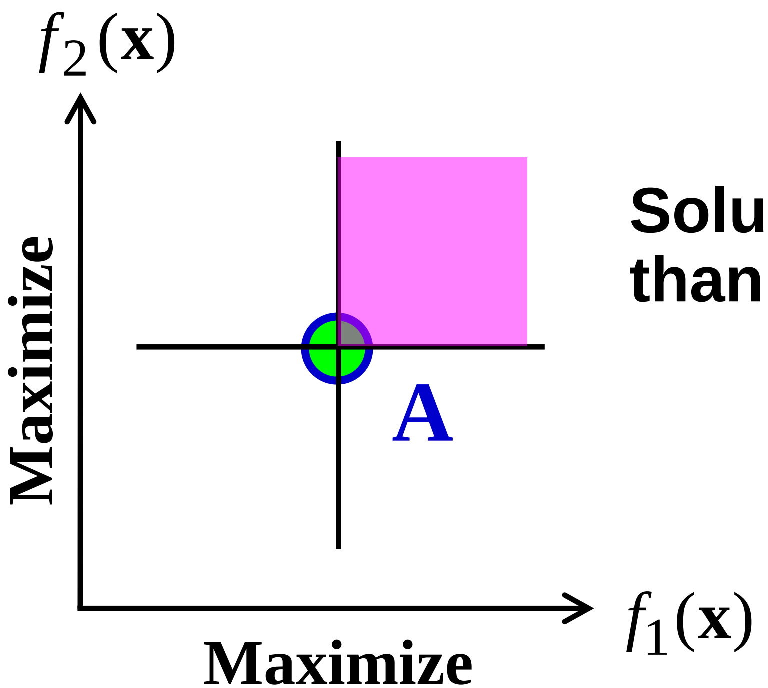


Solutions in this region are better than solution A. (1/4 of the space)

Pareto dominance-based comparison

Better Solution: Four-Objective

$$\textbf{Maximize } \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x}))$$

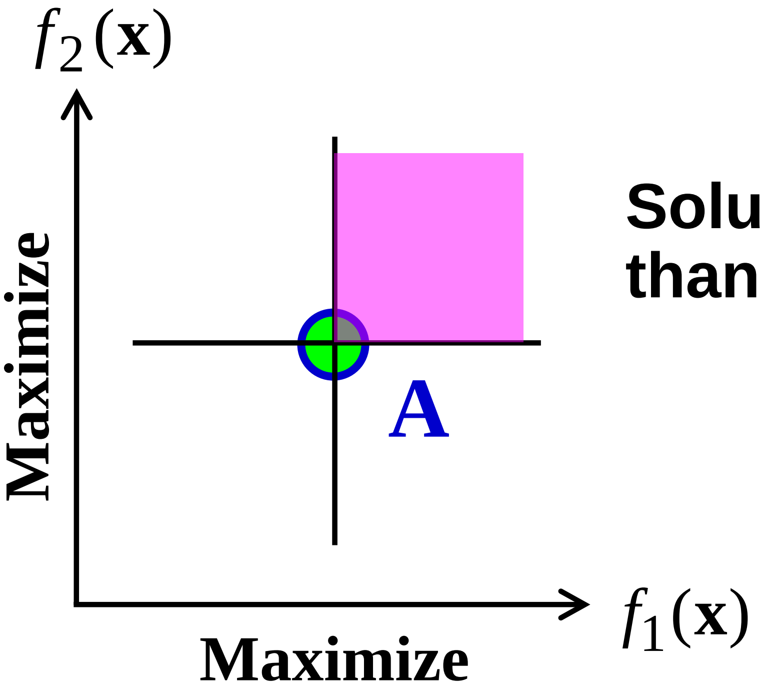


Solutions in this region are better than solution A. (1/16 of the space)

Pareto dominance-based comparison

Better Solution: M-Objective

$$\textbf{Maximize } \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ ...,\ f_M(\boldsymbol{x}))$$

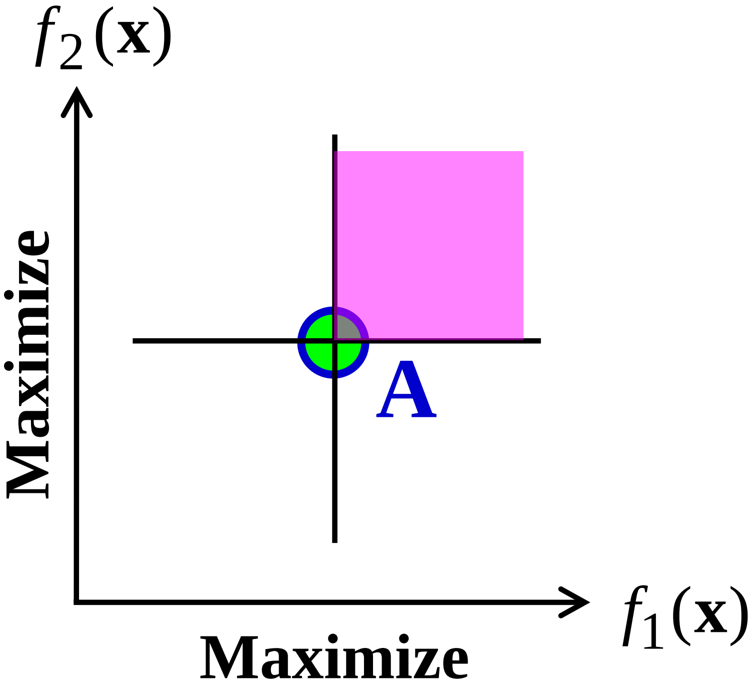


Solutions in this region are better than solution A. (1/2^M of the space)

Pareto dominance-based comparison

Better Solution by Pareto Dominance

Pareto dominance-based comparison

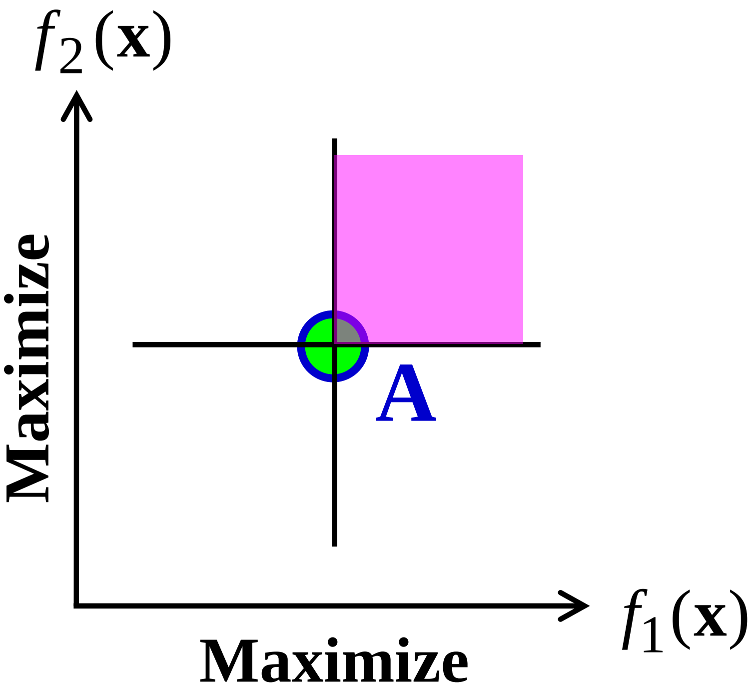


Percentage of the better region

2 objectives	1/4	25%
3 objectives	1/8	13%
4 objectives	1/16	6%
5 objectives	1/32	3%
10 objectives	1/1024	0.1%
15 objectives	1/32768	0.003%
20 objectives	1/1048576	0.0001%

Better Solution by Pareto Dominance

Pareto dominance-based comparison



Percentage of the better region

2 objectives	1/4	25%
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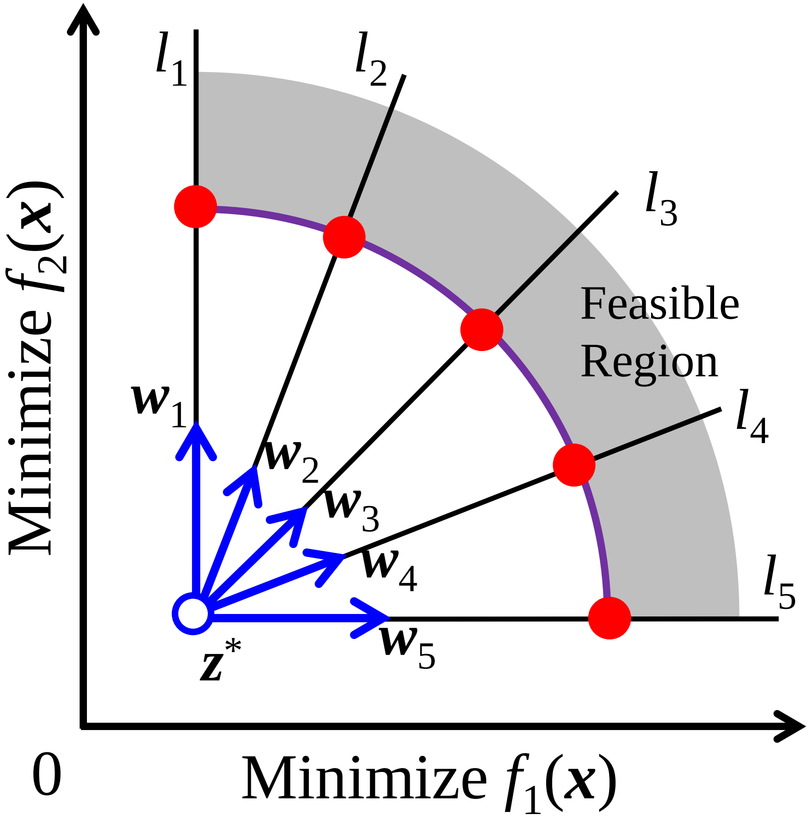
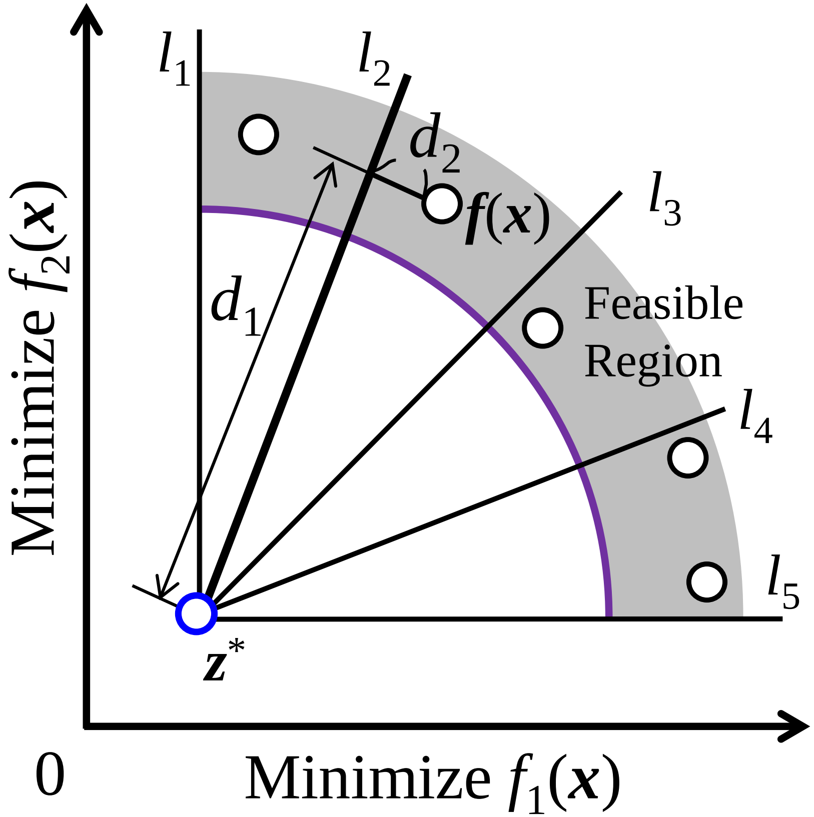
4 objectives	1/16	6%
5 objectives	1/32	3%
10 objectives	1/1024	0.1%
15 objectives	1/32768	0.003%
20 objectives	1/1048576	0.0001%

It is very difficult to find a better solution.

Use of Scalarizing Function (MOEA/D)

Recently MOEA/D has been very popular.
A scalarizing function is used in MOEA/D.





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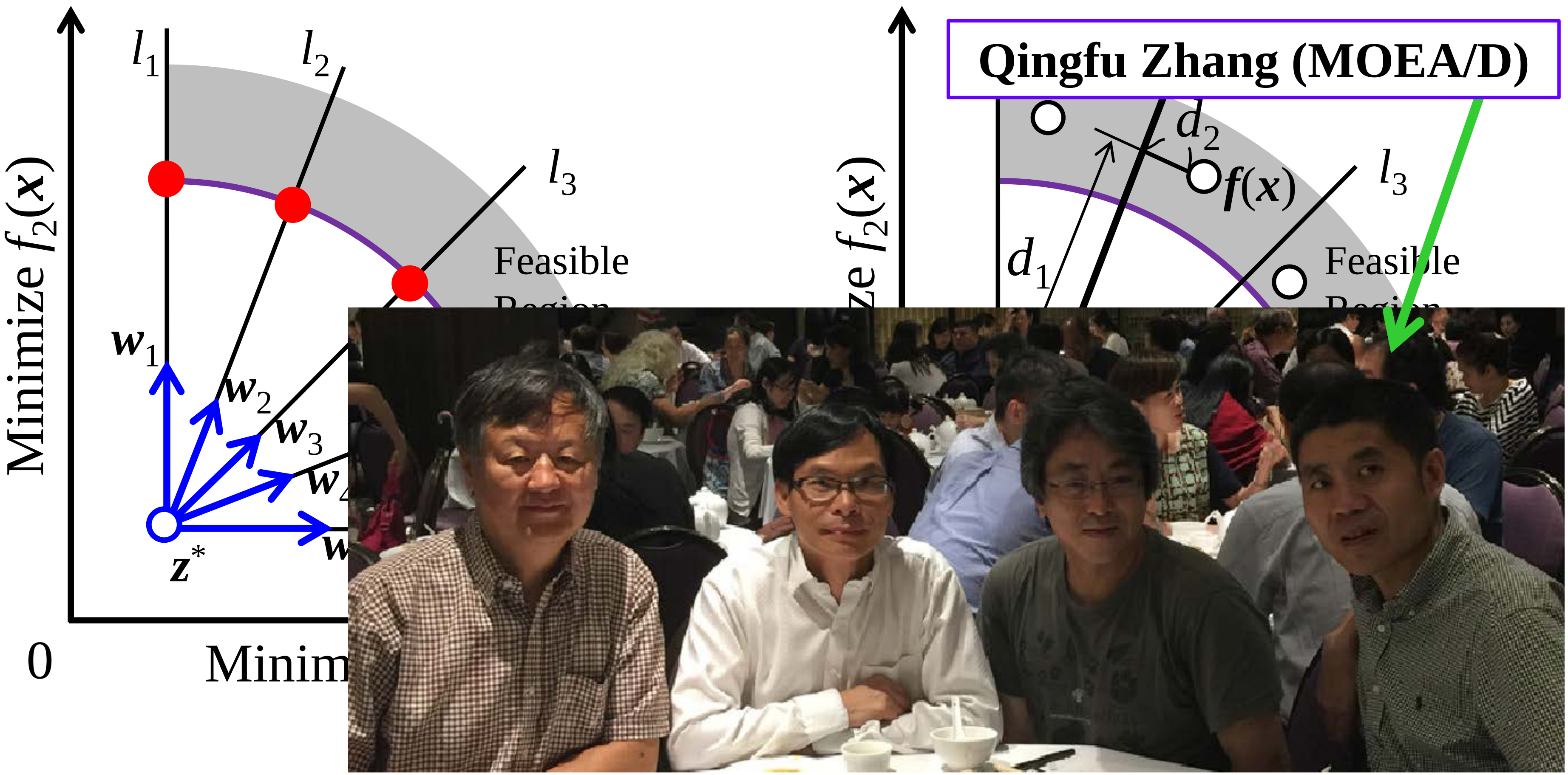
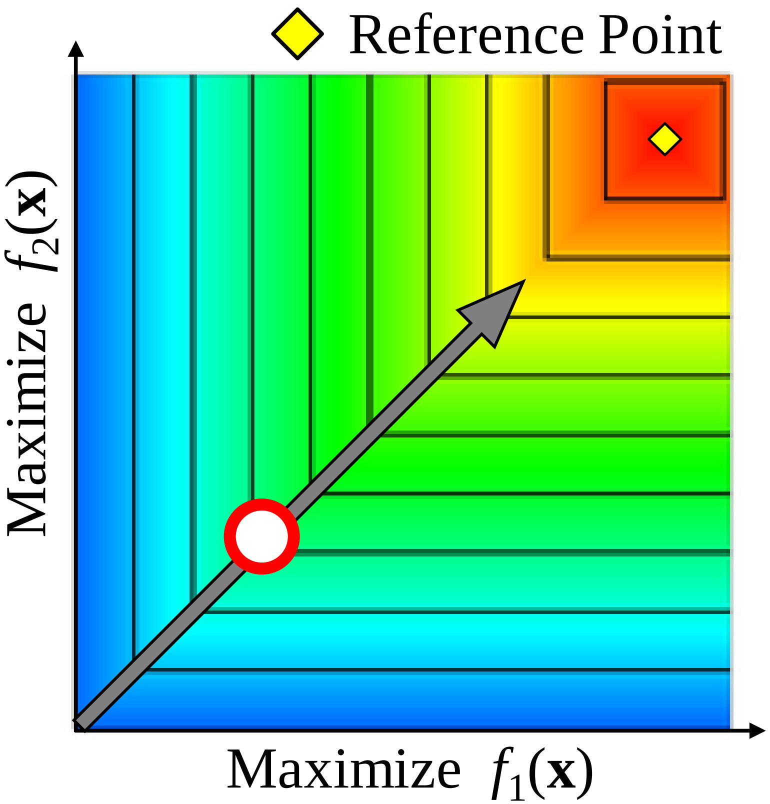


Use of Scalarizing Function

Weighted Tchebycheff

$$g^{TE}(\mathbf{x} \mid \boldsymbol{\lambda}, \mathbf{z}^*) = \max_{i=1,2,\ldots,m} \{\lambda_i \cdot | z_i^* - f_i(\mathbf{x}) |\}$$

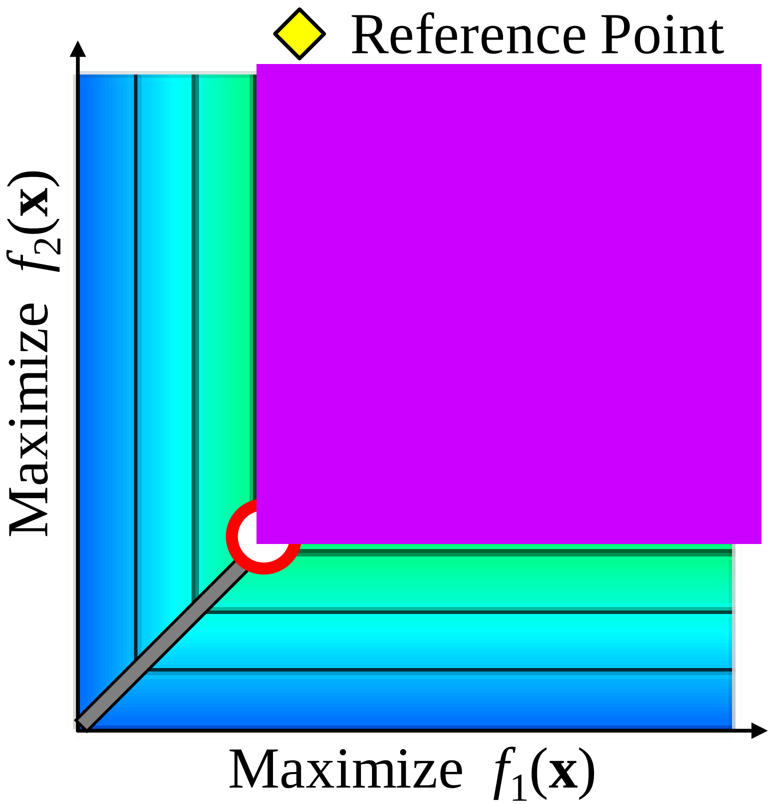


Contour lines of the Tchebycheff function

Use of Scalarizing Function

Weighted Tchebycheff

$$g^{TE}(\mathbf{x}\,|\,\boldsymbol{\lambda}, \mathbf{z}^*) = \max_{i=1,2,\ldots,m} \{\lambda_i \cdot |\, z_i^* - f_i(\mathbf{x})\,|\}$$

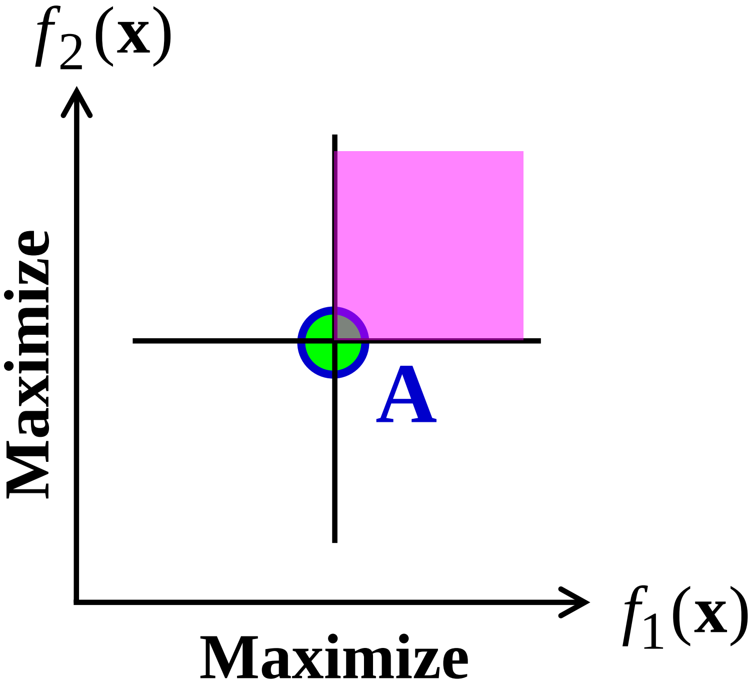


Solutions in this region is better than the current one.

Contour lines of the Tchebycheff function

Use of Scalarizing Function

Weighted Tchebycheff

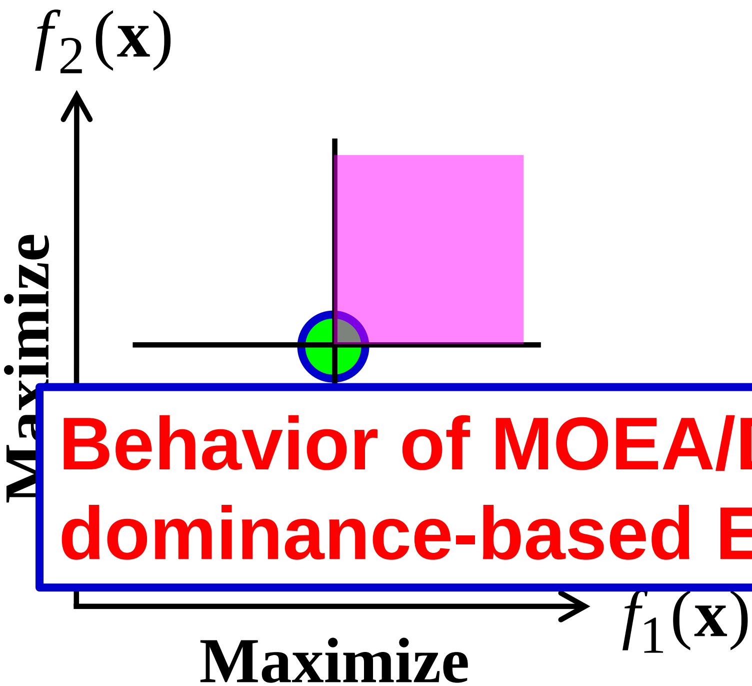


Percentage of the better region

2 objectives	1/4	25%
3 objectives	1/8	13%
4 objectives	1/16	6%
5 objectives	1/32	3%
10 objectives	1/1024	0.1%
15 objectives	1/32768	0.003%
20 objectives	1/1048576	0.0001%

Use of Scalarizing Function

Weighted Tchebycheff



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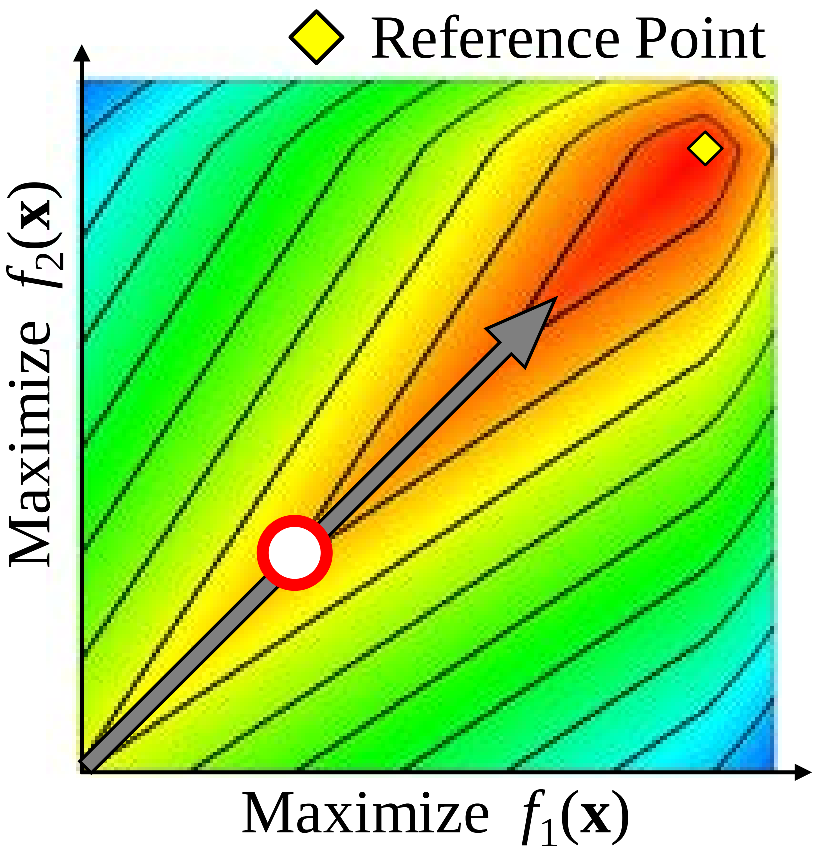
4 objectives	1/16	6%
5 objectives	1/32	3%
15 objectives	1/32768	0.003%
20 objectives	1/1048576	0.0001%

Behavior of MOEA/D-Tch may be similar to Pareto dominance-based EMO algorithms (e.g., NSGA-II).

Use of Scalarizing Function

PBI Function ($\theta = 5$)

$$g^{PBI}(\mathbf{x} \mid \boldsymbol{\lambda}, \mathbf{z}^*) = d_1 + \theta d_2$$



Contour lines of the PBI function

Use of Scalarizing Function

PBI Function ($\theta = 5$)

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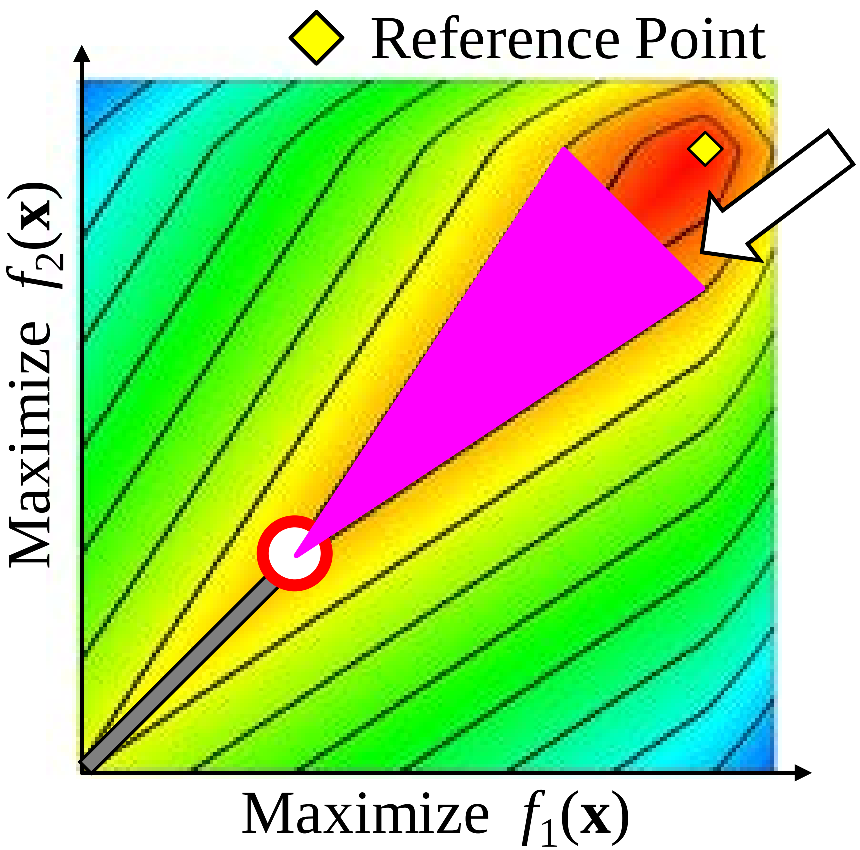


Solutions in this region is better than the current one.

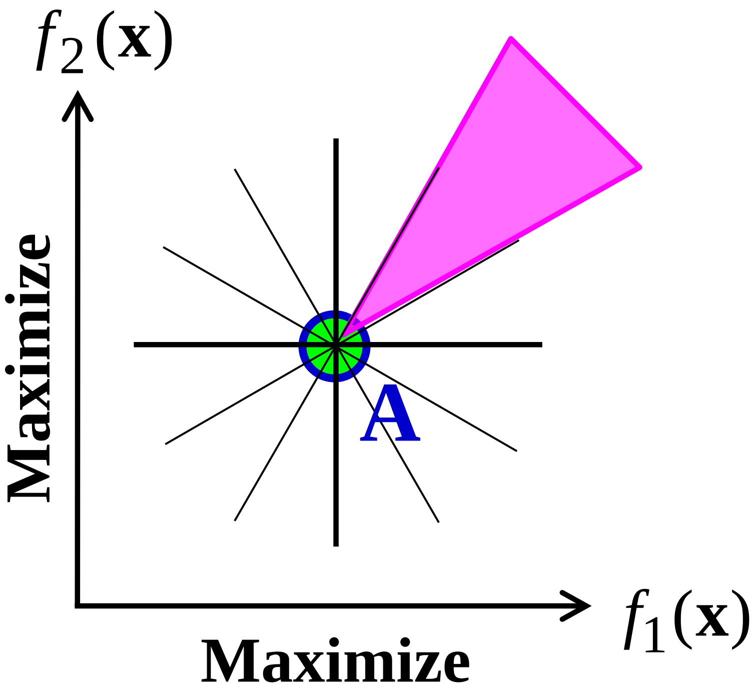
Contour lines of the PBI function

Use of Scalarizing Function

PBI Function ($\theta = 5$)

Very Rough Calculation

Percentage of the better region



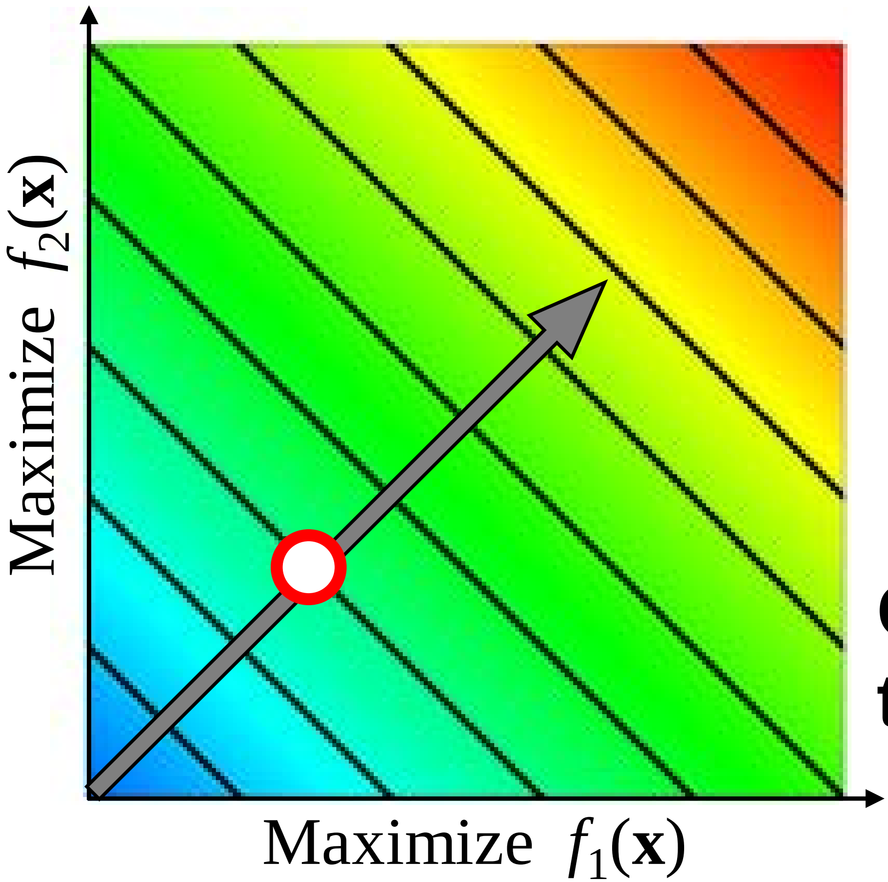
2 objectives	1/12	8%
3 objectives	1/36	3%
4 objectives	1/108	1%
5 objectives	1/324	0.3%
10 objectives	1/78732	0.001%
15 objectives		
20 objectives		

Much smaller than the case of the Pareto dominance.

Use of Scalarizing Function

Weighted Sum

$$g^{WS}(\mathbf{x} \mid \boldsymbol{\lambda}) = \lambda_1 \cdot f_1(\mathbf{x}) + \lambda_2 \cdot f_2(\mathbf{x}) + \ \cdots \ + \lambda_m \cdot f_m(\mathbf{x})$$

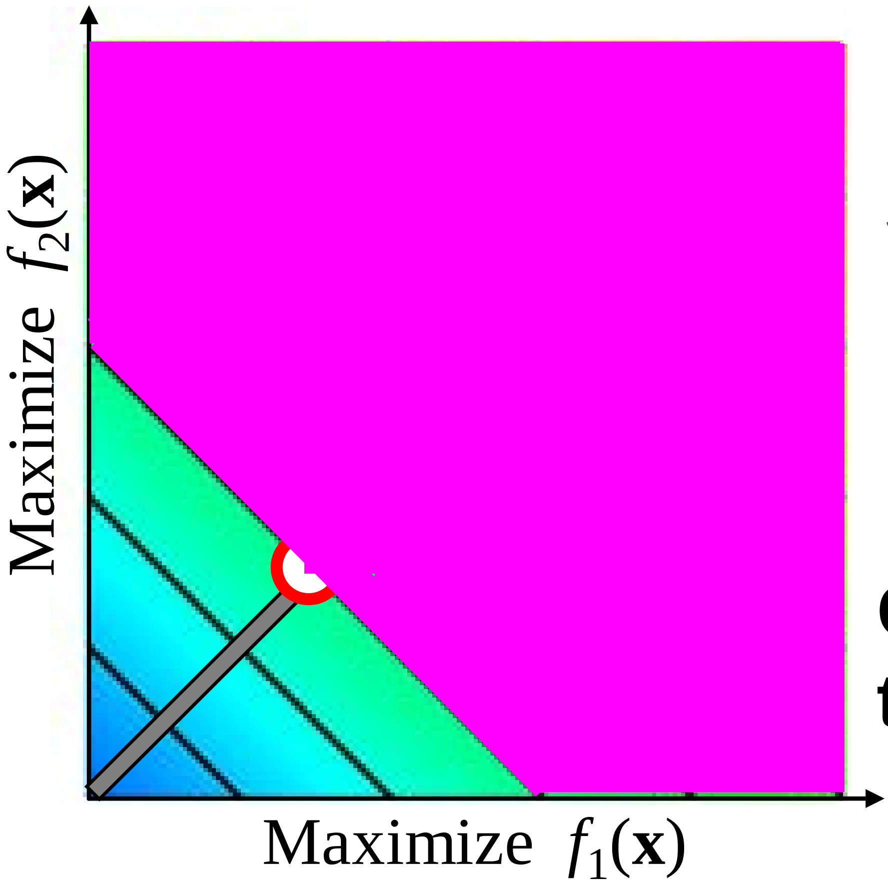


Contour lines of the Weighted sum function

Use of Scalarizing Function

Weighted Sum

$$g^{WS}(\boldsymbol{x}\,|\,\boldsymbol{\lambda}) = \lambda_1 \cdot f_1(\boldsymbol{x}) + \lambda_2 \cdot f_2(\boldsymbol{x}) + \ \cdots \ + \lambda_m \cdot f_m(\boldsymbol{x})$$

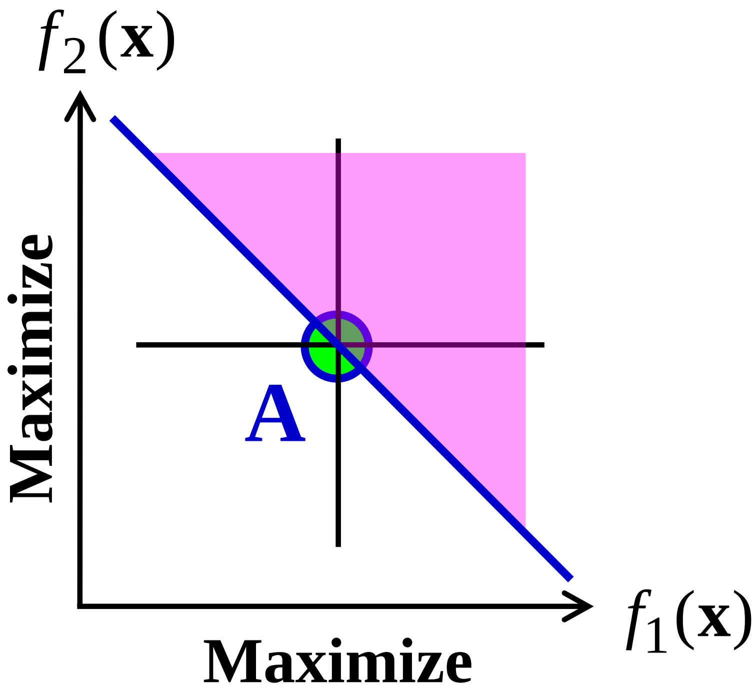


Solutions in this region is better than the current one.

Contour lines of the Weighted sum function

Use of Scalarizing Function

Weighted Sum

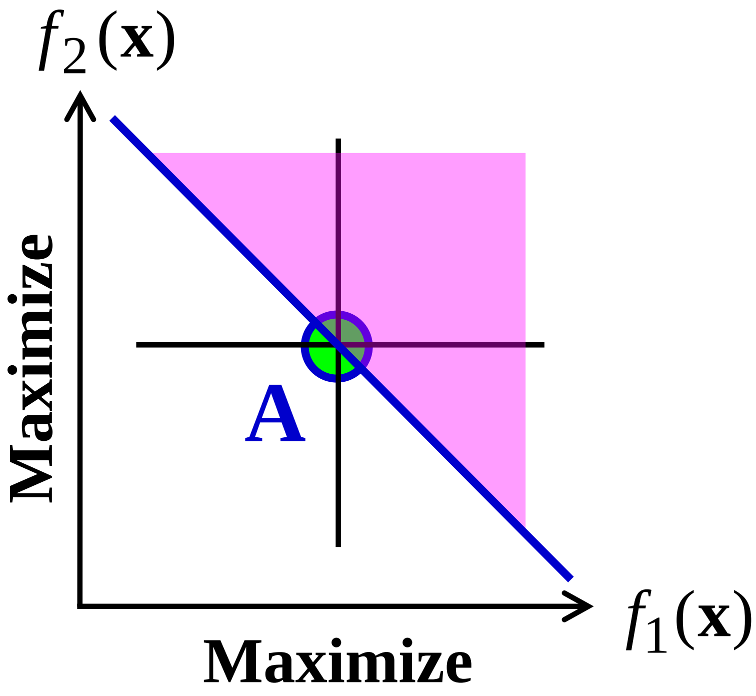


Percentage of the better region

2 objectives	1/2	50%
3 objectives	1/2	50%
4 objectives	1/2	50%
5 objectives	1/2	50%
10 objectives	1/2	50%
15 objectives	1/2	50%
20 objectives	1/2	50%

Use of Scalarizing Function

Weighted Sum



Percentage of the better region

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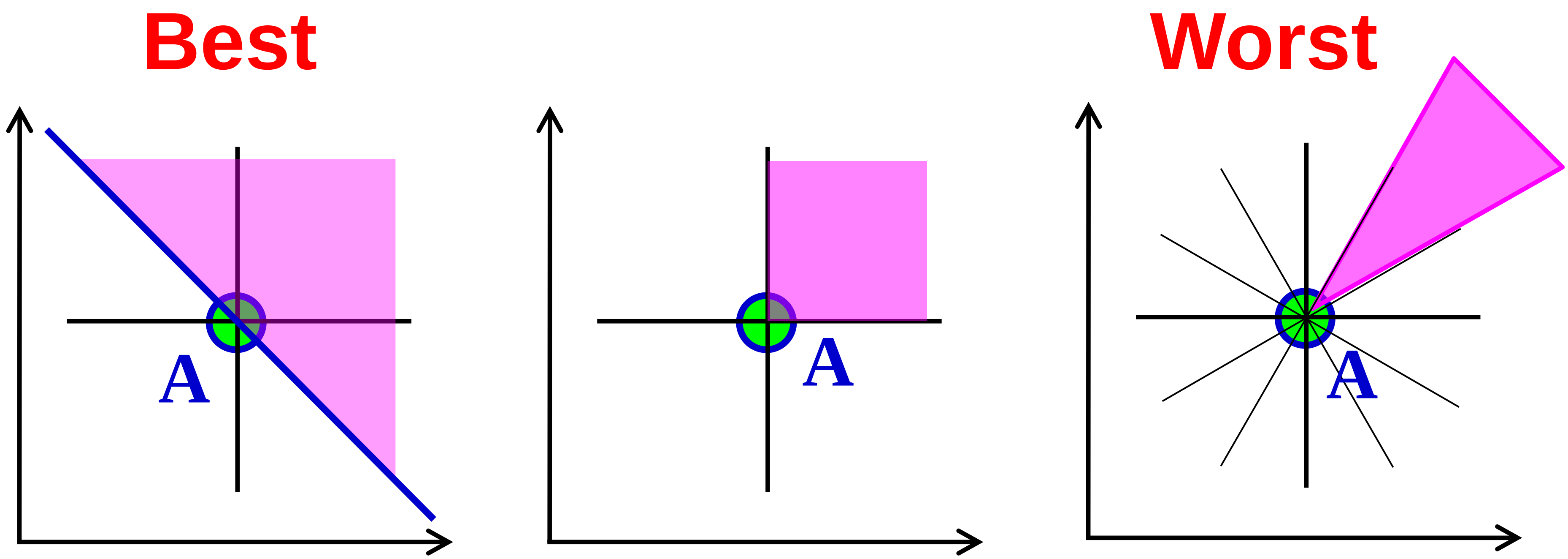
3 objectives	1/2	50%
4 objectives	1/2	50%
5 objectives	1/2	50%
10 objectives	1/2	50%
15 objectives	1/2	50%
20 objectives	1/2	50%

Always a half of the objective space is better.

Expected Performance of EMO Algorithms on Many-Objective Problems



Weighted Sum (MOEA/D-WS)	Pareto Dominance (NSGA-II) Tchebycheff (MOEA/D-Tch)	PBI Function (MOEA/D-PBI) ($\theta = 5$)

Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

Test Problems:

500-item knapsack problems with 2-10 objectives

Algorithms:

NSGA-II

MOEA/D with WS (Weighted Sum)

MOEA/D with Tchebycheff

MOEA/D with PBI ($\theta = 5$)

Performance Indicator:

Hypervolume

Expected difficulties are observed.

Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

Average Hyper-Volume Value

(Normalized by the Result of the MOEA/D-WS)

EMO Algorithm	2-Obj	4-Obj	6-Obj	8-Obj	10-Obj
MOEA/D: WS	100.0	100.0	100.0	100.0	100.0
MOEA/D: Tchebycheff	100.7	99.7	94.0	90.1	87.7
NSGA-II	96.5	86.2	77.8	72.0	65.5
MOEA/D: PBI (5)	100.9	89.3	73.8	67.4	61.9

Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

For 2-objective problems, MOEA/D-PBI is the best.
No large differences among the four algorithms

EMO Algorithm	2-Obj	4-Obj	6-Obj	8-Obj	10-Obj
MOEA/D: WS	100.0	100.0	100.0	100.0	100.0
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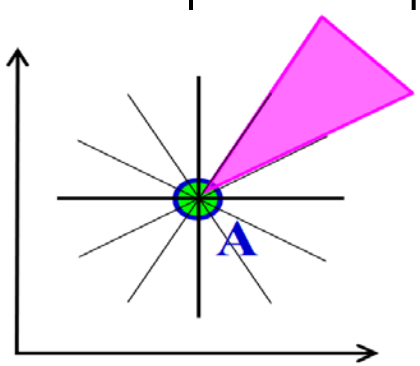

Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

For 6-10 objectives, MOEA/D-PBI is the worst.
Large differences among the algorithms.

EMO Algorithm	2-Obj	4-Obj	6-Obj	8-Obj	10-Obj
MOEA/D: WS	100.0	100.0	100.0	100.0	100.0
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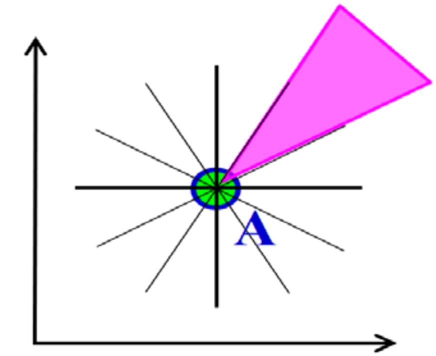

EMO Algorithm			6-Obj	8-Obj	10-Obj
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Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

For 6-10 objectives, MOEA/D-Tchebycheff and NSGA-II did not work well.

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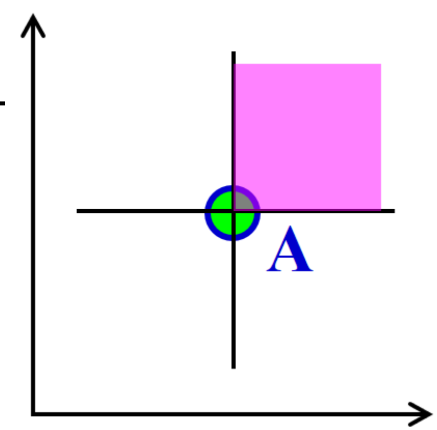

Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)

For 6-10 objectives, MOEA/D-WS is the best.

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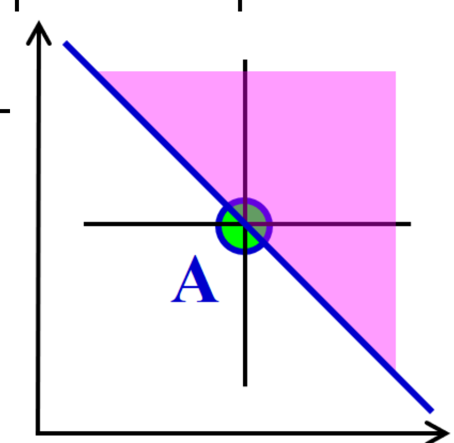
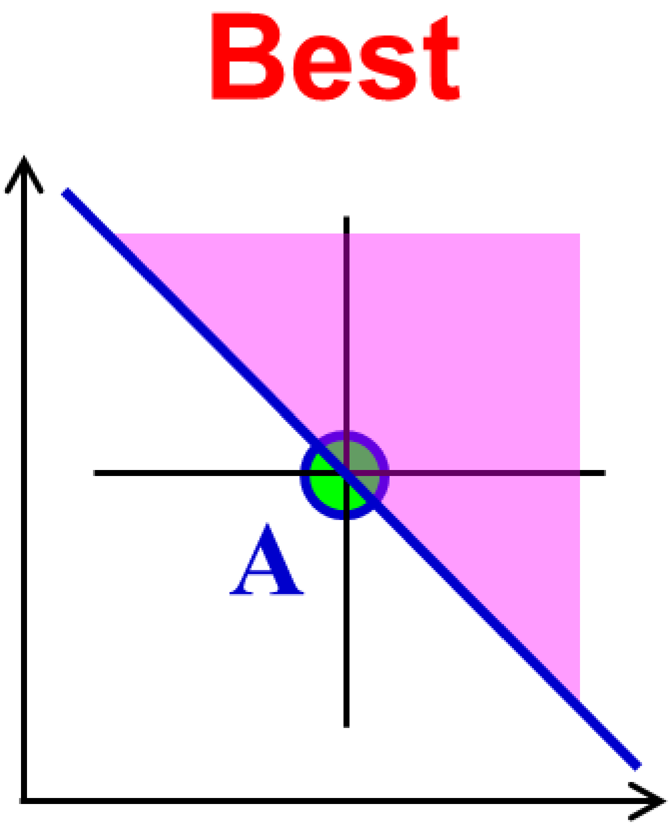


Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)



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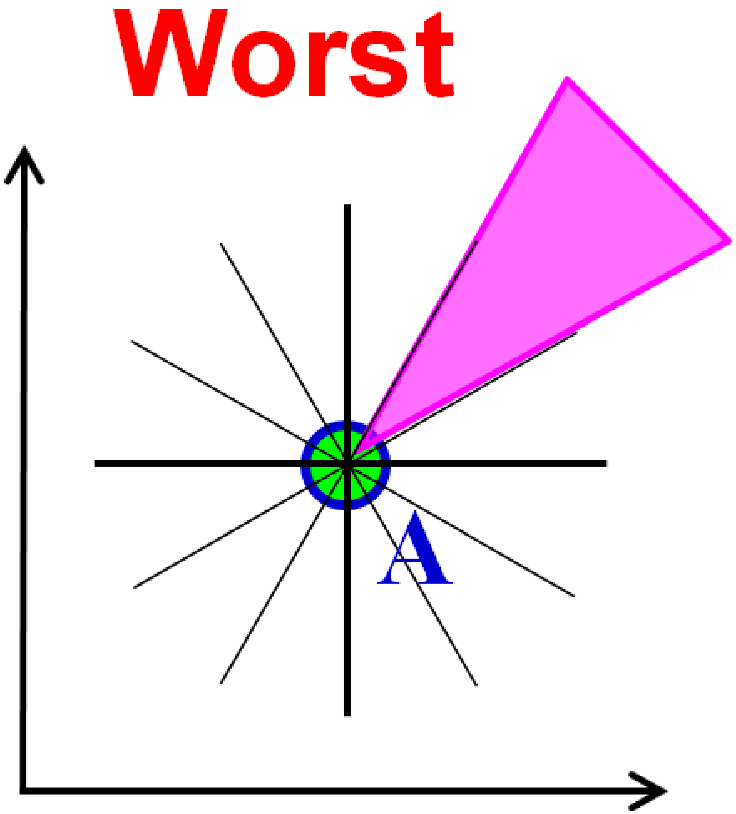
Our Results on Knapsack Problems

Ishibuchi et al. IEEE TECV (2015)



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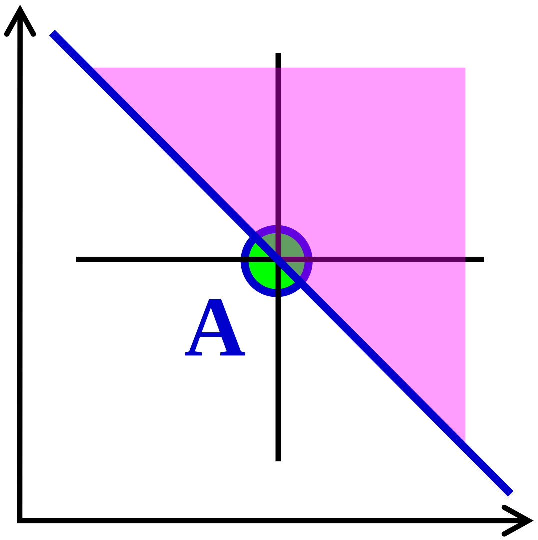
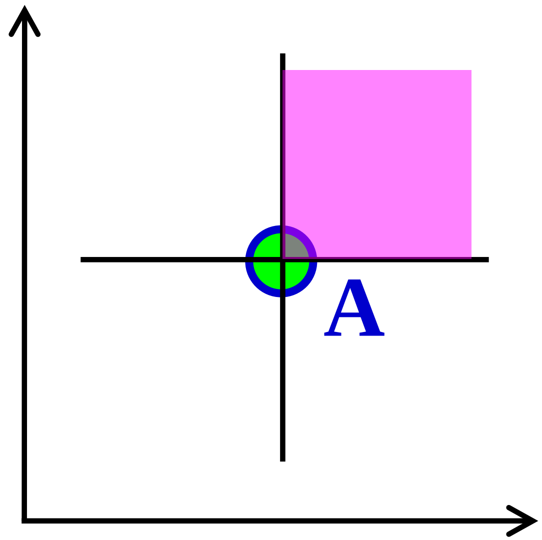
100.9	89.3	73.8	67.4	61.9

Expected Performance of EMO Algorithms on Many-Objective Problems

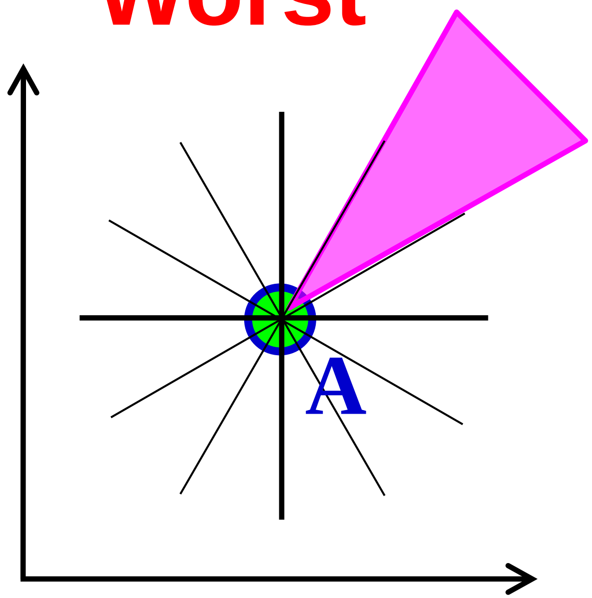
Best **Worst**



Weighted Sum (MOEA/D-WS)



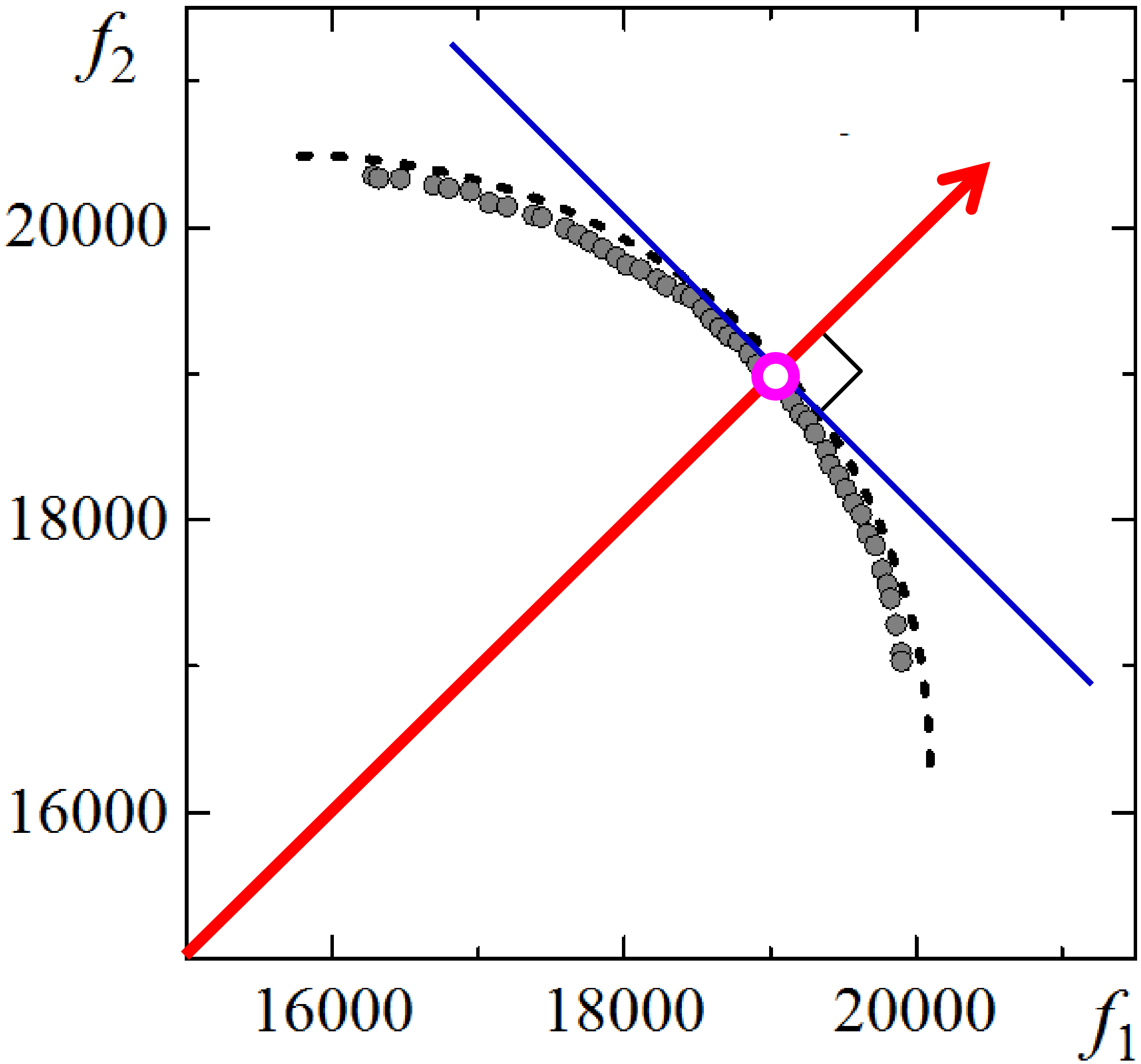
Tchebycheff (MOEA/D-Tch)



PBI Function (MOEA/D-PBI) (θ = 5)

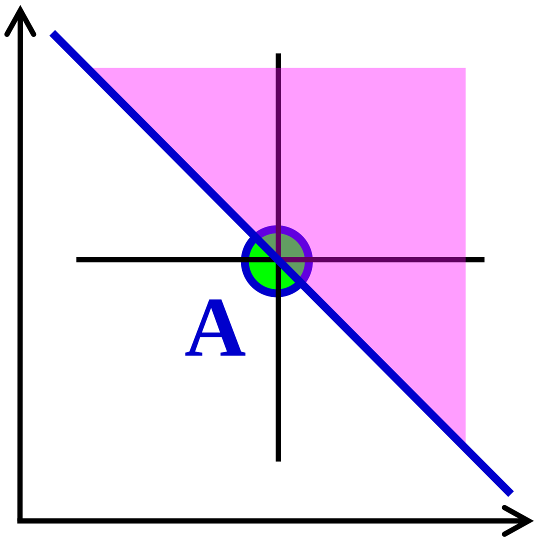
Multi-Objective Knapsack Problems

WS works well for the convex Pareto front



Expected Performance of EMO Algorithms on Many-Objective Problems

Best



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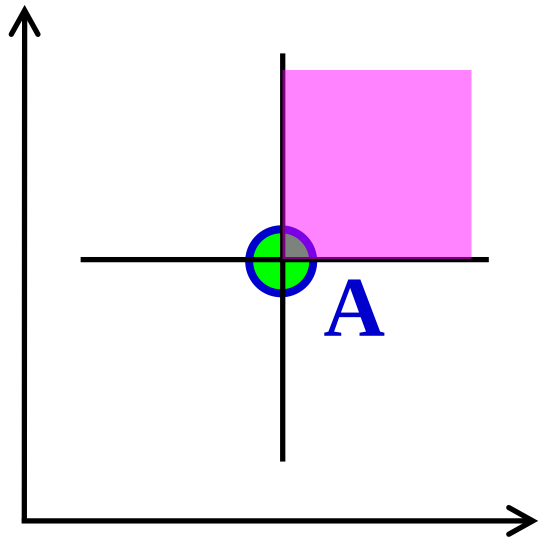
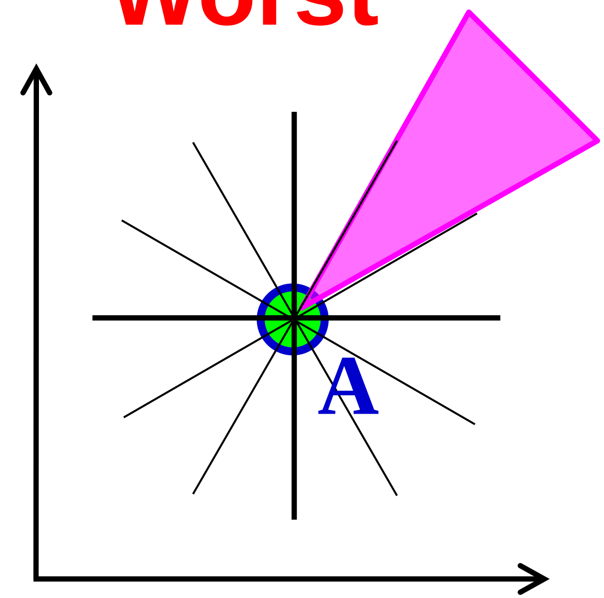


Tchebycheff (MOEA/D-Tch)

Worst



PBI Function (MOEA/D-PBI) (θ = 5)

Our Results on DTLZ Test Problems

Ishibuchi et al. IEEE TECV (2017)

Test Problems:

DTLZ1 - DTLZ4 Problems with 5-10 objectives

Algorithms:

NSGA-II
MOEA/D with WS (Weighted Sum)
MOEA/D with Tchebycheff
MOEA/D with PBI ($\theta = 5$)
NSGA-III
MOEA/DD

Performance Indicator:

Hypervolume

Totally different results are obtained.

Our Results on DTLZ Test Problems

Ishibuchi et al. IEEE TECV (2017)

Average Hyper-Volume Value

Problem	*M*	NSGA-III	MOEA/DD	PBI	Tch	WS	NSGA-II
DTLZ 1	5	1.57677	1.57794	1.57768	1.51186	0.50052	0.00000
	8	2.13770	2.13730	2.13620	2.05463	0.96246	0.00000
	10	2.59280	2.59260	2.59220	2.51973	1.07913	0.00000
DTLZ 2	5	1.30317	1.30778	1.30728	1.14598	0.61944	0.67442
	8	1.96916	1.97862	1.97817	1.35469	0.68315	0.00004
	10	2.50878	2.51509	2.51500	1.69045	0.83883	0.00000
DTLZ 3	5	1.29894	1.30638	1.30398	1.14475	0.60143	0.00000
	8	1.95007	1.97162	1.74240	1.33166	0.66684	0.00000
	10	2.50727	2.51445	2.50933	1.69956	0.80348	0.00000
DTLZ 4	5	1.30839	1.30876	1.20680	1.00426	0.42941	1.00881
	8	1.98025	1.98083	1.86439	1.35100	0.71296	0.00000
	10	2.51524	2.51532	2.43536	1.56890	0.95488	0.00000

Tchebycheff is better than WS

Average Hyper-Volume Value

Problem	M	NSGA-III	MOEA/DD	PBI	Tch	WS	NSGA-II
DTLZ 1	5	1.57677	1.57794	1.57768	1.51186	0.50052	0.00000
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PBI is better than Tchebycheff

Average Hyper-Volume Value

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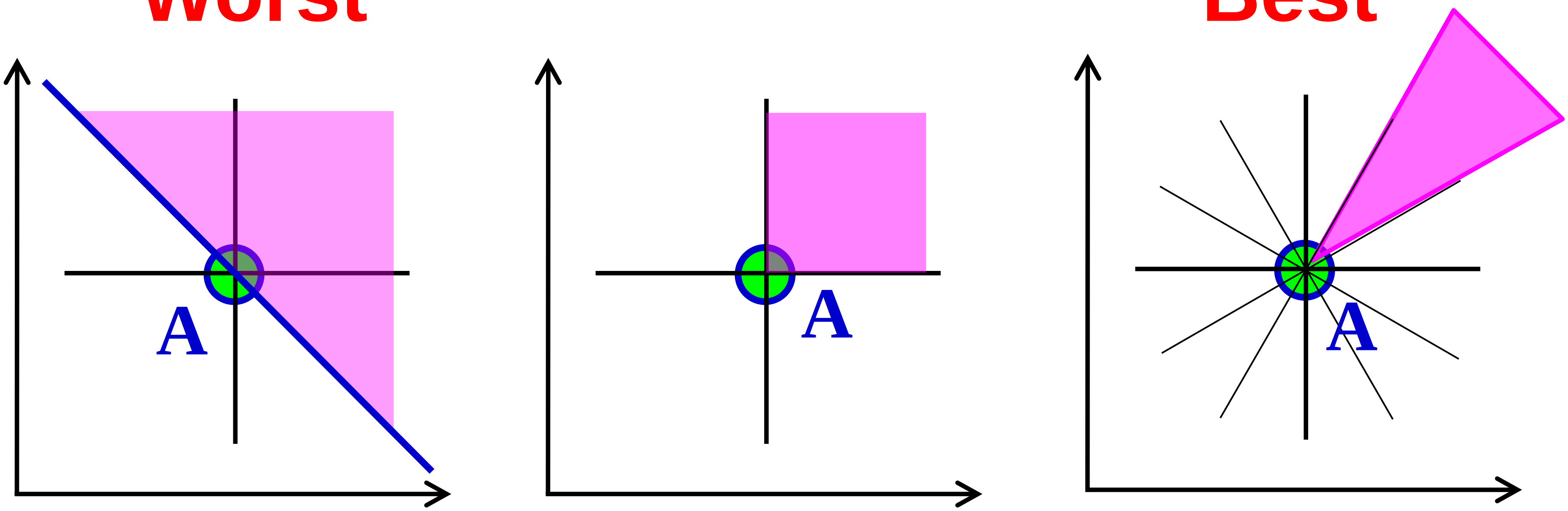
	10	2.51524	2.51532	2.43536	1.56890	0.95488	0.00000

Results on DTLZ Test Problems

Totally different from the expected results

Worst | **Best**



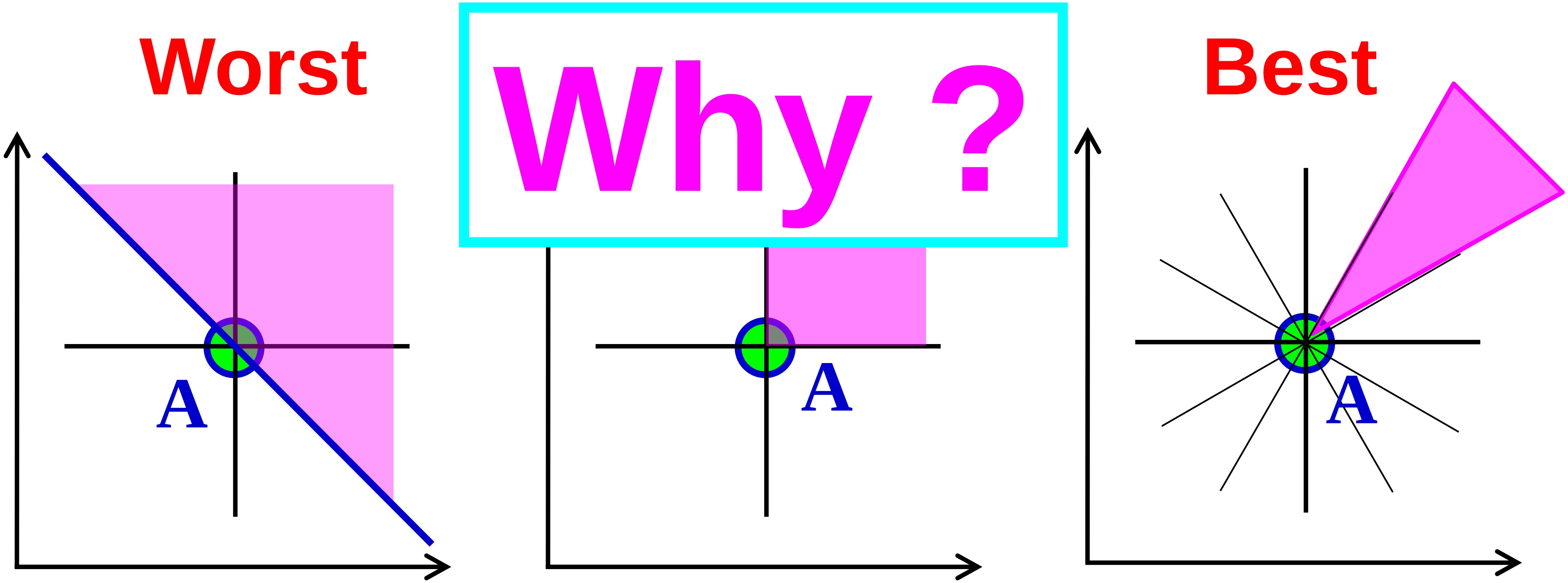
Weighted Sum (MOEA/D-WS)

Tchebycheff (MOEA/D-Tch)

PBI Function (MOEA/D-PBI) ($\theta = 5$)

Results on DTLZ Test Problems

Totally different from the expected results



Weighted Sum (MOEA/D-WS)

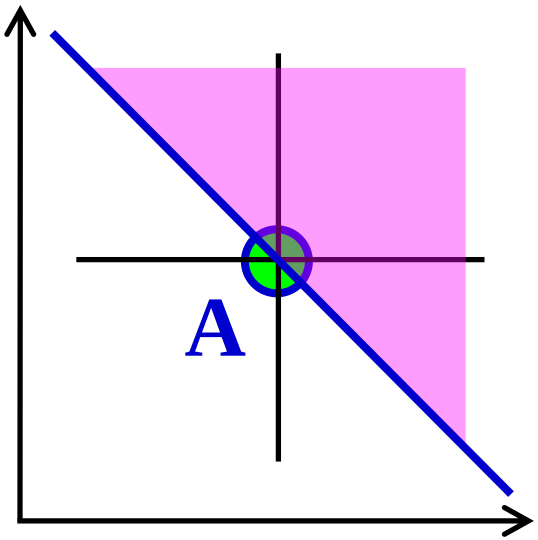
Tchebycheff (MOEA/D-Tch)

PBI Function (MOEA/D-PBI) ($\theta = 5$)

Results on DTLZ Test Problems

Totally different from the expected results

Worst

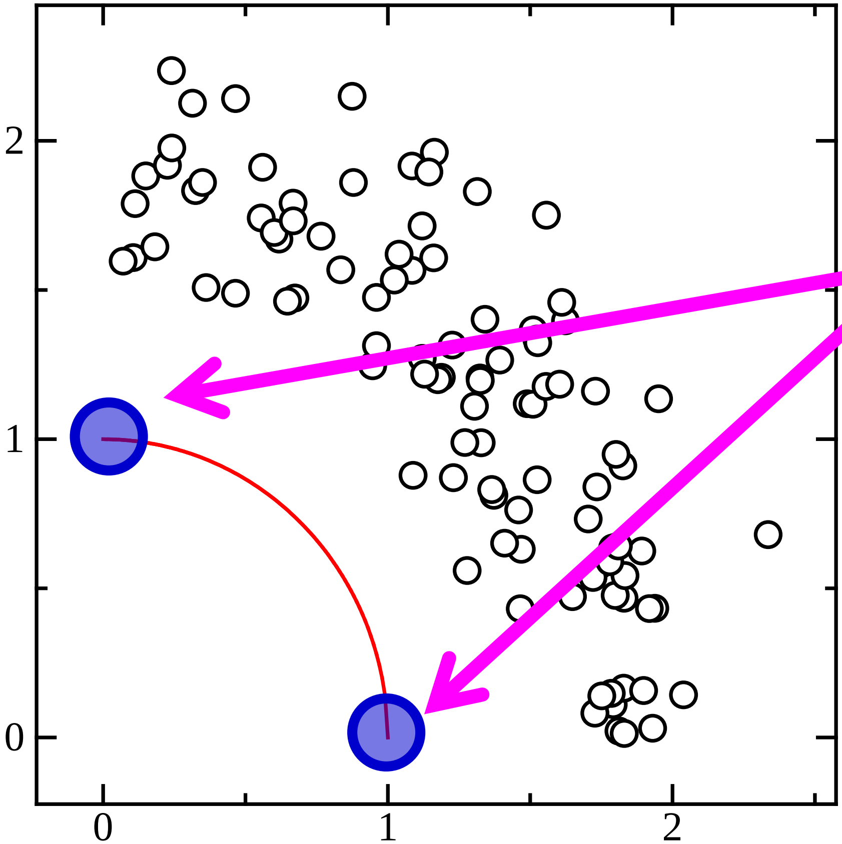


Weighted Sum
(MOEA/D-WS)

Why ?

DTLZ problems have concave Pareto fronts
==> Weighted sum cannot handle concave Pareto fronts

DTLZ2 (Minimization Problem)



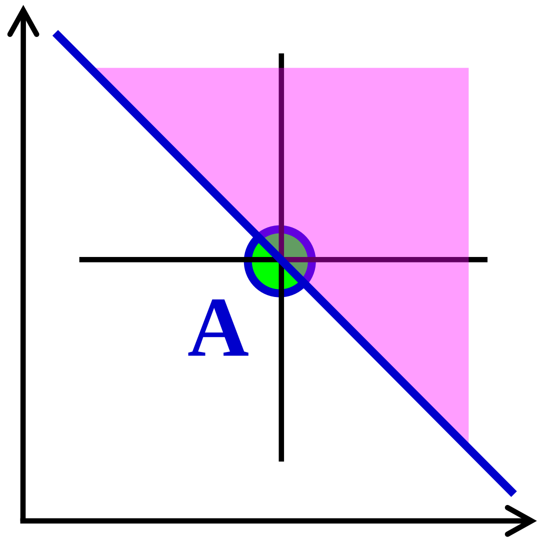
Obtained solutions by MOEA/D-WS

Pareto front and Initial Solutions

Results on DTLZ Test Problems

Totally different from the expected results

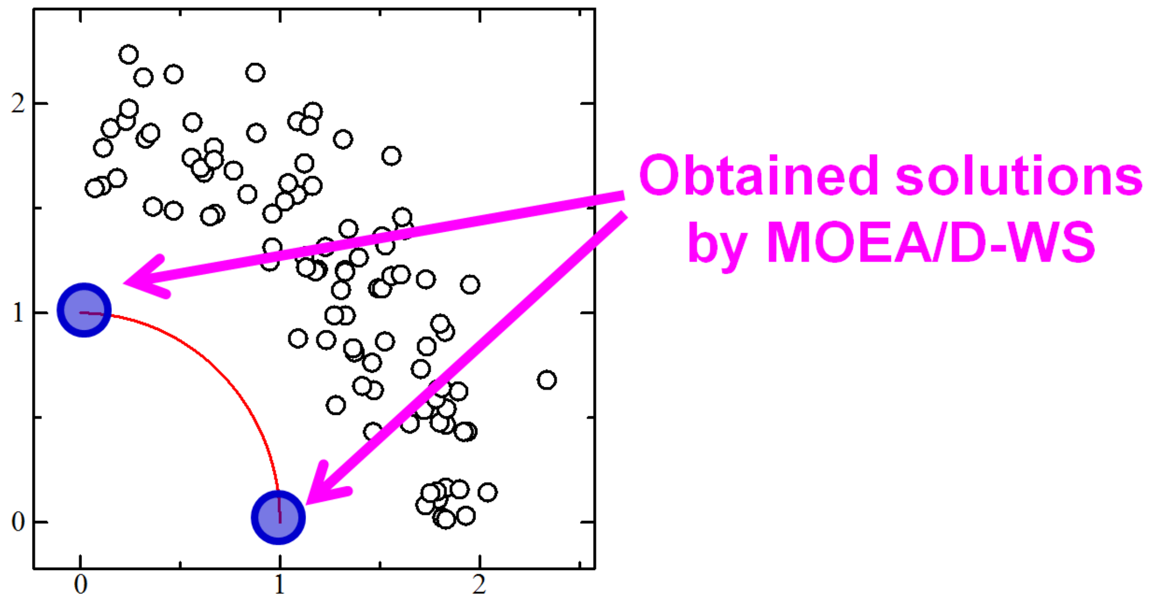
Worst



Weighted Sum (MOEA/D-WS)

Why ?

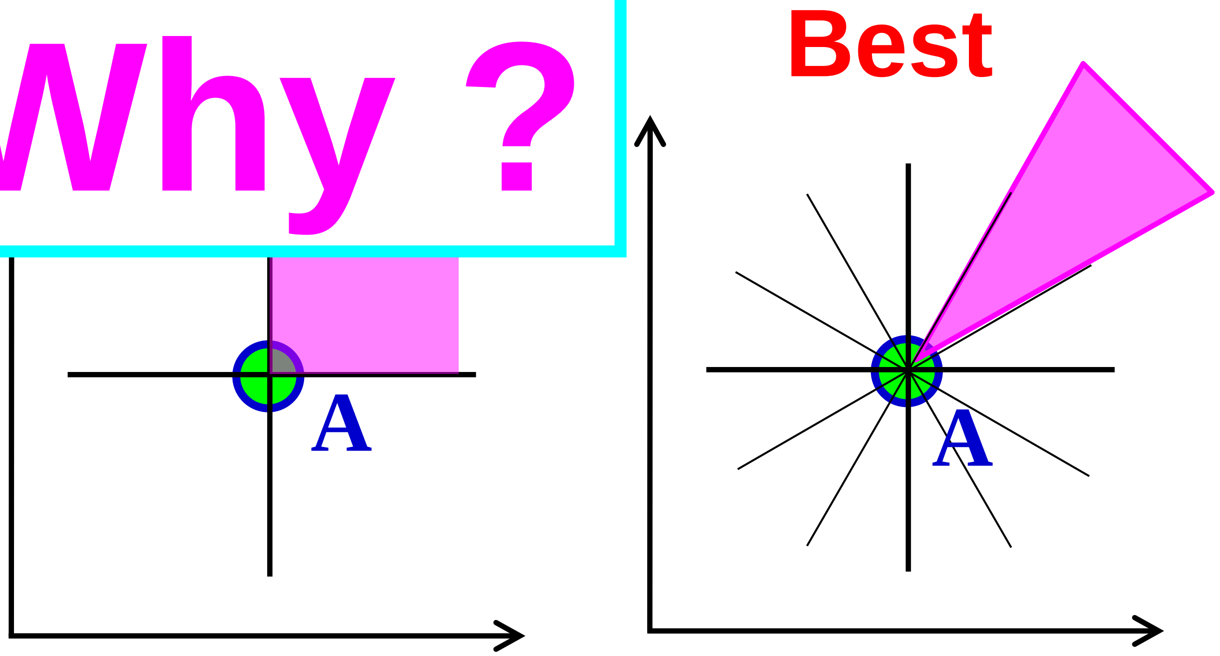
==> Because of the concave shape of the Pareto fronts !



Results on DTLZ Test Problems

Totally different from the expected results

Why ?

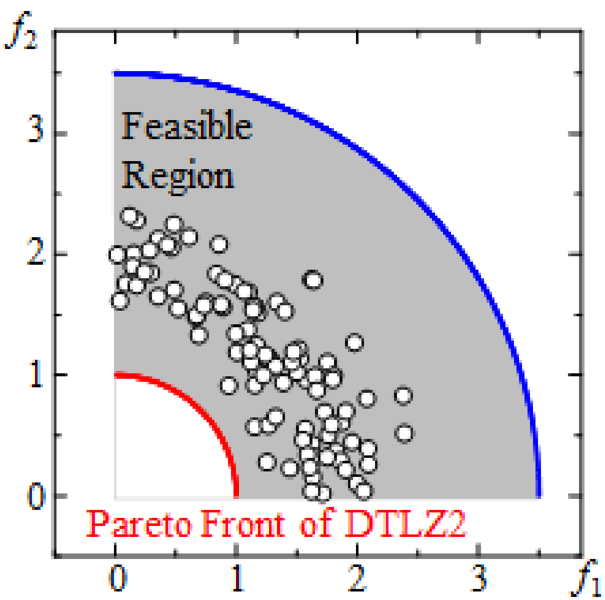


Tchebycheff (MOEA/D-Tch)	PBI Function (MOEA/D-PBI) ($\theta = 5$)

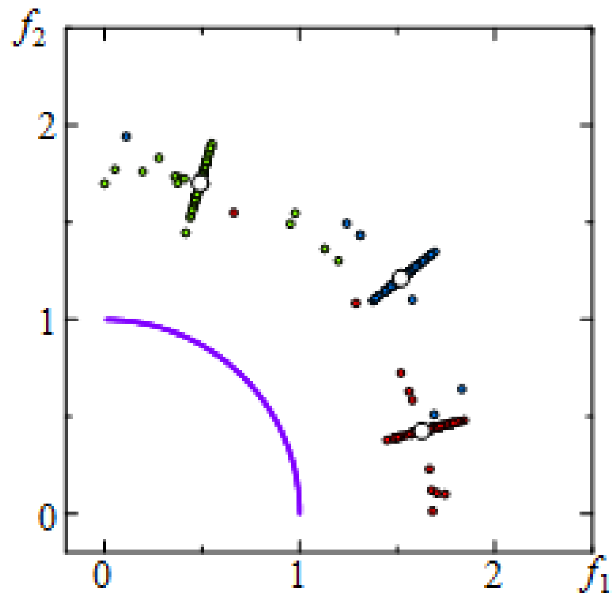
Reason

DTLZ test problems are very easy

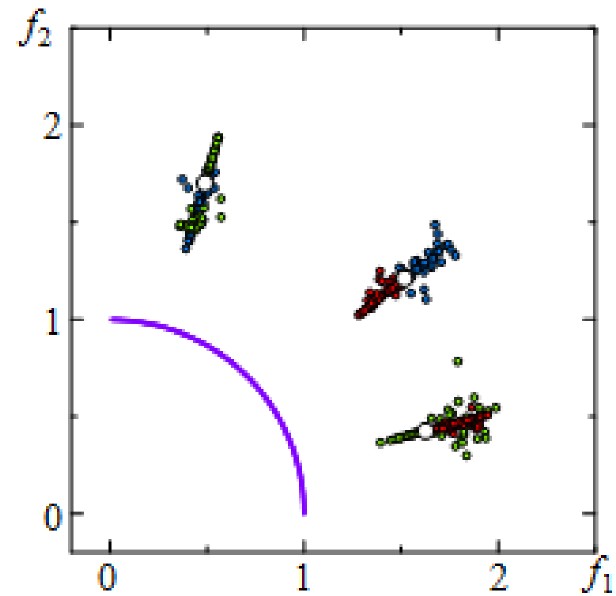
DTLZ2



Feasible Region and Initial Solutions



Generated Solutions by Mutation

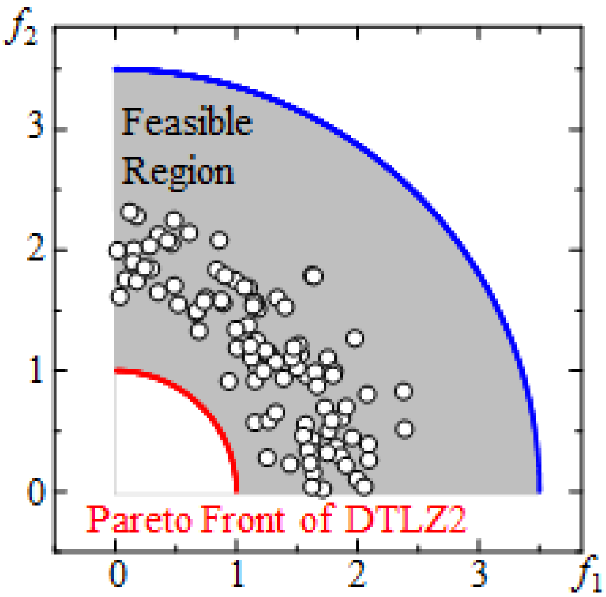


Generated Solutions by Crossover

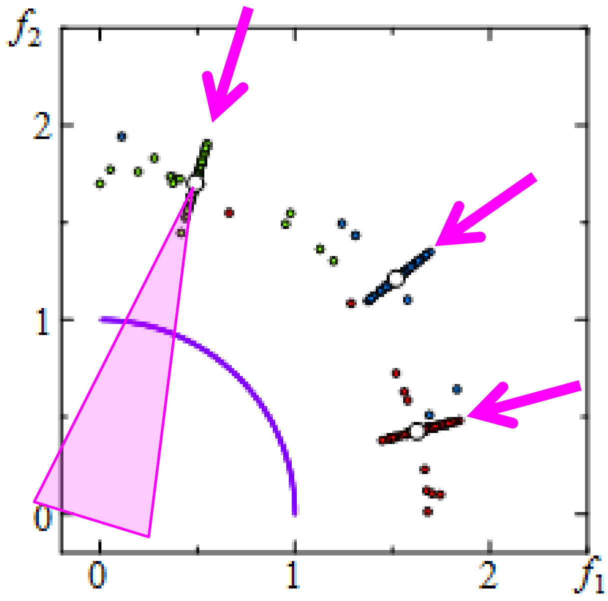
Reason

It is easy to find better solution.

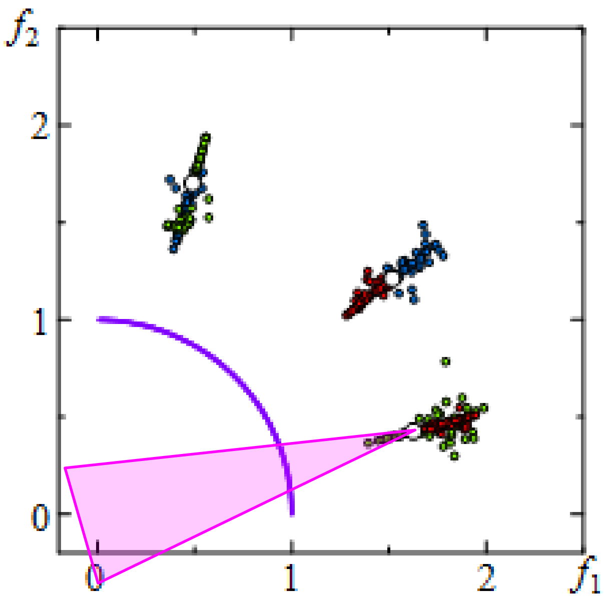
DTLZ2



Feasible Region and Initial Solutions



Generated Solutions by Mutation

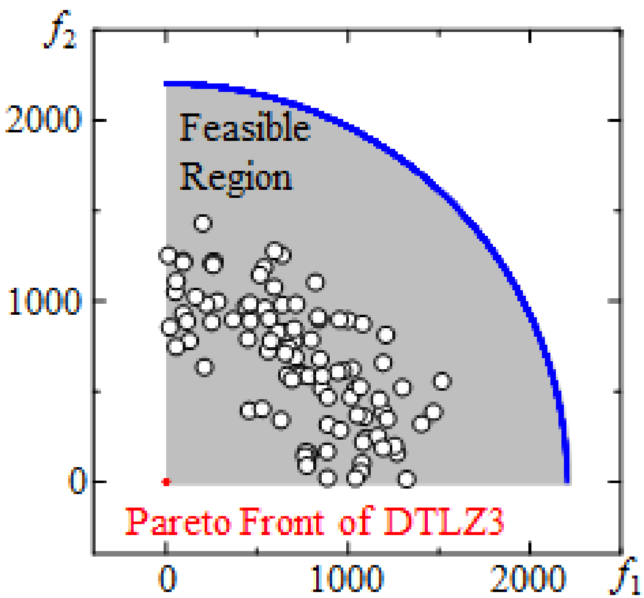


Generated Solutions by Crossover

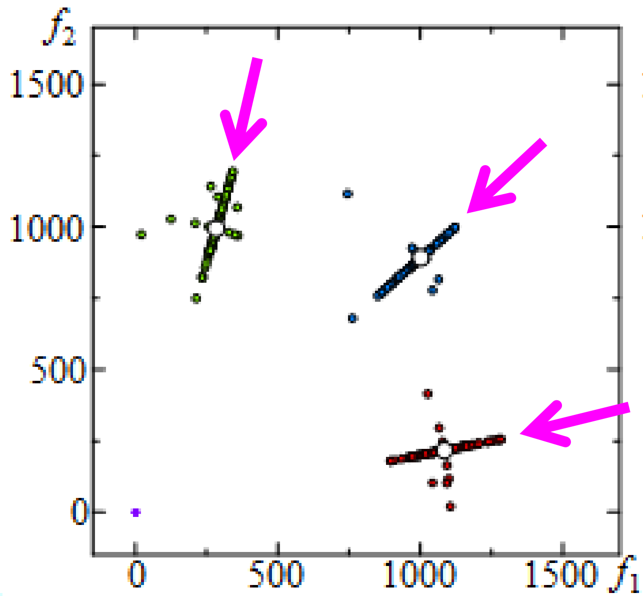
Reason

DTLZ test problems are very easy

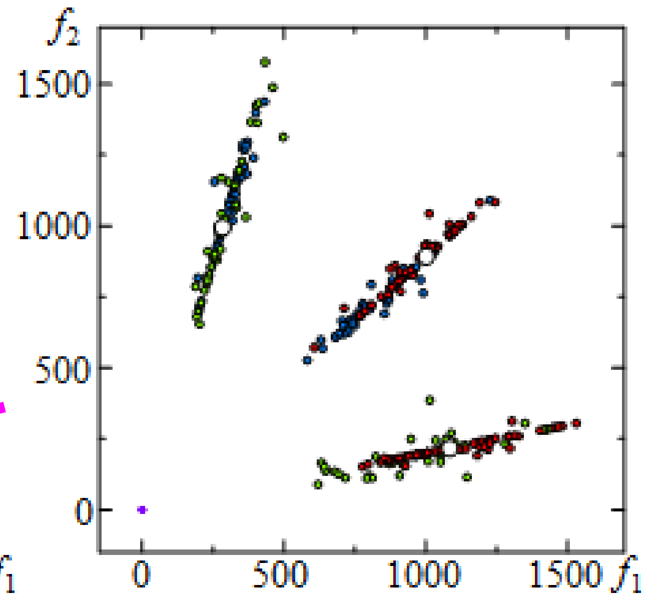
DTLZ3



Feasible Region and Initial Solutions



Generated Solutions by Mutation

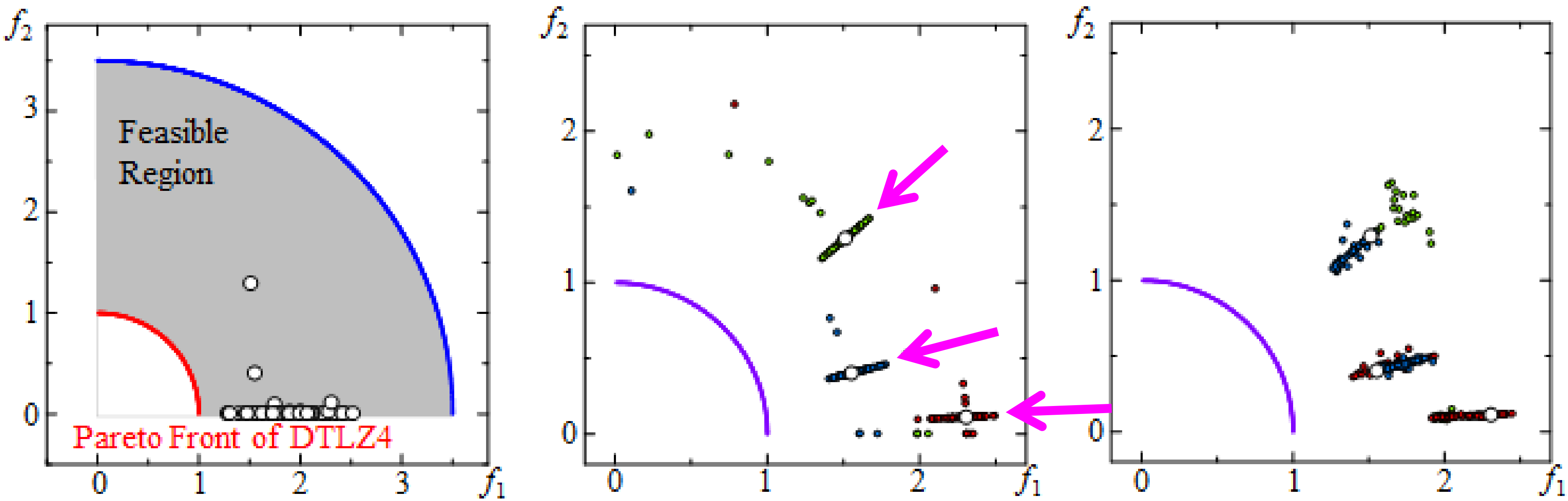


Generated Solutions by Crossover

Reason

DTLZ test problems are very easy

DTLZ4



Feasible Region and Initial Solutions

Generated Solutions by Mutation

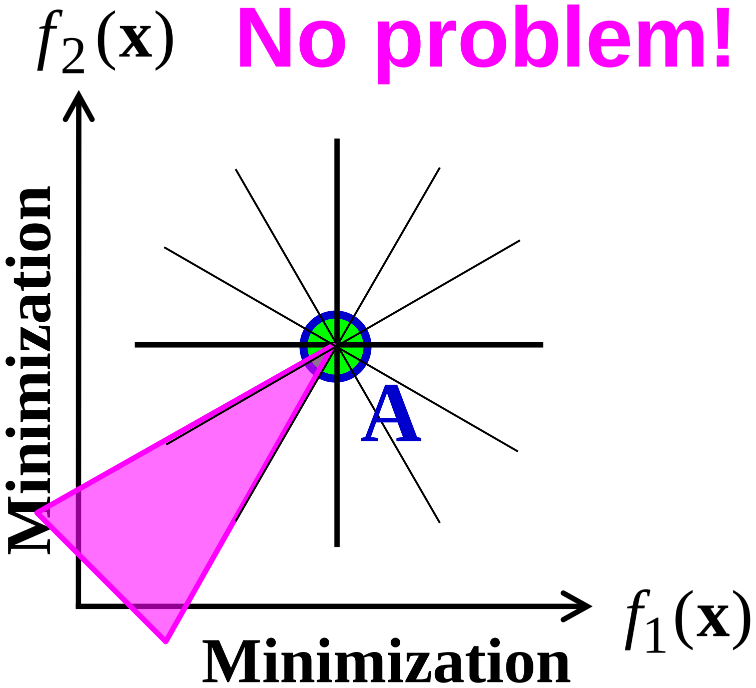
Generated Solutions by Crossover

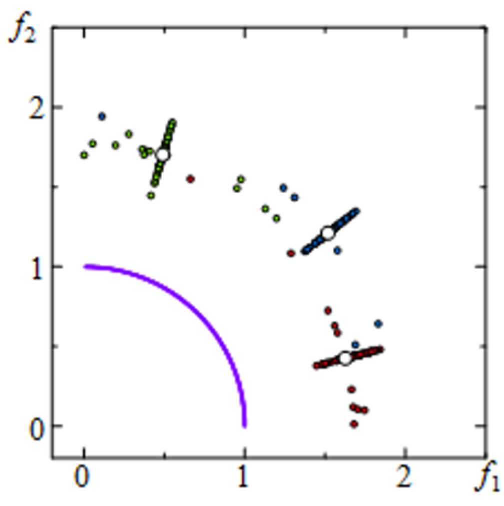
The best results were obtained from the PBI function

Percentage of the better region is very small.

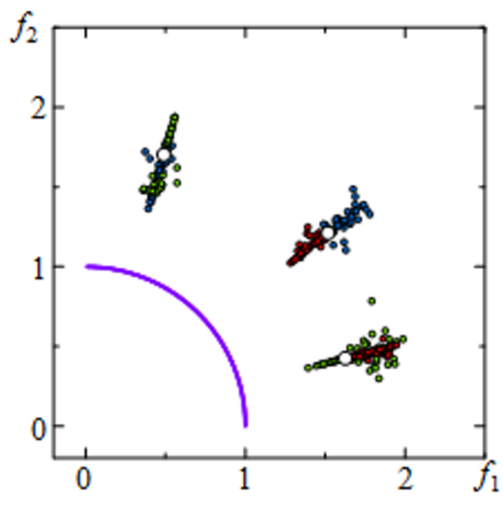
5 objectives	1/324	0.3%
10 objectives	1/78732	0.001%

No problem!





Generated Solutions
by Mutation



Generated Solutions
by Crossover

Today's Plan

Difficulties in Evolutionary Many-Objective Optimization Studies

1. **Difficulties related to many-objective search**
2. **Difficulties related to test problems**
3. **Difficulties related to performance evaluation**

Typical Scenario of Many-Objective Optimization Papers

Motivation:

- Many-objective optimization problems are difficult.
- New algorithms are needed.

Proposal:

- We propose a new high-performance algorithm.

Computational Experiments:

- Better results are obtained by the proposed algorithm than the existing ones on DTLZ 1-4 and WFG 1-9 problems.

Test Problems in Recent Many-Objective Papers

Publication Year	Proposed Algorithm	Test Problems	Number of Objectives
2014	NSGA-III	DTLZ 1-4 WFG 6-7 S-DTLZ 1-2	3, 5, 8, 10, 15 3, 5, 8, 10, 15 3, 5, 8, 10, 15
2015	I-DBEA	DTLZ 1-4 DTLZ5(I, M) WFG 1-9	3, 5, 8, 10, 15 3, 5, 8, 10, 15 3, 5, 10, 15
2015	MOEA/DD	DTLZ 1-4 WFG 1-9	3, 5, 8, 10, 15 3, 5, 8, 10
2016	MOEA/D-DU EFR-RR	DTLZ 1-4, 7 WFG 1-9 S-DTLZ 1-2	2, 5, 8, 10, 13 2, 5, 8, 10, 13 2, 5, 8, 10, 13
2016	θ-DEA	DTLZ 1-4, 7 WFG 1-9 S-DTLZ 1-2	3, 5, 8, 10, 15 3, 5, 8, 10, 15 3, 5, 8, 10, 15

High-Performance Evolutionary Many-Objective Algorithms

2007 MOEA/D

2014 NSGA-III

2015 I-DBEA

2015 MOEA/DD

2016 θ - DEA

Better Results on DTLZ and WFG

(New algorithms are better than old ones).

Typical Scenario of Many-Objective Optimization Papers

Motivation:

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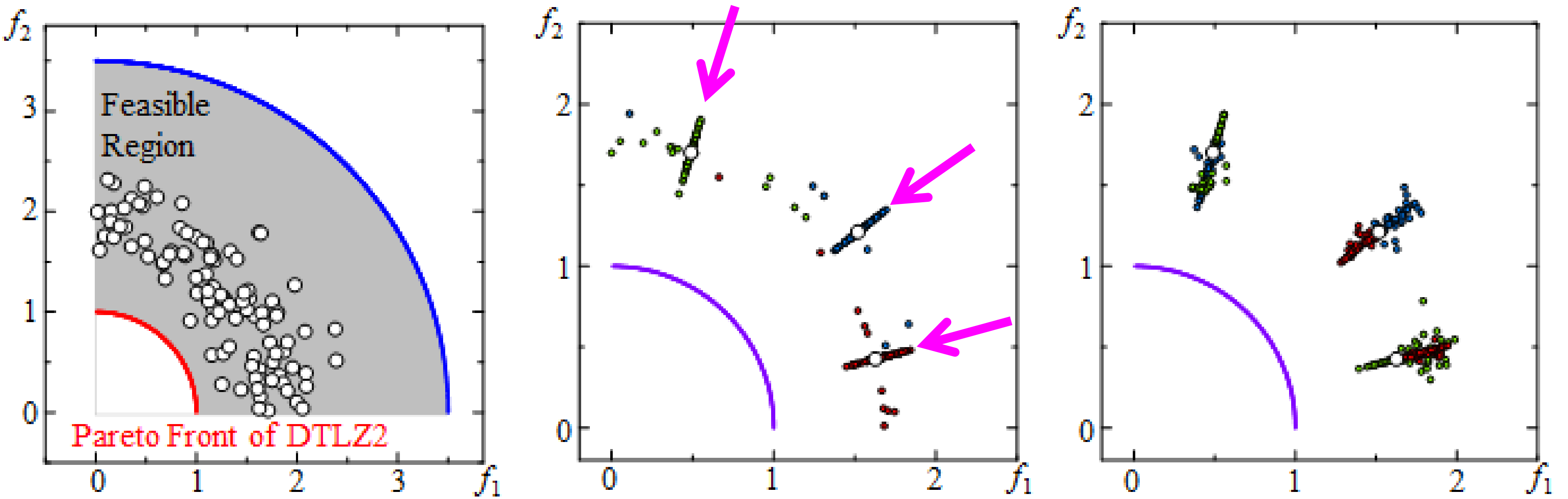
Computational Experiments:

- Better results are obtained by the proposed algorithm than the existing ones on <u>DTLZ 1-4 and WFG 1-9 problems</u>.

Test problems are easy and have special features.

Special Feature: Better new solutions can be easily created by genetic operators

DTLZ2

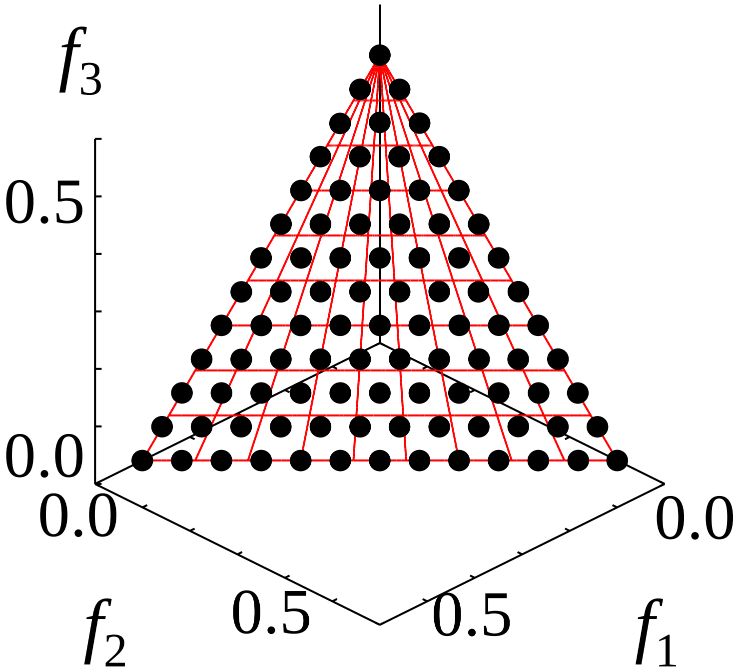


Feasible Region and Initial Solutions

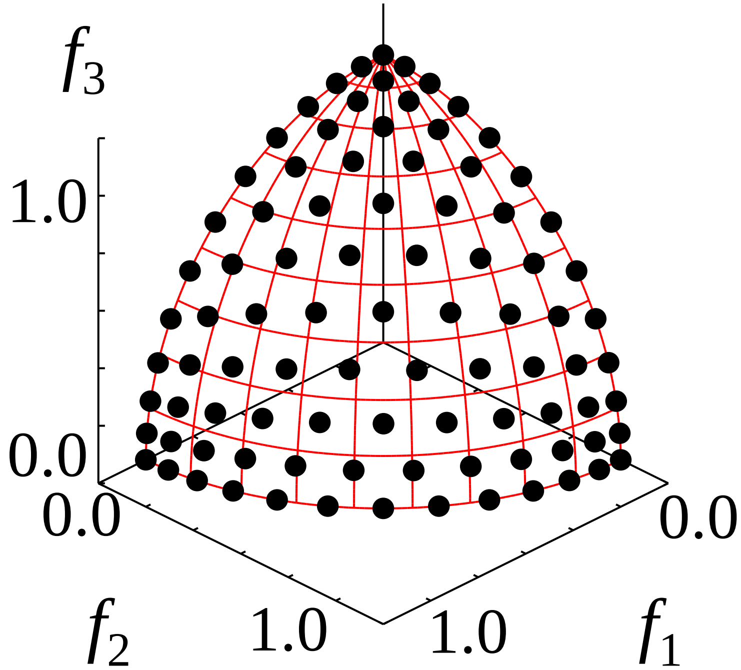
Generated Solutions by Mutation

Generated Solutions by Crossover

Special Feature: DTLZ 1-4 and WFG 4-9 have triangular Pareto fronts



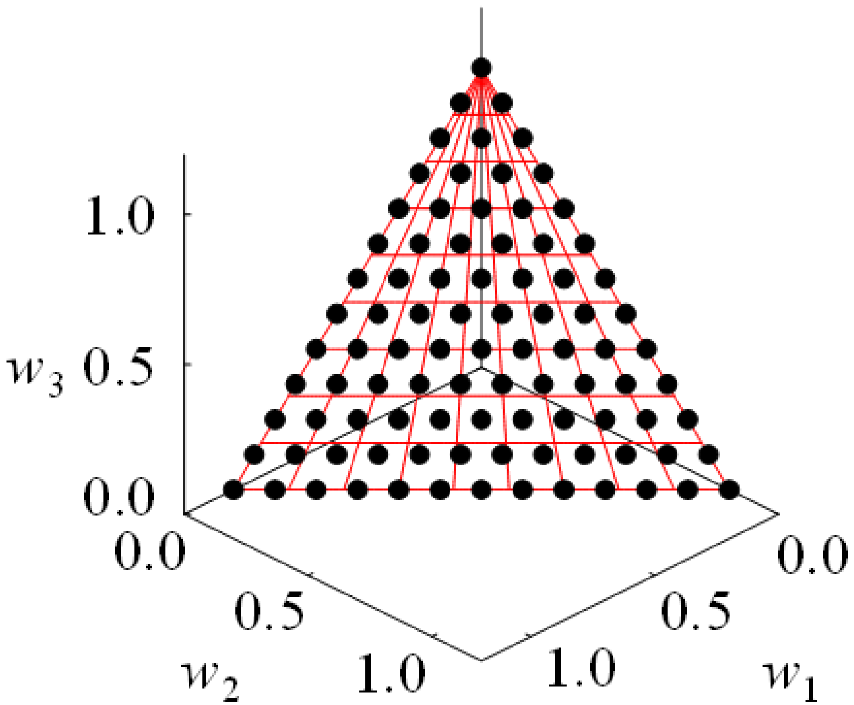
DTLZ 1



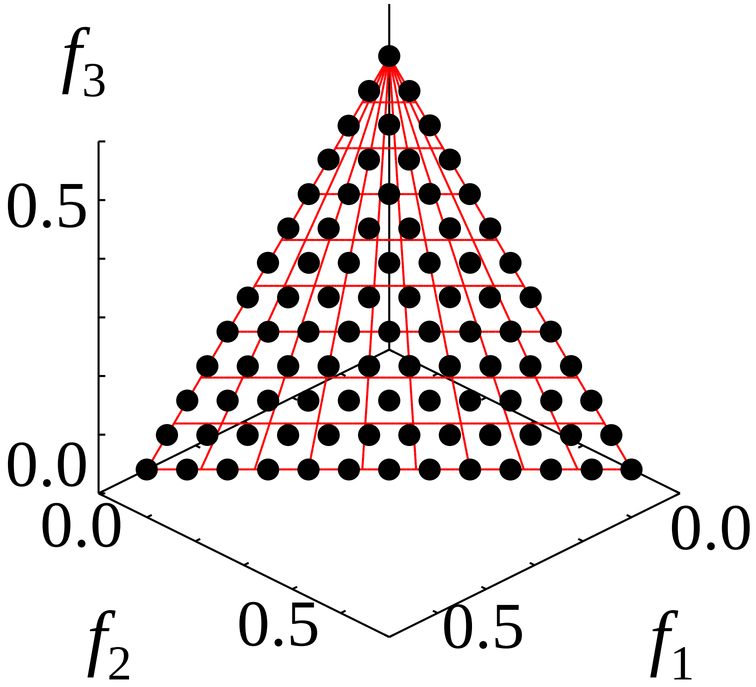
DTLZ 2

MOEA/D and Test Problems

MOEA/D looks perfect for DTLZ



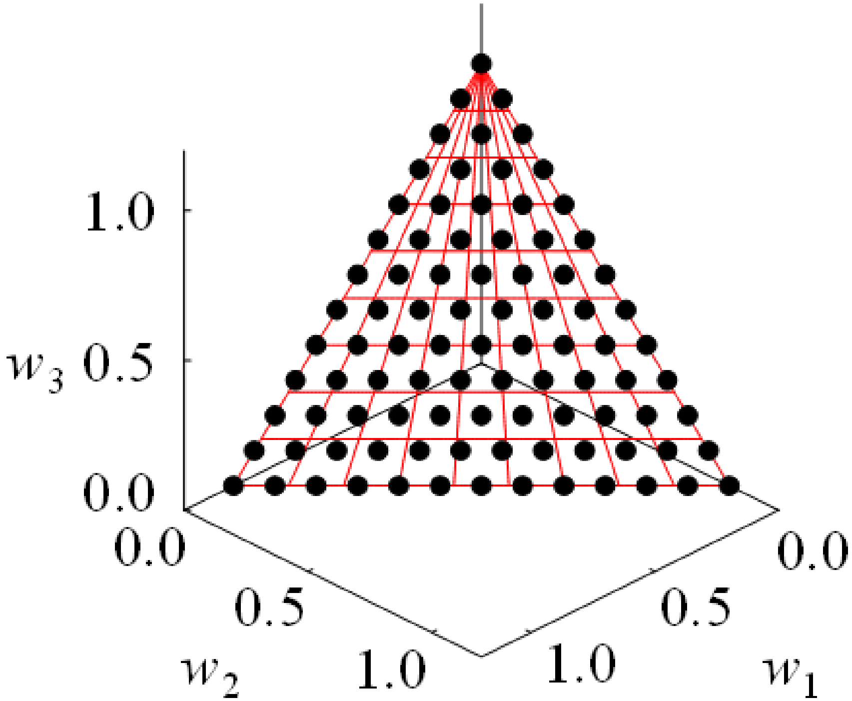
Weight Vectors



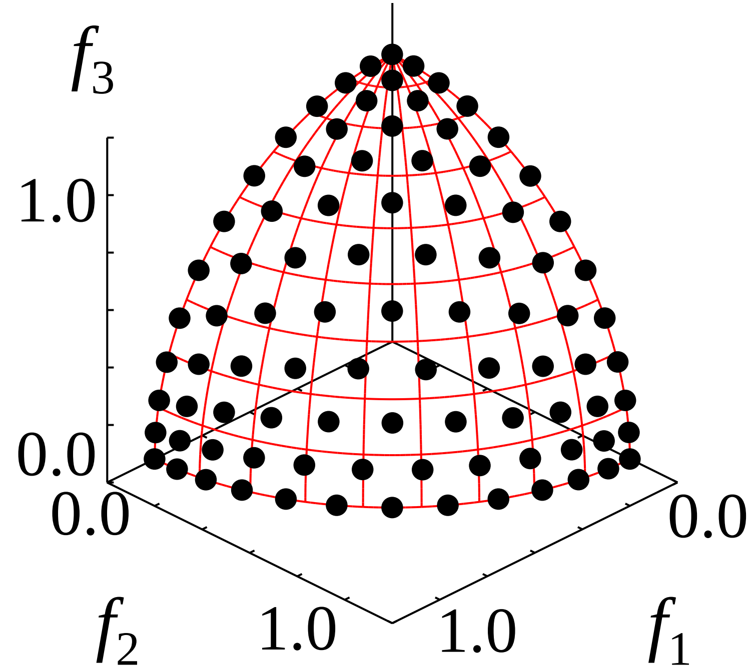
Pareto front (DTLZ 1)

MOEA/D and Test Problems

MOEA/D looks perfect for DTLZ



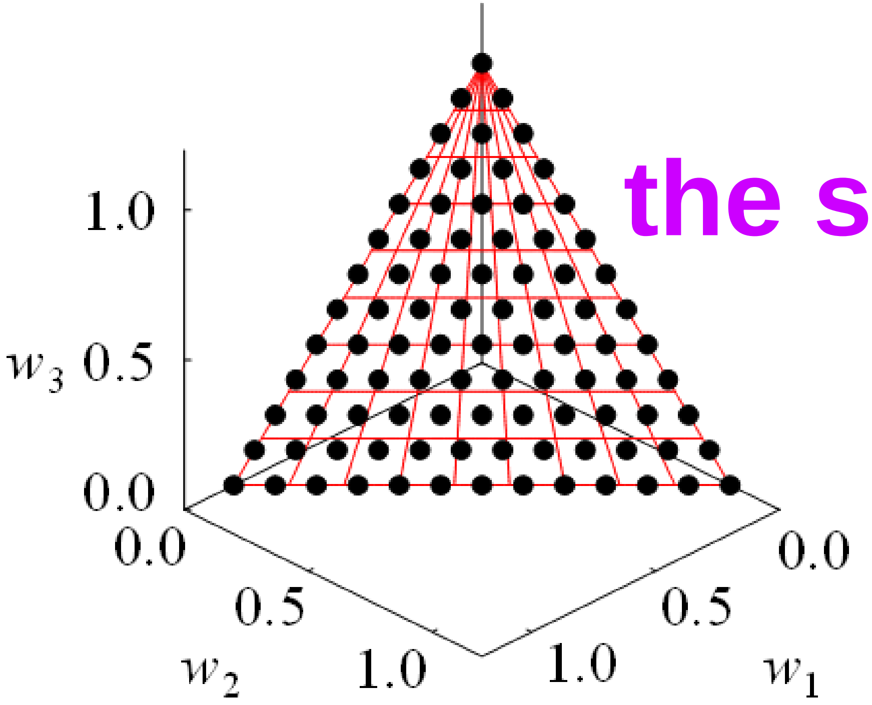
Weight Vectors



Pareto front (DTLZ 2)

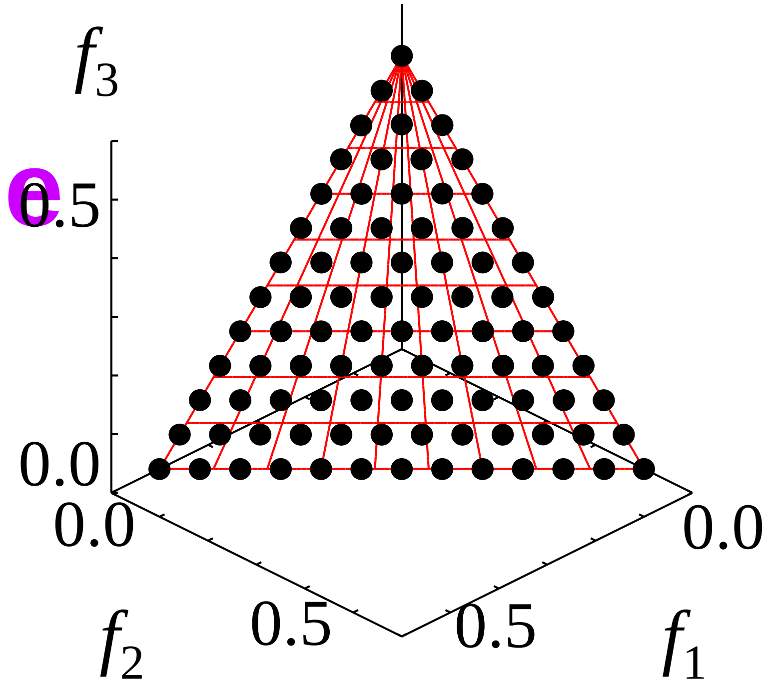
Shape of the Pareto front for MOEA/D:

The point is whether the shape of the Pareto front is similar to the shape of the weight vector distribution.



Weight Vectors

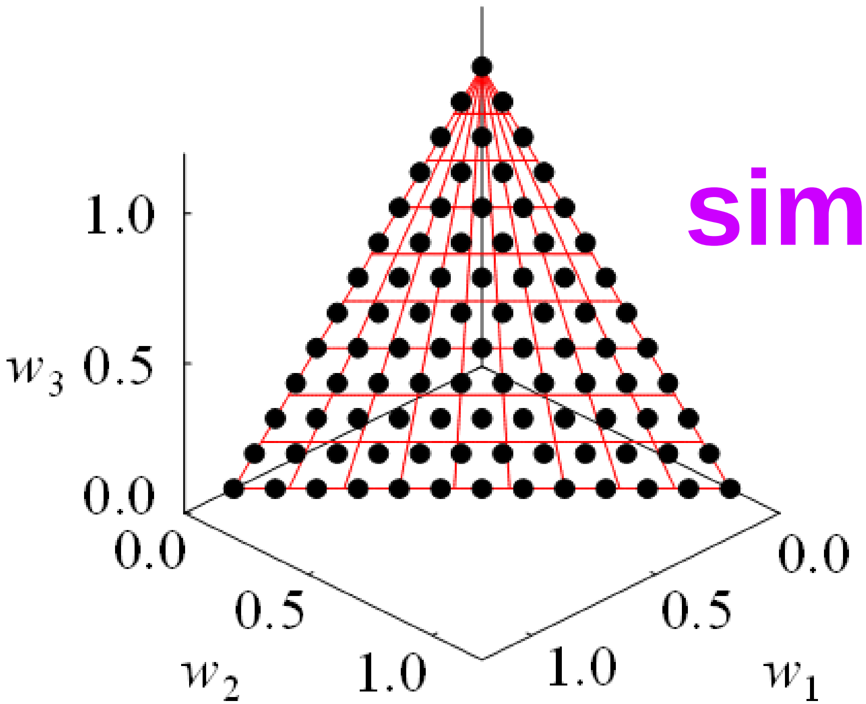
the same



Pareto front (DTLZ 1)

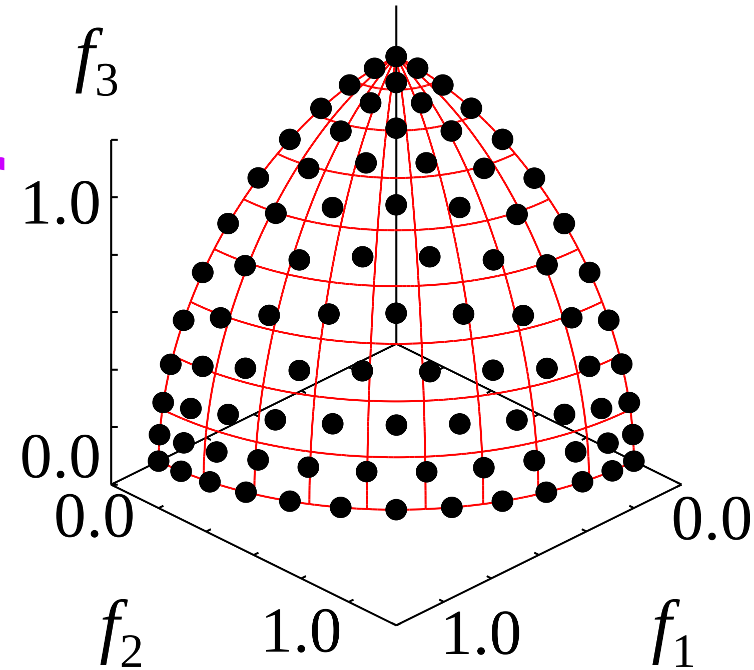
Shape of the Pareto front for MOEA/D:

The point is **whether the shape of the Pareto front is similar to the shape of the weight vector distribution.**



Weight Vectors

similar



Pareto front (DTLZ 2)

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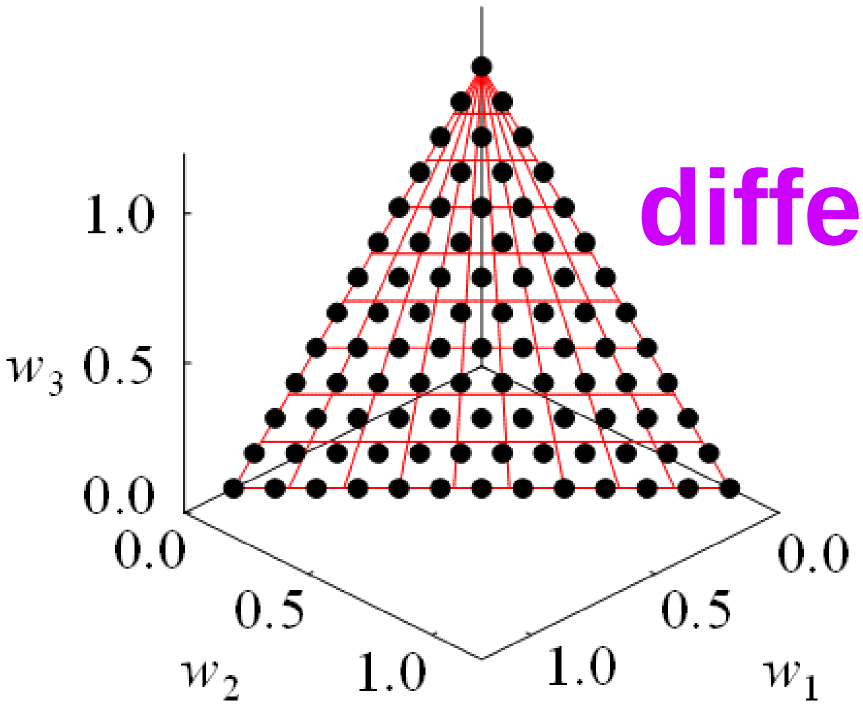
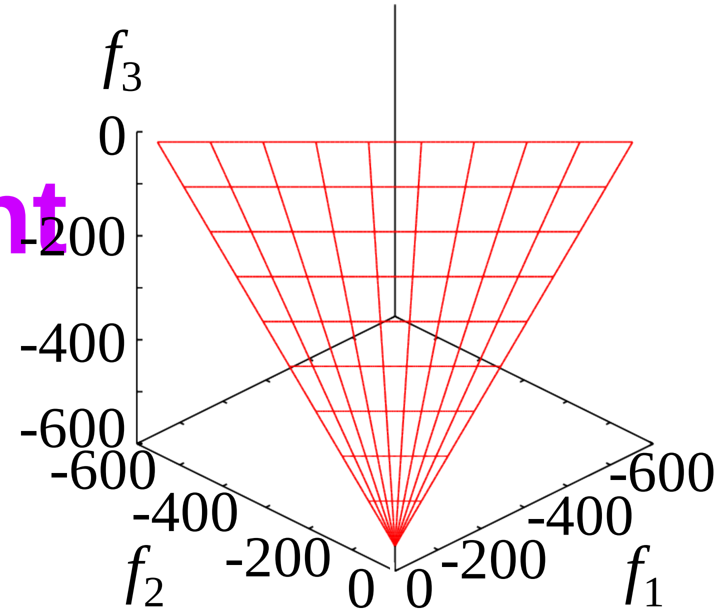


different

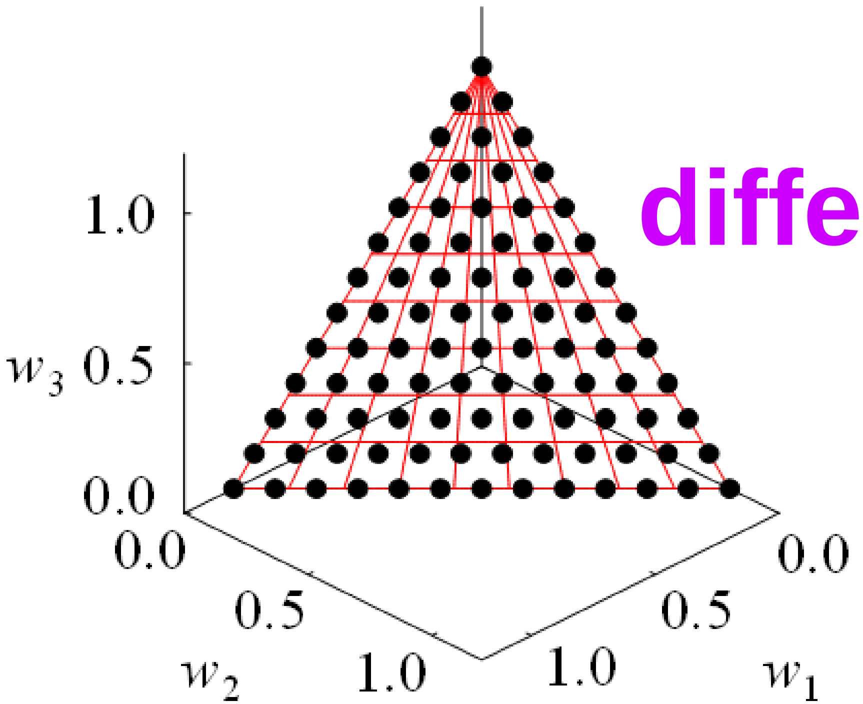


Weight Vectors

Pareto front (Minus-DTLZ 1)

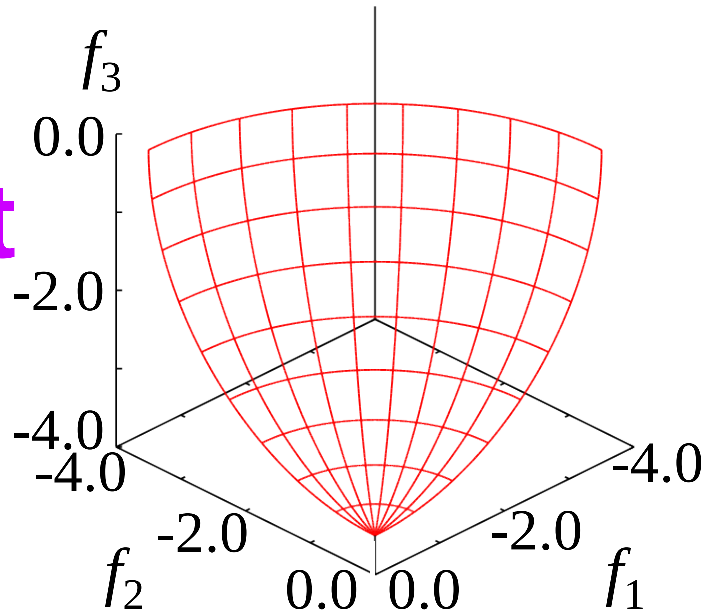
Shape of the Pareto front for MOEA/D:

The point is whether the shape of the Pareto front is similar to the shape of the weight vector distribution.



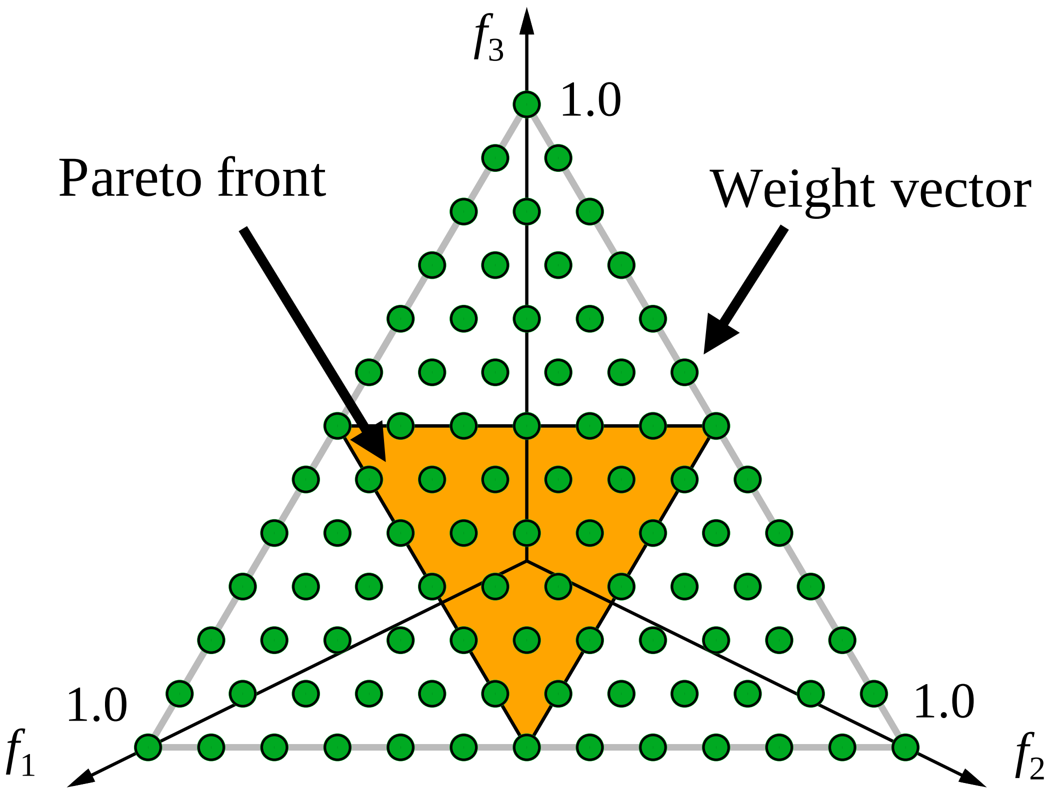
Weight Vectors

different

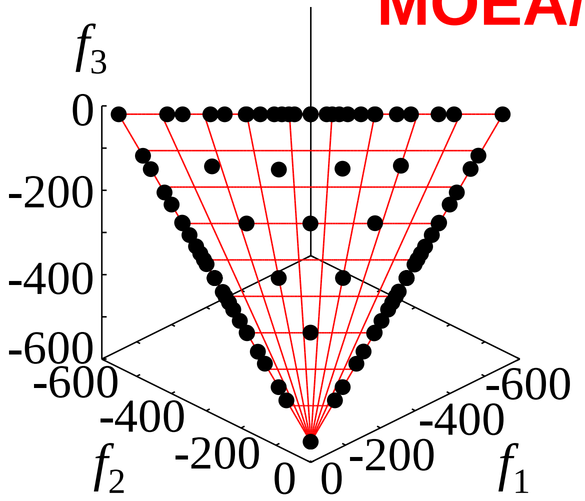


Pareto front (Minus-DTLZ 2)

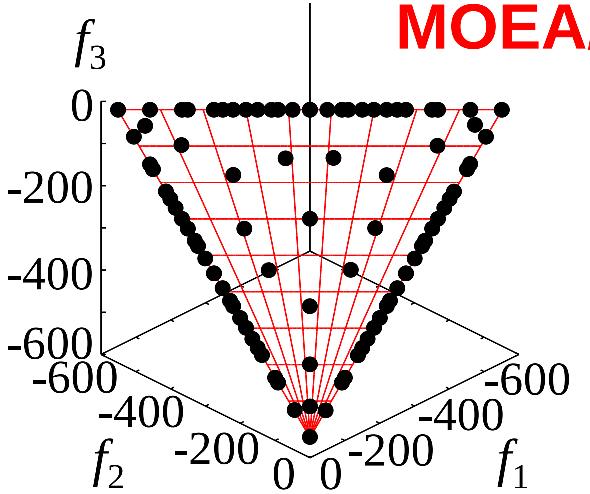
Experimental Results on (- 1) x DTLZ1



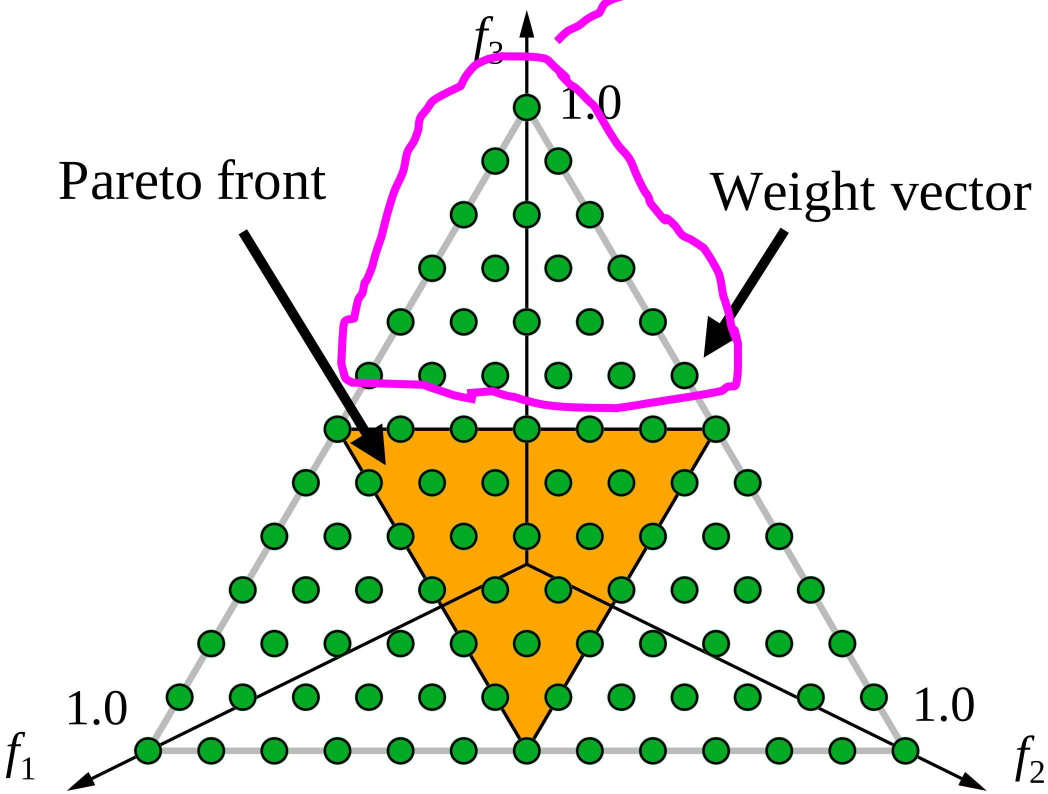
MOEA/D-PBI

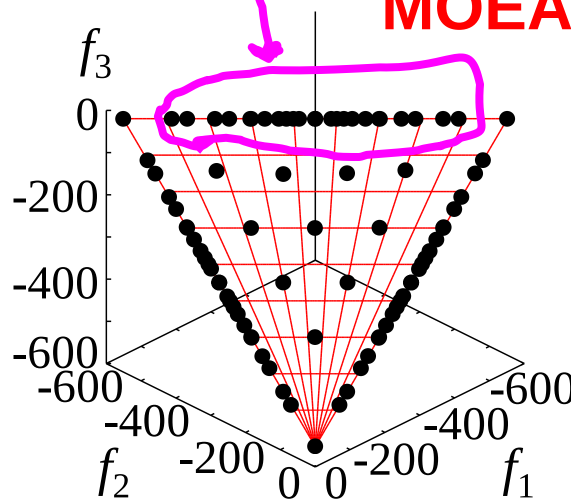


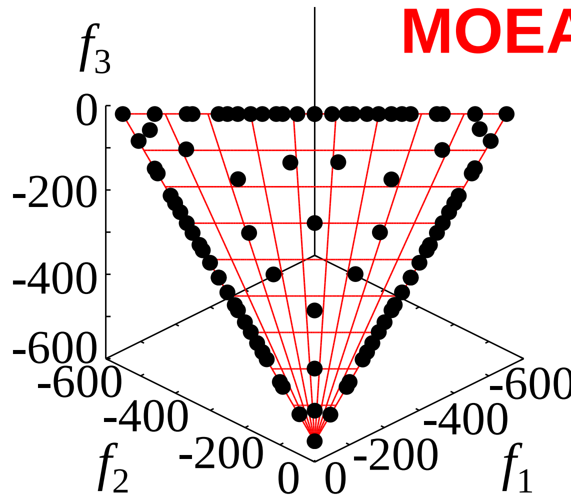
MOEA/D-Tch



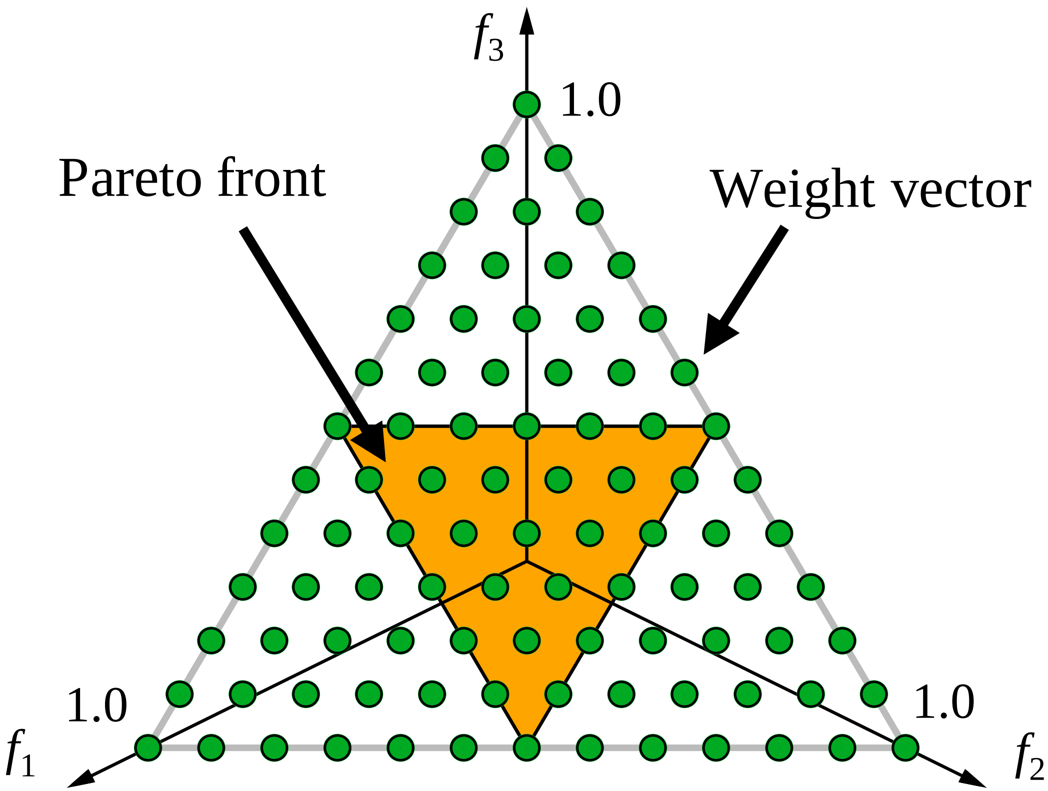
Experimental Results on (- 1) x DTLZ1



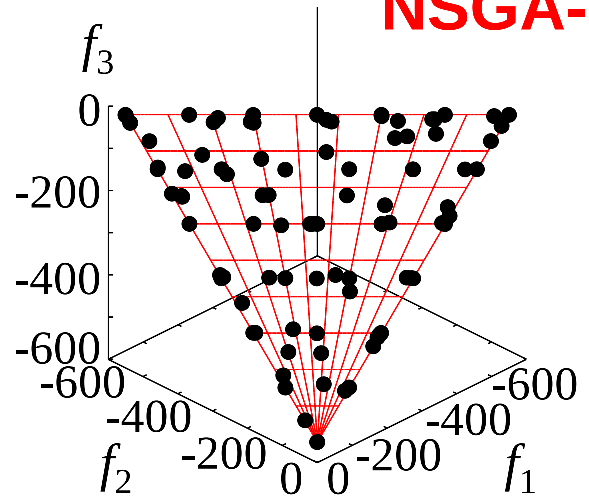




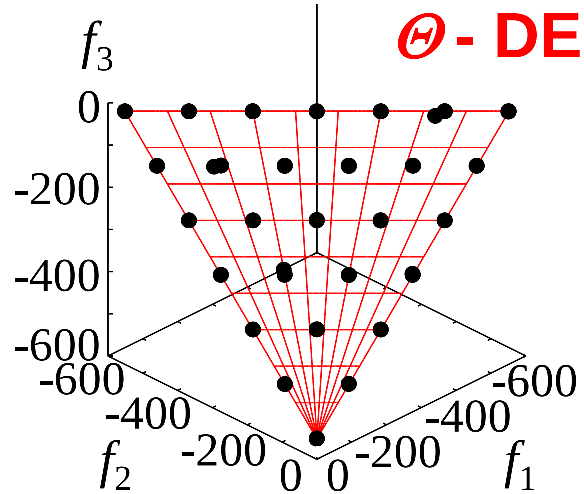
Experimental Results on (- 1) x DTLZ1



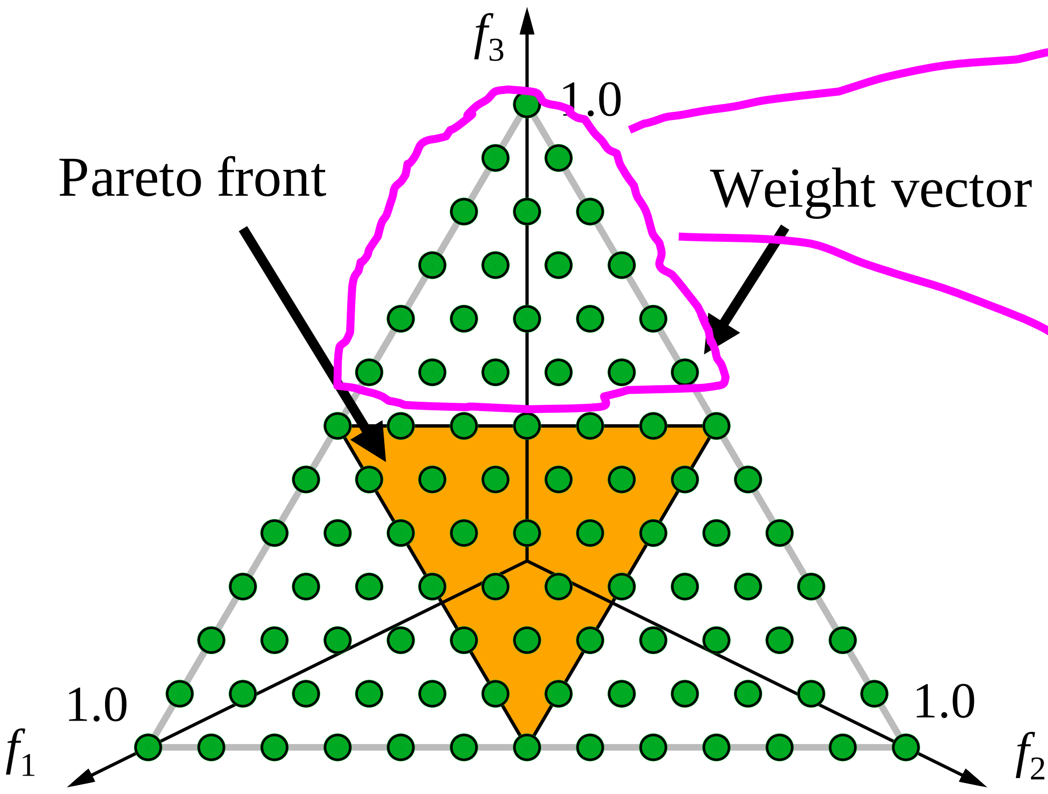
NSGA-III

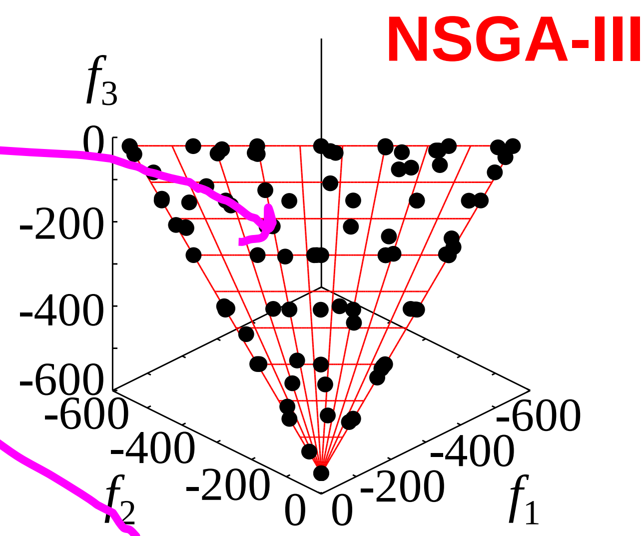


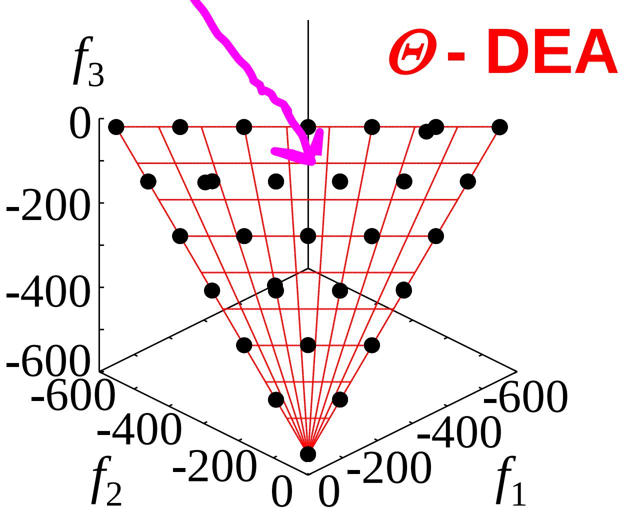
Θ - DEA



Experimental Results on (- 1) x DTLZ1







Our Results on Minus-DTLZ Test Problems

Ishibuchi et al. IEEE TECV (2017)

Average Hyper-Volume Value

Problem	*M*	NSGA-III	MOEA/DD	PBI	Tch	WS	NSGA-II
Minus	5	0.01265	0.00972	0.01739	0.01208	0.00083	0.01520
DTLZ 1	8	5.227E-05	0.881E-05	0.598E-05	3.215E-05	0.139E-05	3.568E-05
	10	1.185E-06	0.100E-06	0.079E-06	0.620E-06	0.025E-06	0.765E-06
Minus	5	0.13957	0.08794	0.15984	0.15556	0.14930	0.17147
DTLZ 2	8	4.454E-03	2.690E-03	5.978E-03	0.459E-03	1.560E-03	4.585E-03
	10	6.308E-04	1.836E-04	5.199E-04	0.052E-04	0.640E-04	3.797E-04
Minus	5	0.12951	0.08190	0.15902	0.15199	0.14891	0.16472
DTLZ 3	8	0.00414	0.00255	0.00596	0.00050	0.00156	0.00390
	10	0.00054	0.00018	0.00052	0.00001	0.00006	0.00033
Minus	5	0.12326	0.07242	0.12296	0.14878	0.14881	0.16970
DTLZ 3	8	4.582E-03	2.198E-03	2.020E-03	0.485E-03	1.563E-03	3.886E-03
	10	6.065E-04	2.569E-04	2.333E-04	0.043E-04	0.642E-04	3.006E-04

Experimental Results
(Hypervolume)

DTLZ and WFG

MOEA/D (1997)

NSGA-III (2014)

MOEA/DD (2015)

θ-DEA (2016)

Better

(-1) x DTLZ and (-1) x WFG

Better

MOEA/D (1997)

NSGA-III (2014)

MOEA/DD (2015)

θ-DEA (2016)

Experimental Results (Hypervolume)

DTLZ and WFG

MOEA/D (1997)

NSGA-III (2014)

MOEA/DD (2015)

θ-DEA (2016)

Better

(-1) x DTLZ and (-1) x WFG

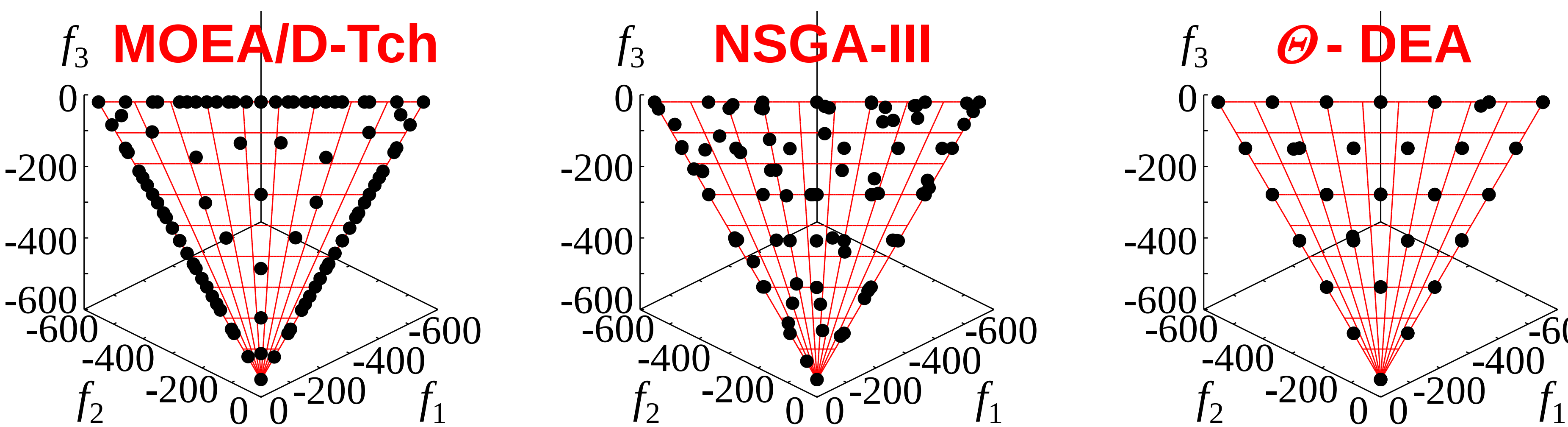
Better

MOEA/D (1997)

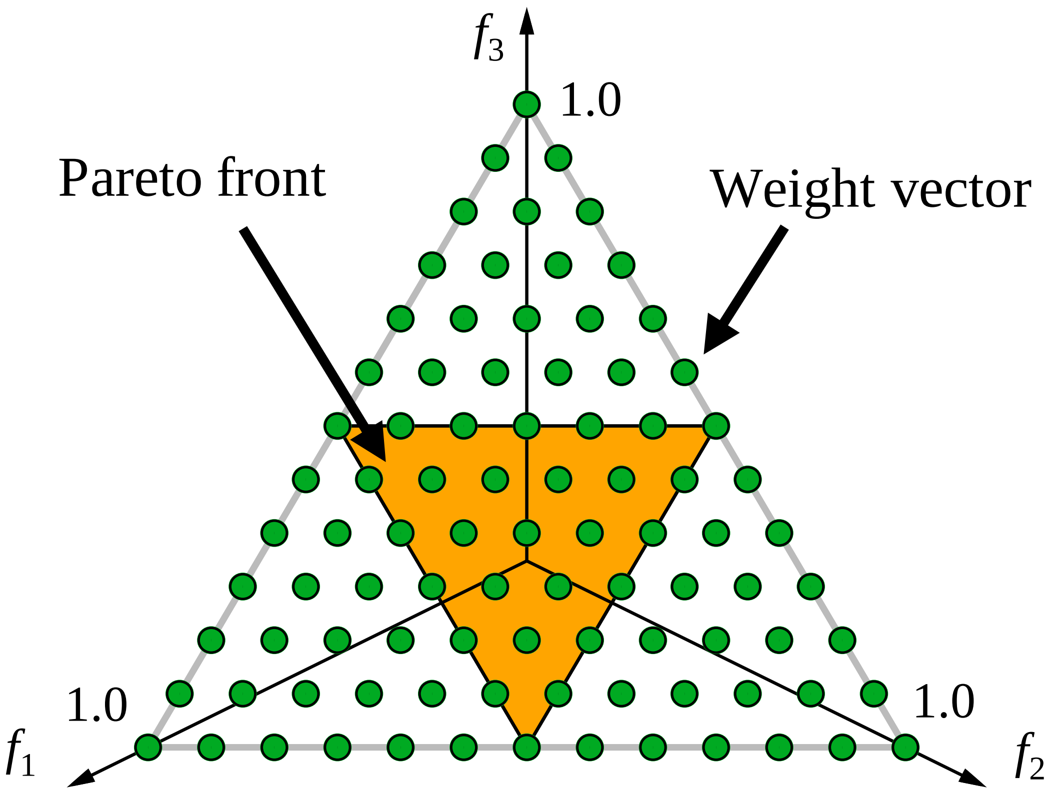
NSGA-III (2014)

MOEA/DD (2015)

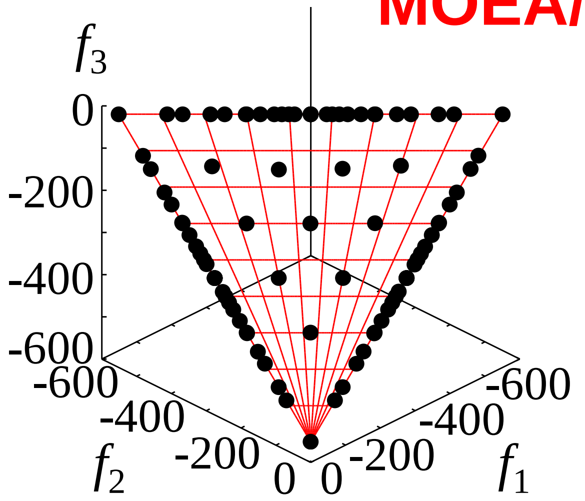
θ-DEA (2016)



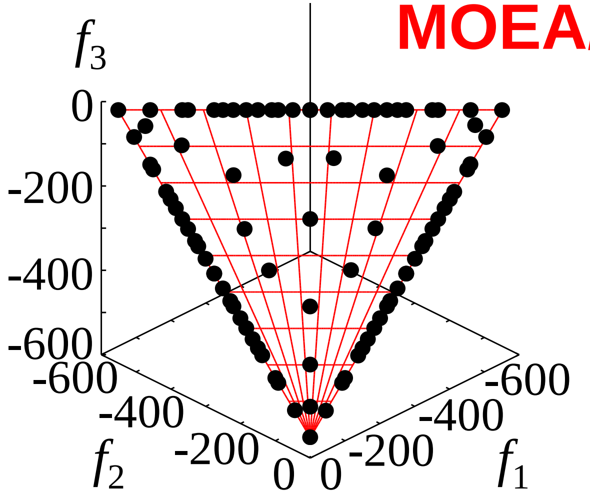
Adaptation of weight vectors is an important research topic in MOEA/D.



MOEA/D-PBI



MOEA/D-Tch



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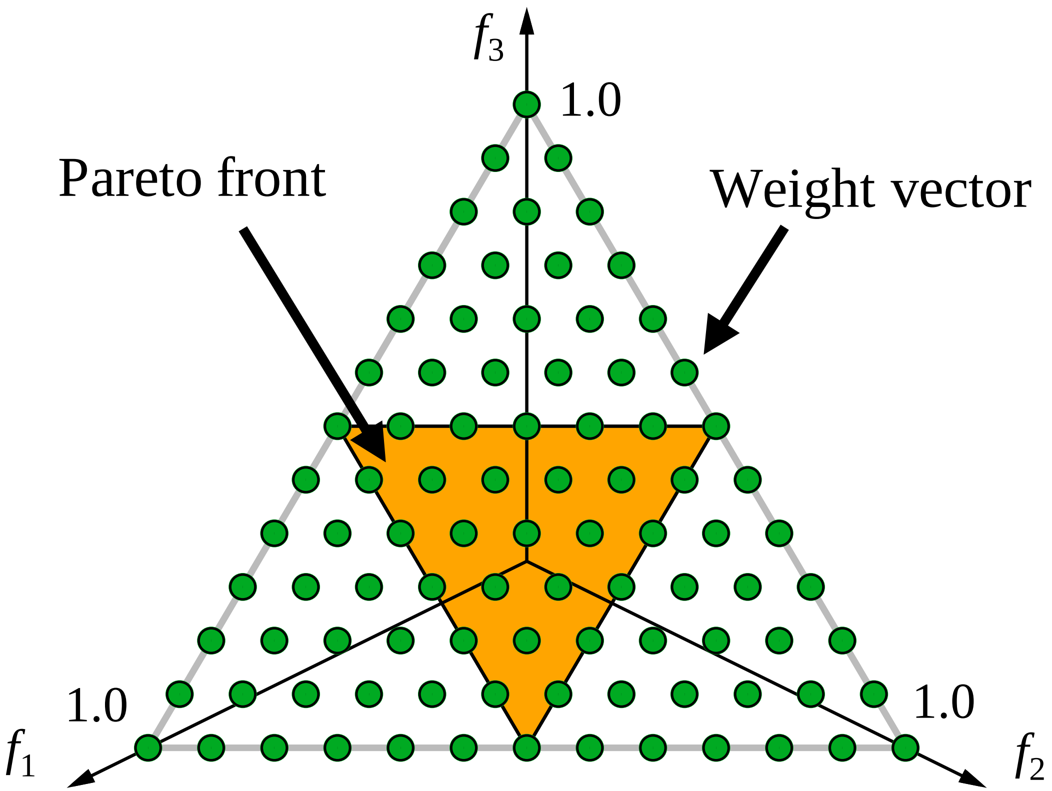
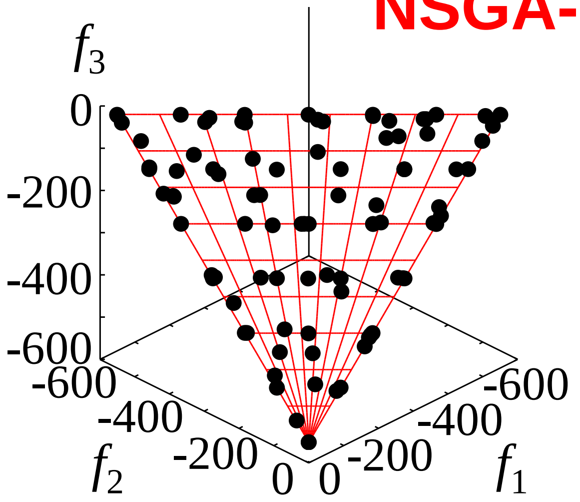
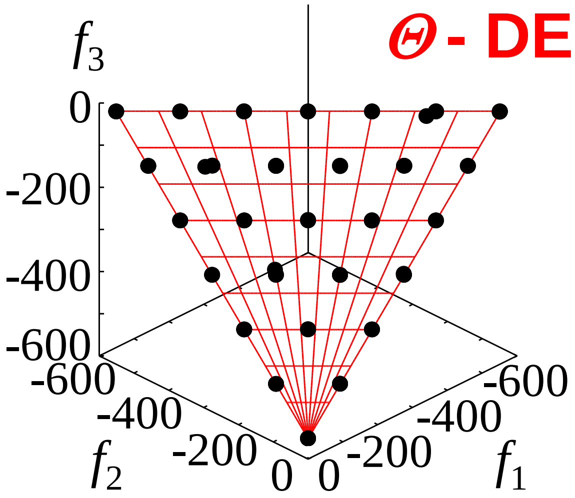


NSGA-III



Θ - DEA



Adaptation of reference vectors is an important research topic in MOEA/D.

Big Question:

What is a good distribution of 200 reference vectors in a 10-dimensional objective space? We need 10 million solutions to cover the entire Pareto front.

***k*-Objective Problem**	$5^{(k-1)}$
2-Objective Problem	5
3-Objective Problem	25
10-Objective Problem	10 million

Today's Plan

Difficulties in Evolutionary Many-Objective Optimization Studies

1. **Difficulties related to many-objective search**
2. **Difficulties related to test problems**
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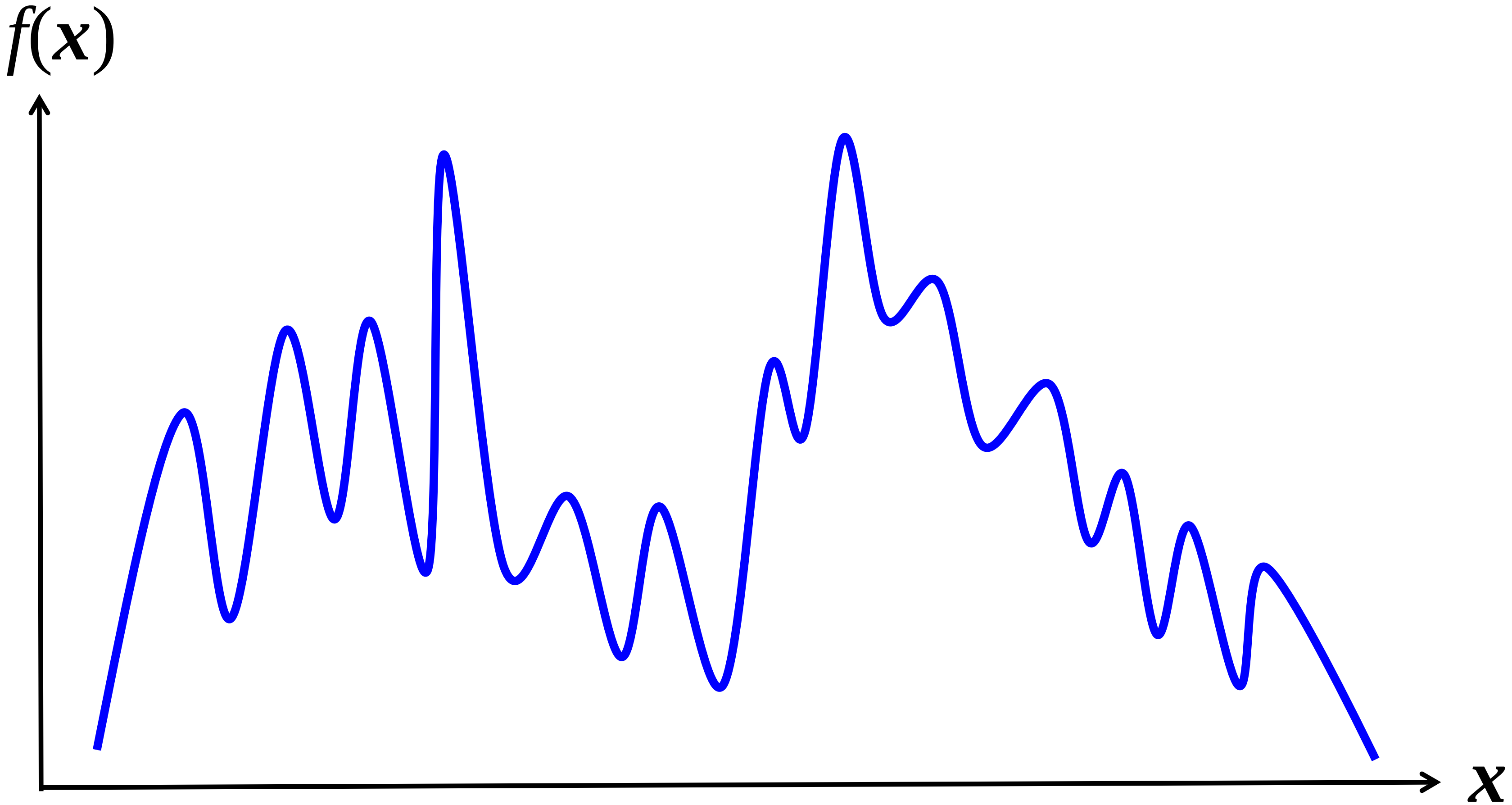
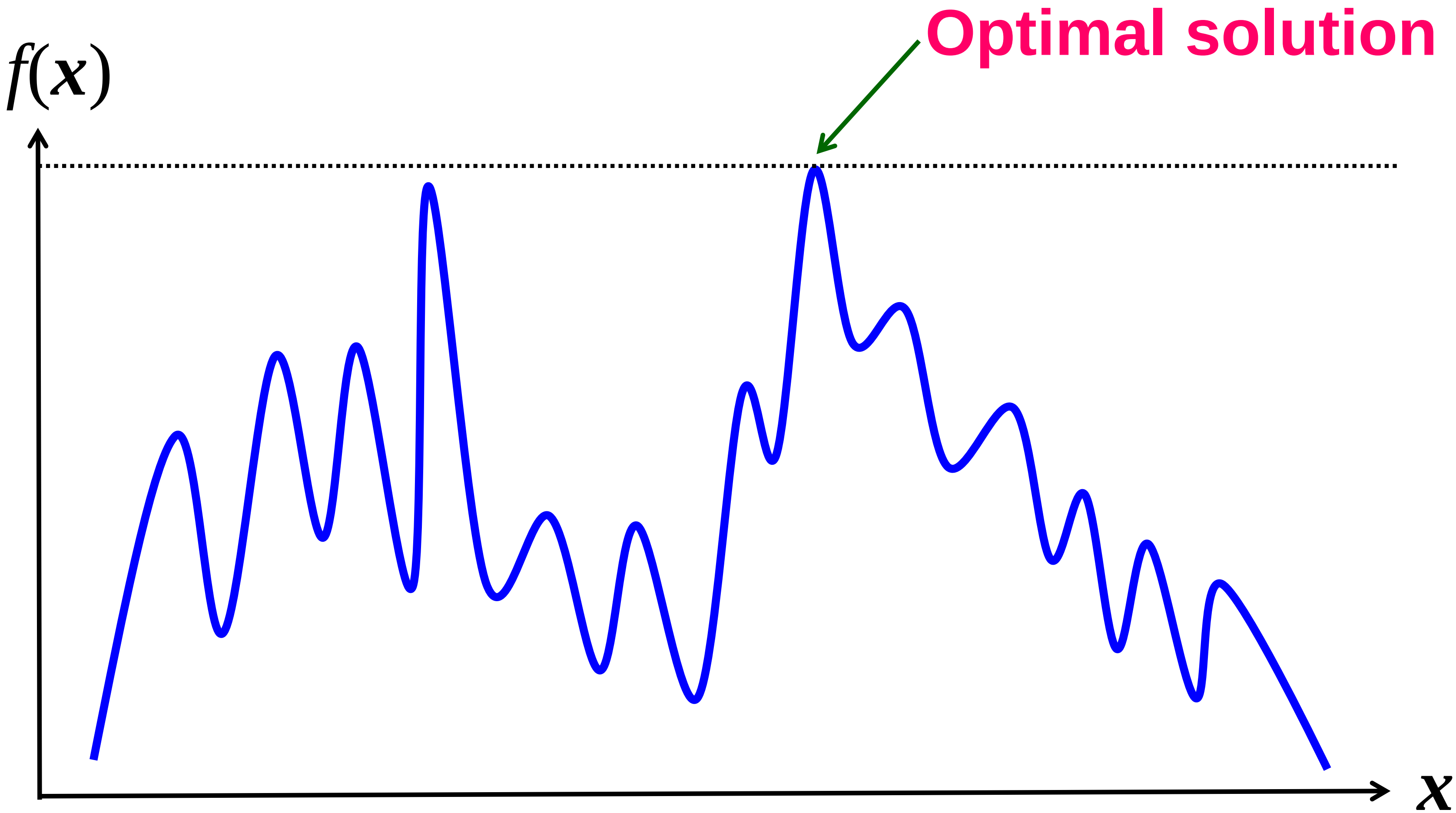
Single-objective Optimization

Single-Objective Optimization: **Maximize** $f(\boldsymbol{x})$



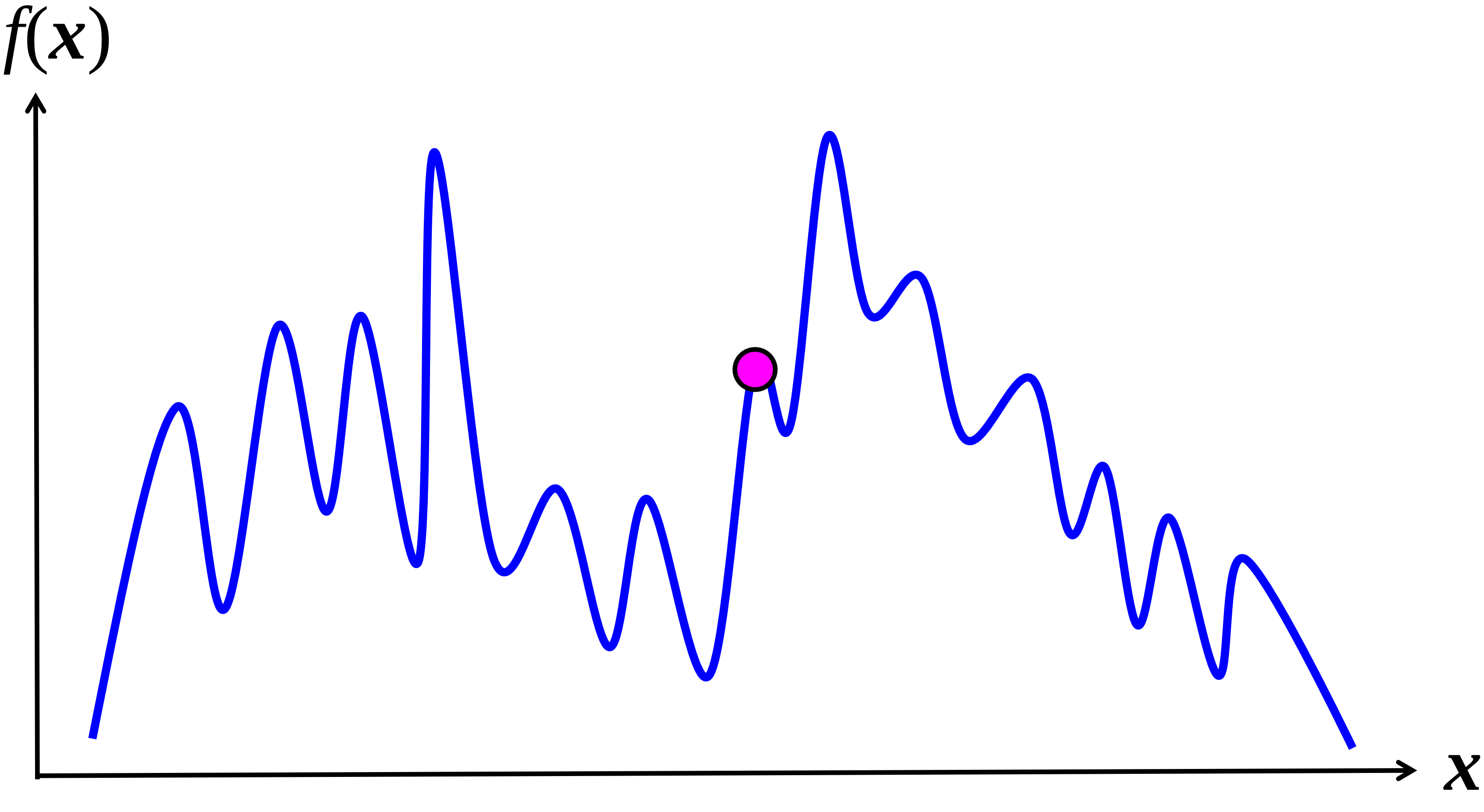
Single-objective Optimization

Single-Objective Optimization: **Maximize** $f(\boldsymbol{x})$



Single-objective Optimization

The final result of optimization is a single solution.
Comparison of solutions is easy.



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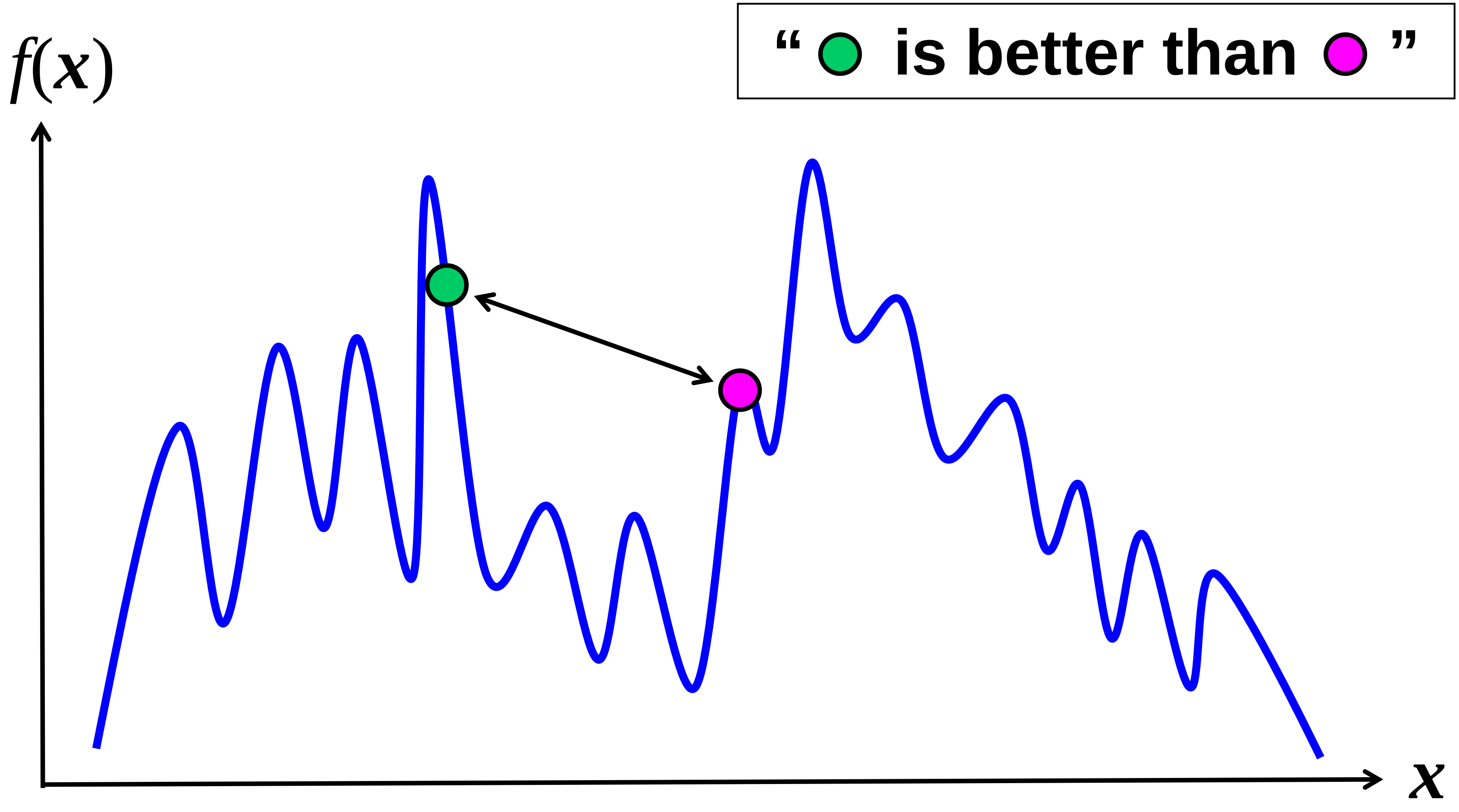
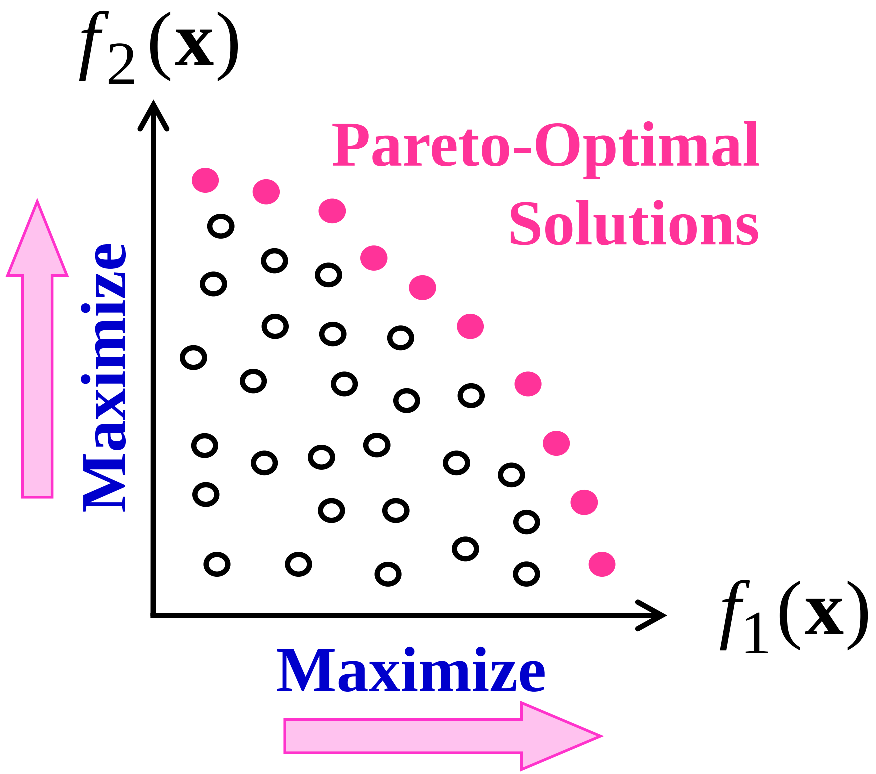


Two-objective Optimization

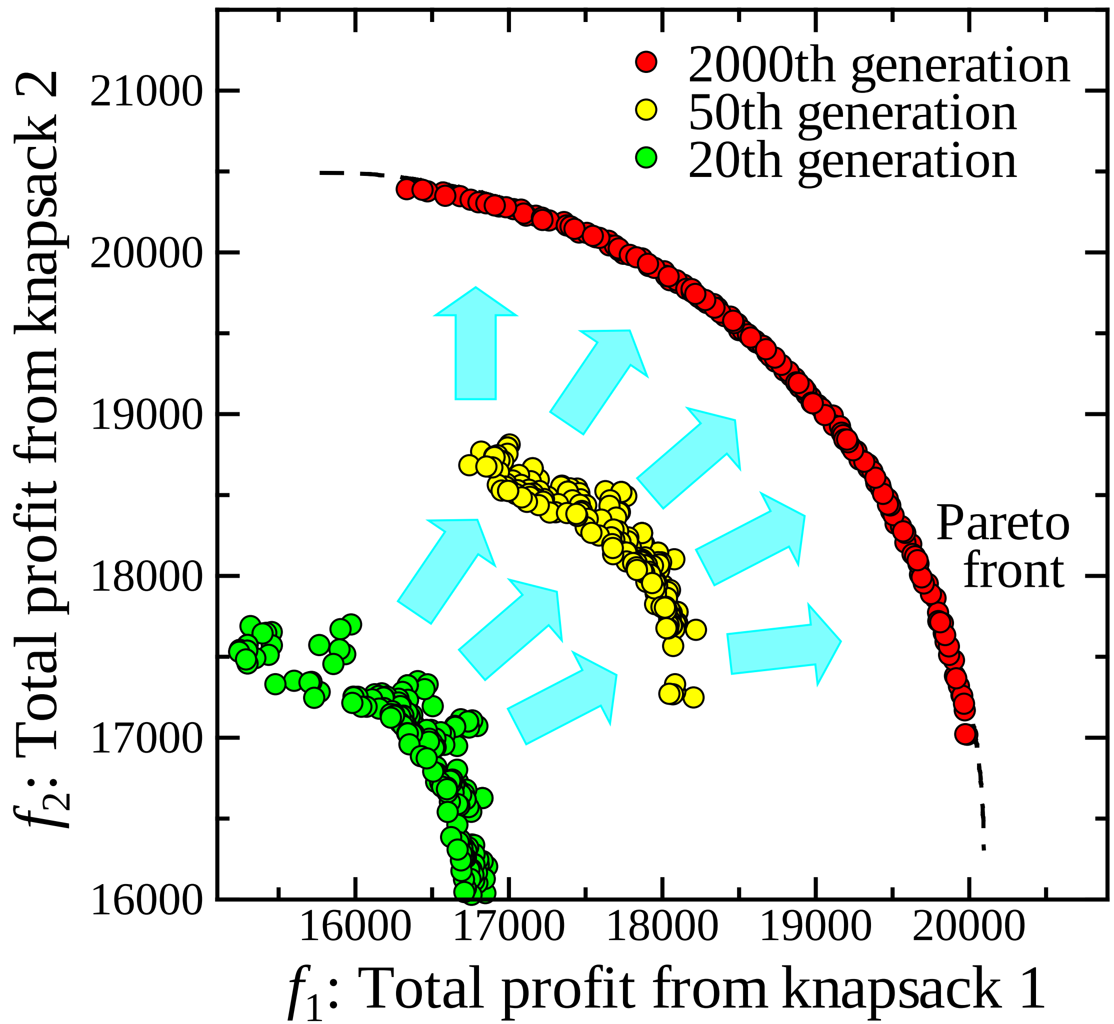
Two-Objective Optimization Problem:

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})$



Two-objective Optimization

The final result of optimization is a solution set.

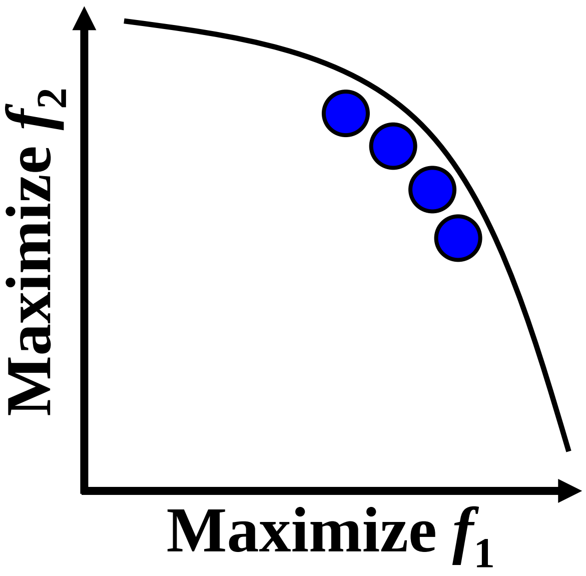


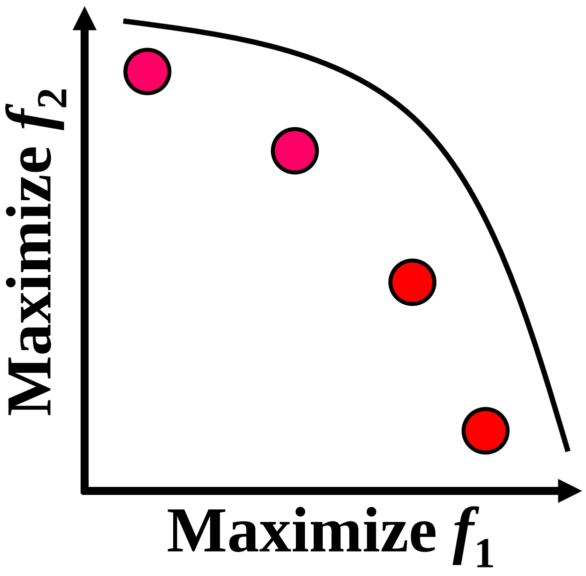
Two-objective Optimization

The final result of optimization is a solution set.

Comparison of solution sets is not easy.

Which is a better solution set?

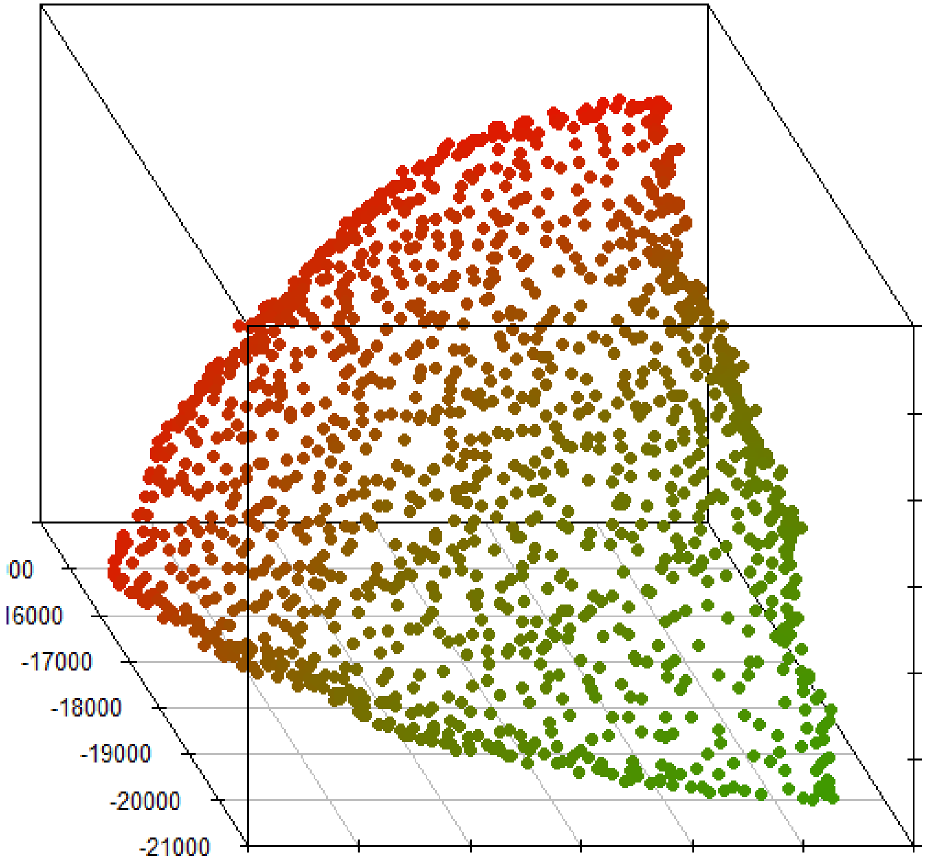




Three-objective Optimization

Three-Objective Optimization Problem:

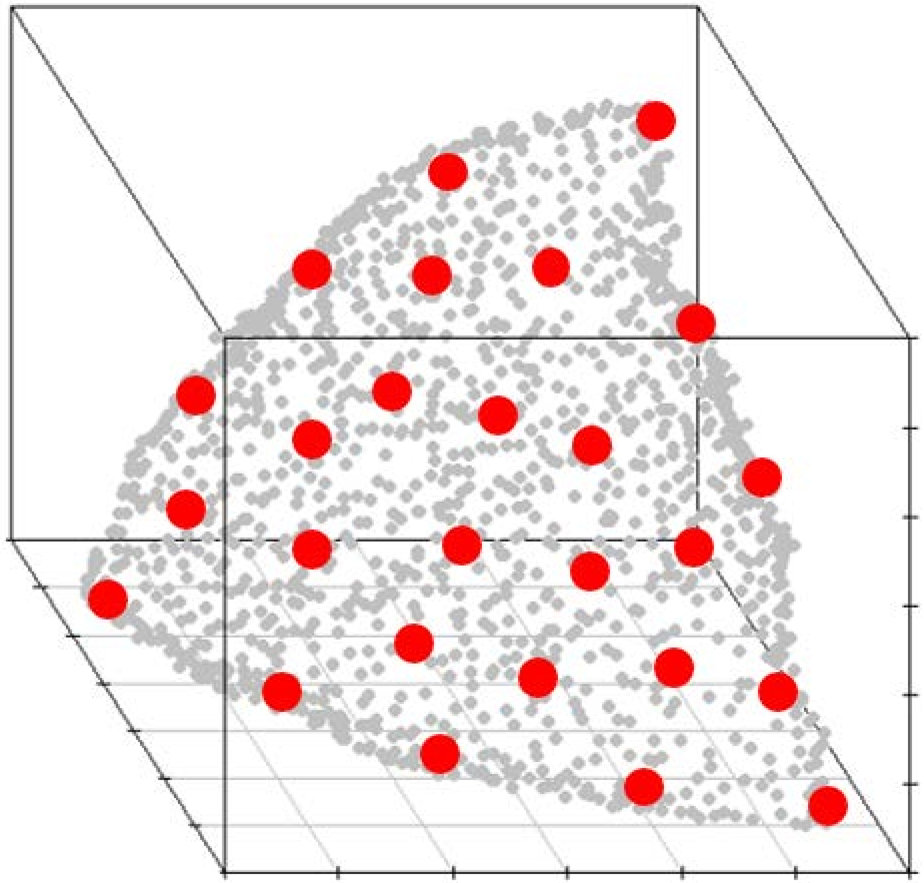
Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x})$



Three-objective Optimization

The final result of optimization is a solution set:

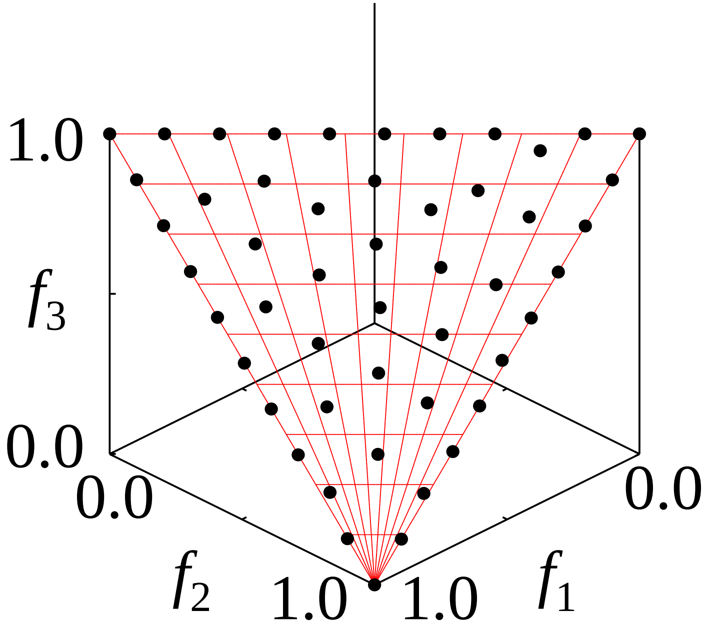
A set of solutions on the tradeoff surface.



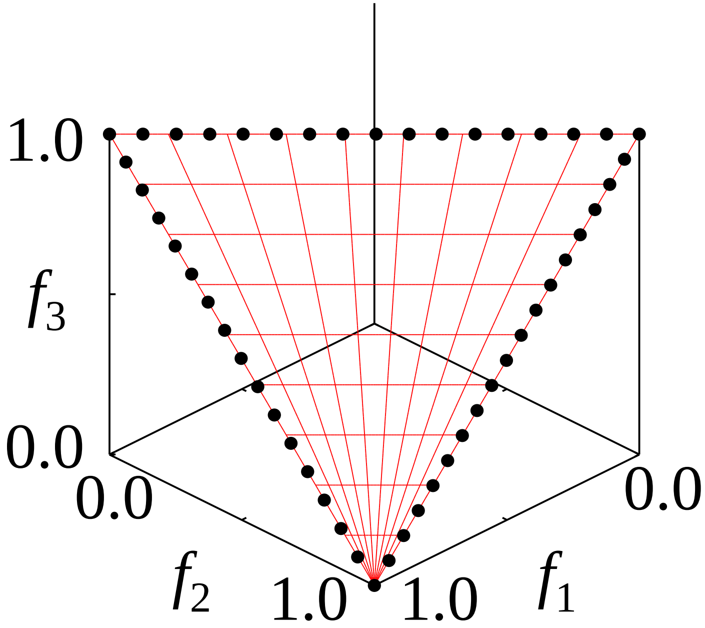
Three-objective Optimization

The final result of optimization is a solution set.
Comparison of solution sets is difficult:

Which is a better solution set?



(a)



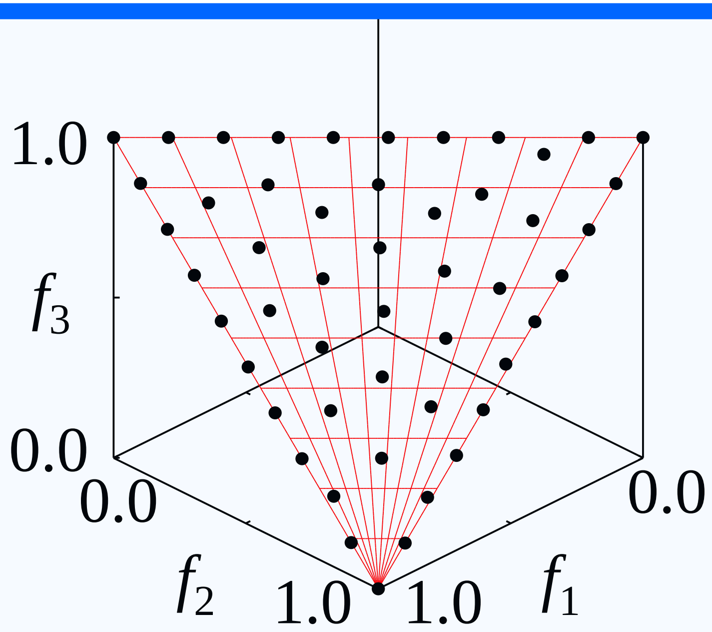
(b)

Three-objective Optimization

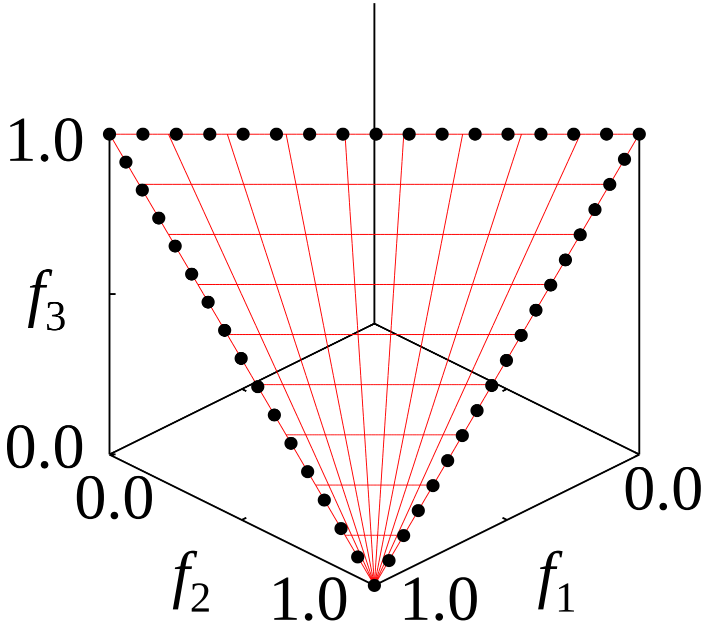
The final result of optimization is a solution set.

Comparison of solution sets is difficult:

Which is a better solution set?



(a) **Better**

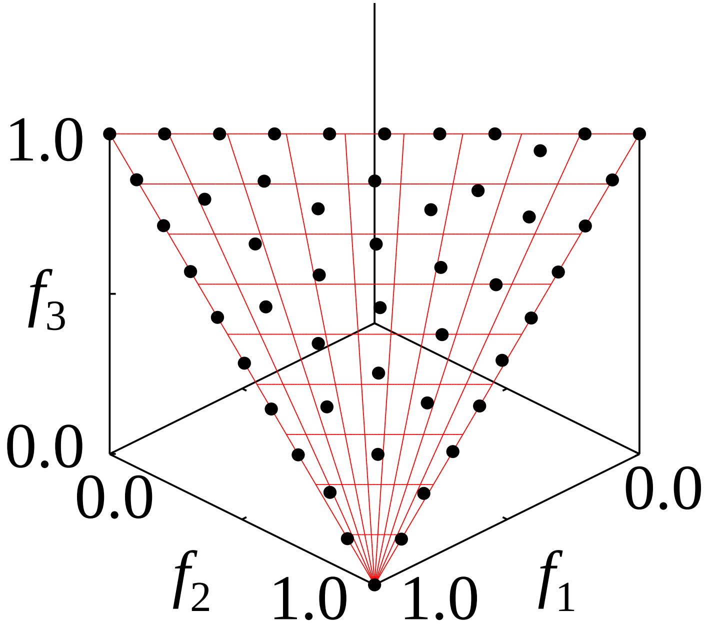


(b)

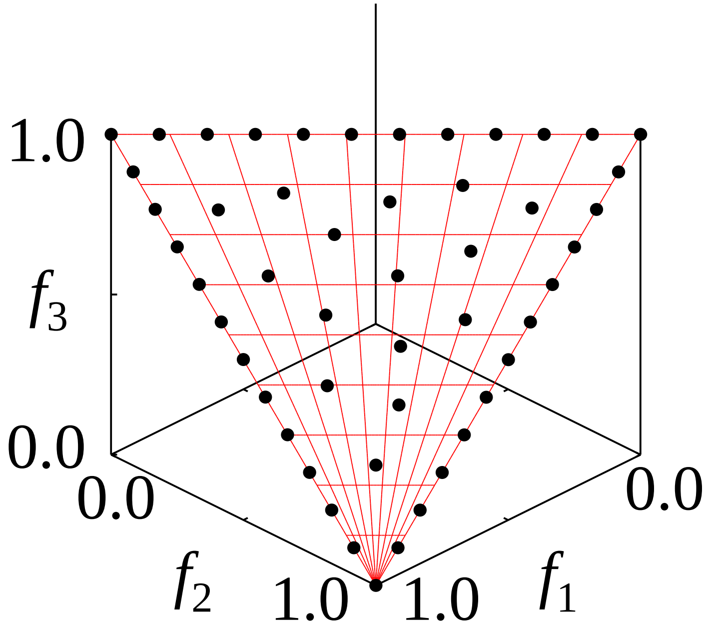
Three-objective Optimization

The final result of optimization is a solution set.
Comparison of solution sets is difficult:

Which is a better solution set?



(a)



(b)

Many-objective Optimization

Single-Objective Optimization: Maximize $f(\boldsymbol{x})$

Multi-Objective Optimization:

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})$

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x})$

Many-Objective Optimization:

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x})$

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x}),\ f_5(\boldsymbol{x})$

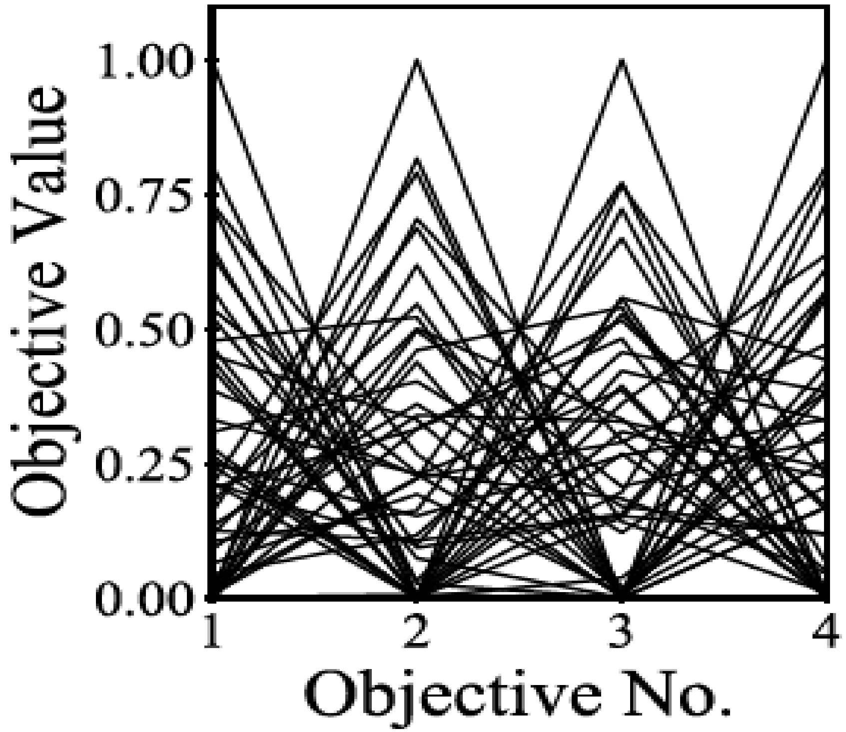
Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x}),\ f_5(\boldsymbol{x}),\ f_6(\boldsymbol{x})$

Four-objective Optimization

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ f_3(\boldsymbol{x}),\ f_4(\boldsymbol{x})$

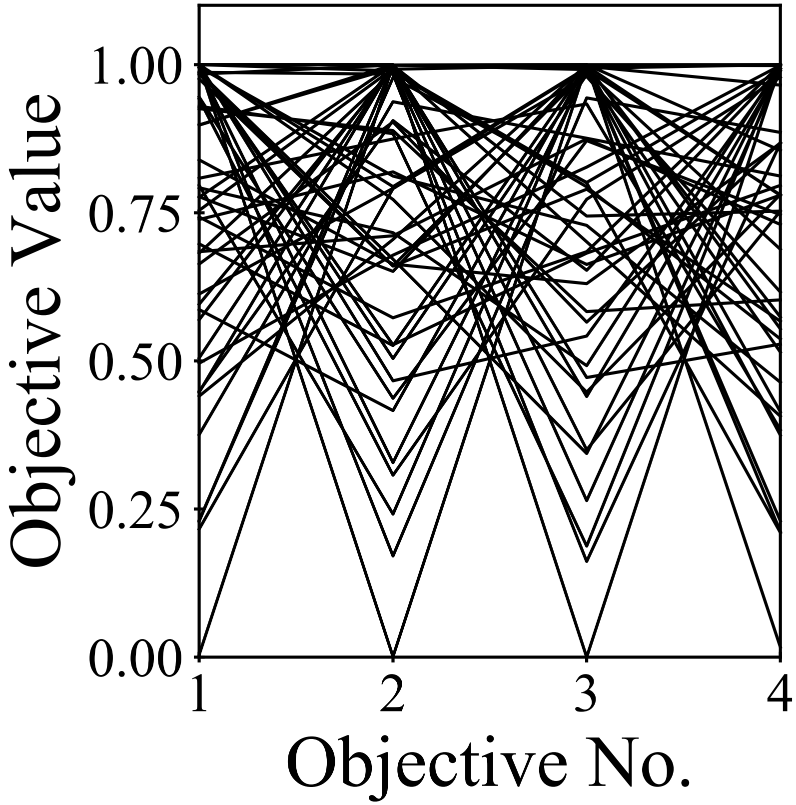
The final result of optimization is a solution set.

Examination of a solution set is not easy.

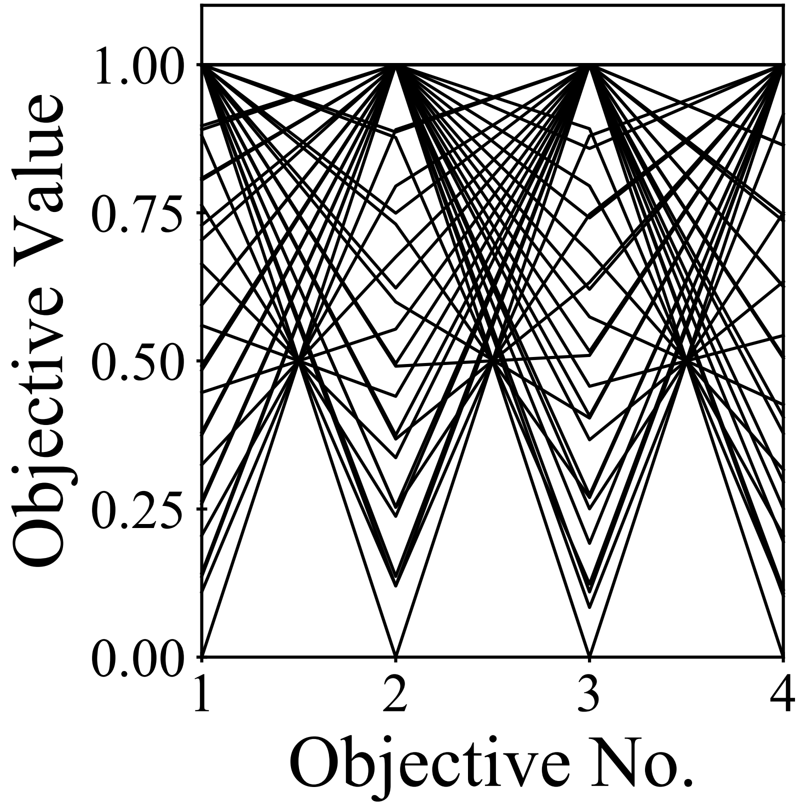


Four-objective Optimization

Which is the better solution set?



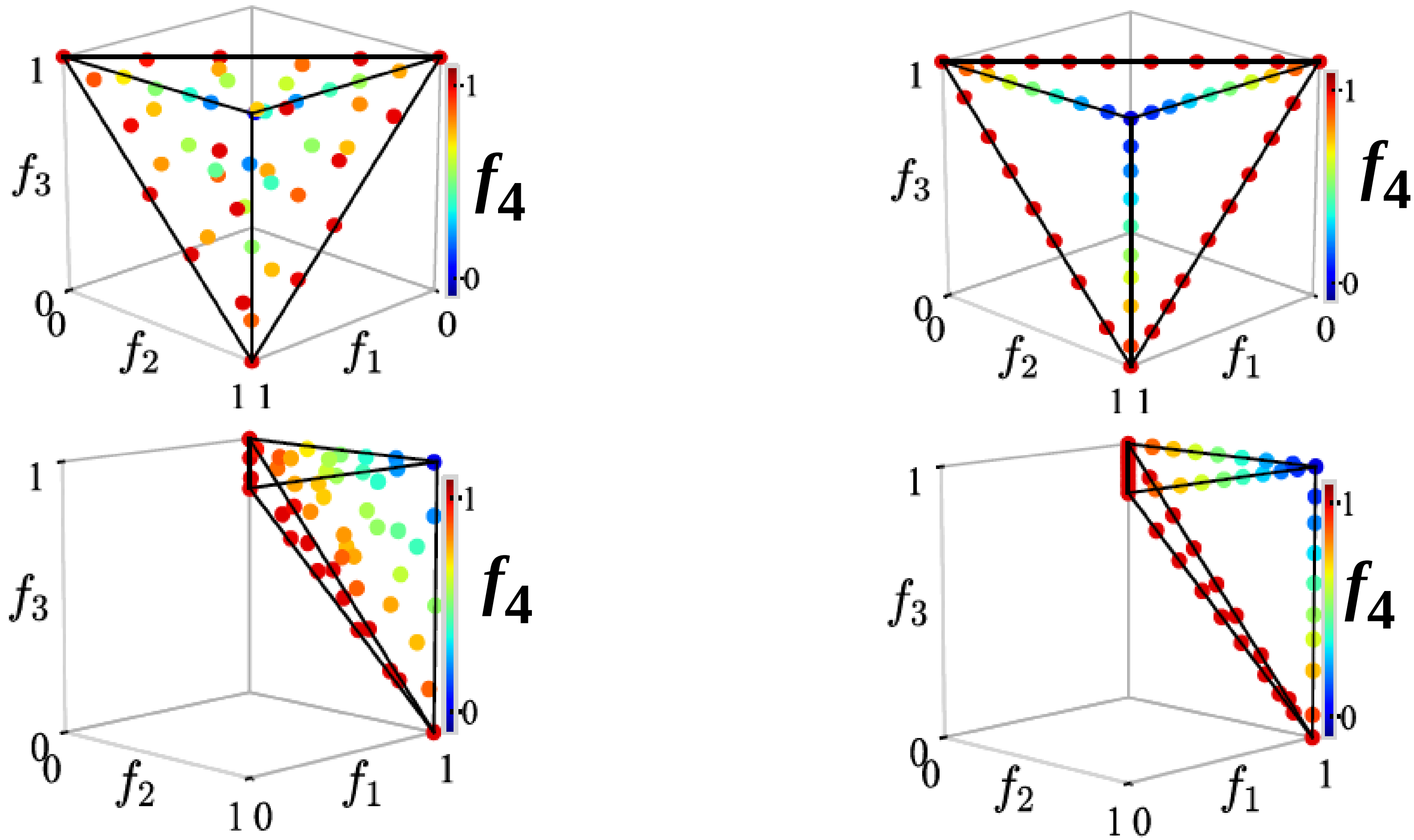
(a)



(b)

Four-objective Optimization

Which is the better solution set?



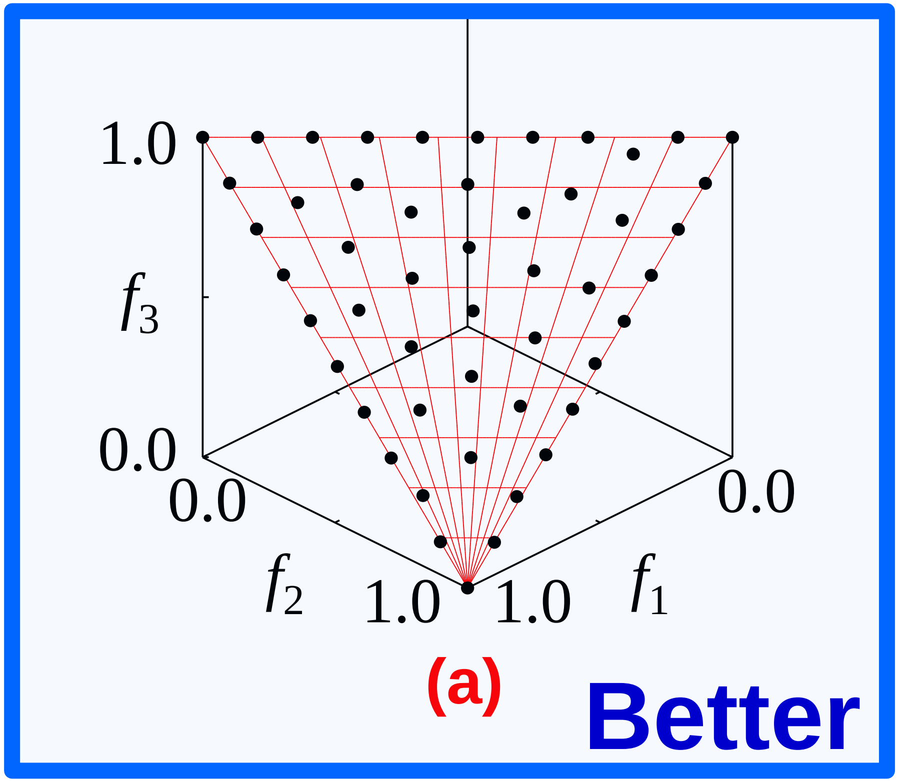
(a) (b)

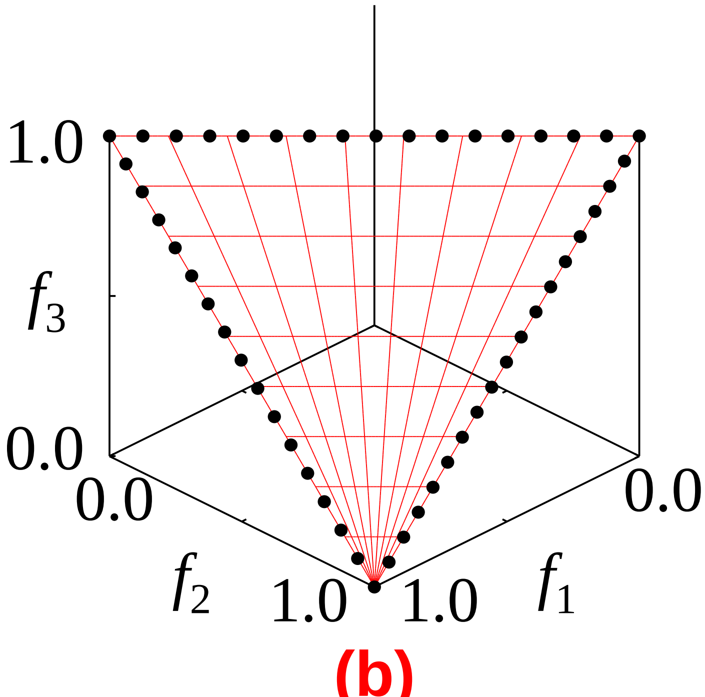
Three-objective Optimization

The final result of optimization is a solution set.

Comparison of solution sets is difficult:

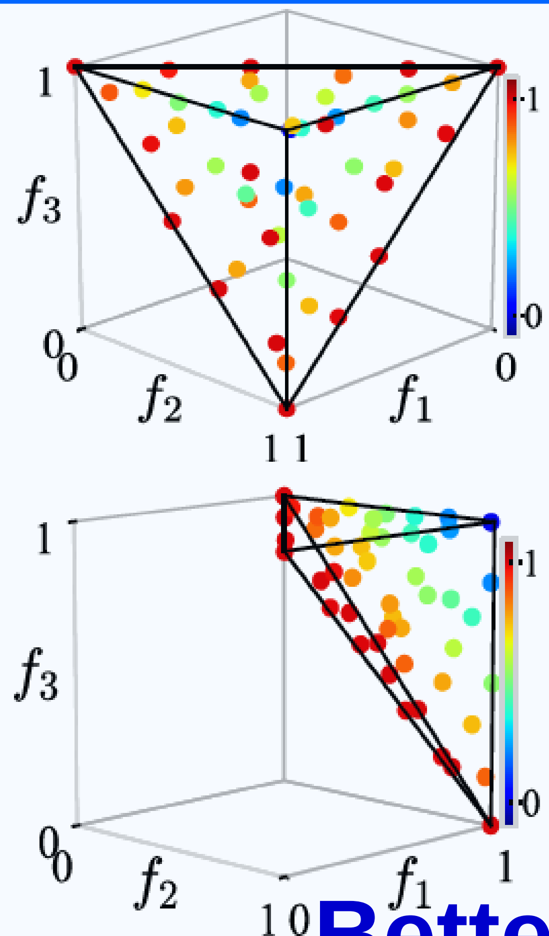
Which is a better solution set?





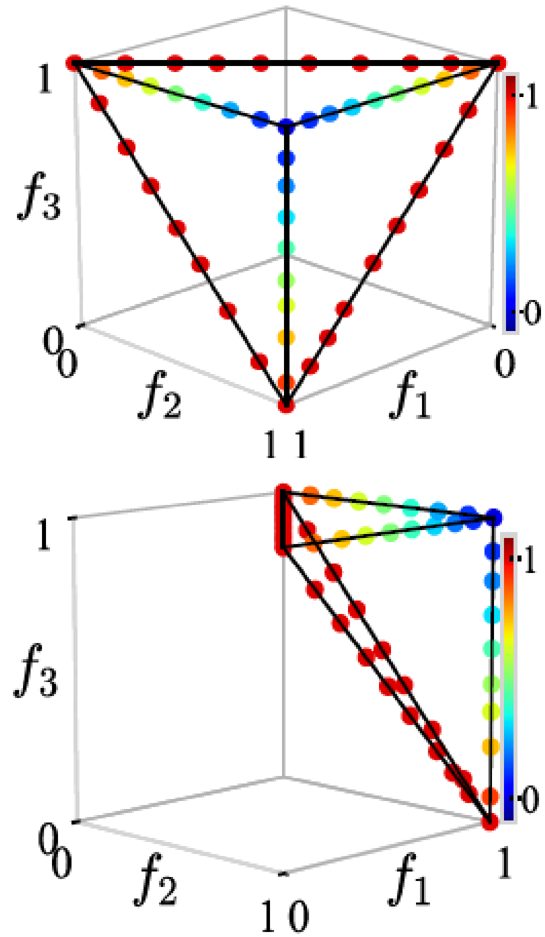
Four-objective Optimization

Which is the better solution set?



Better

(a)



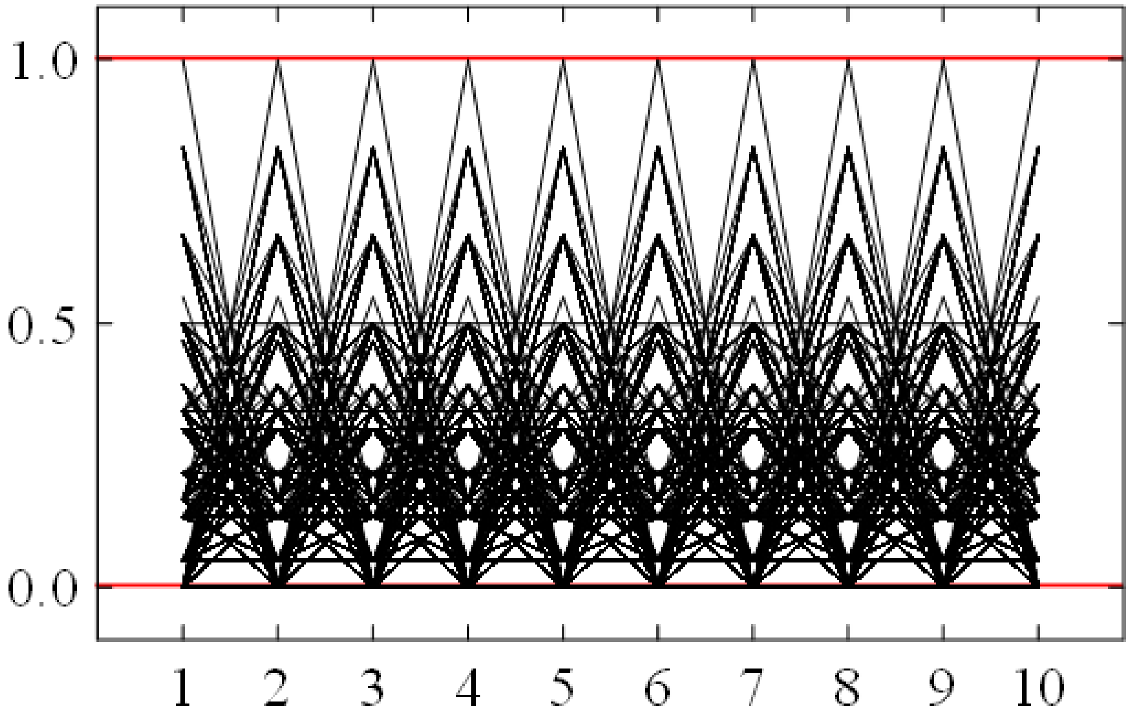
(b)

Ten-objective Optimization

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ ...,\ f_{10}(\boldsymbol{x})$

The final result of optimization is a solution set.

Examination of a solution set is not easy.

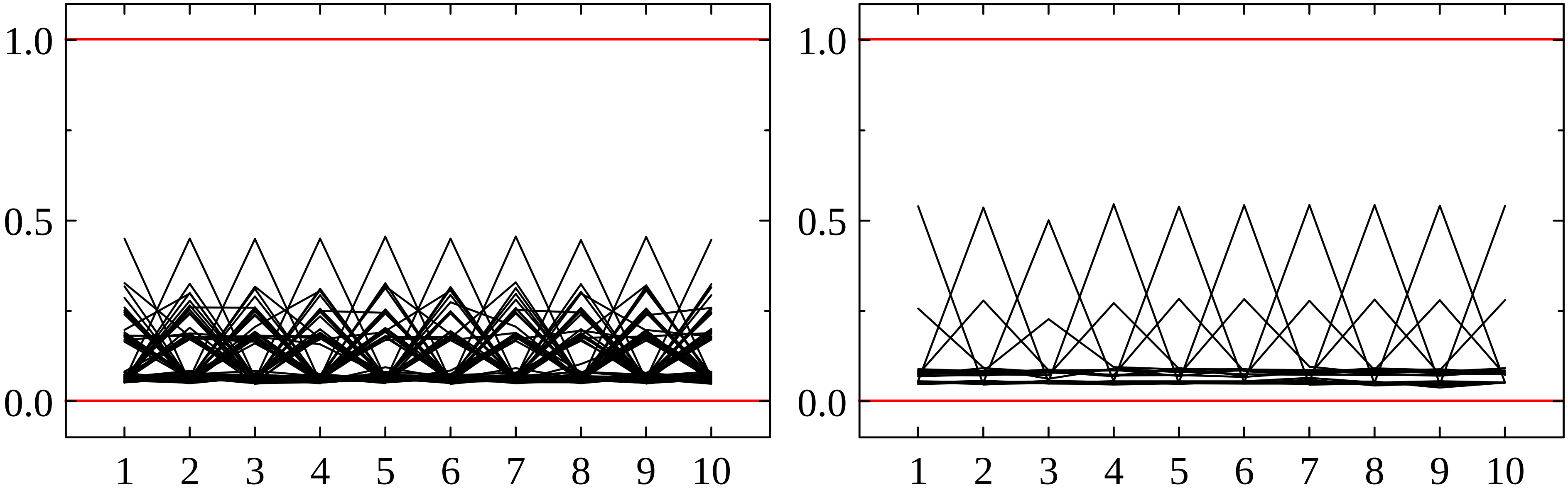


Ten-objective Optimization

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}),\ ...,\ f_{10}(\boldsymbol{x})$

The final result of optimization is a solution set.

Comparison of solution sets is very difficult.



Performance Indicators

Frequently-Used Performance Indicators

1. Hypervolume Indicator

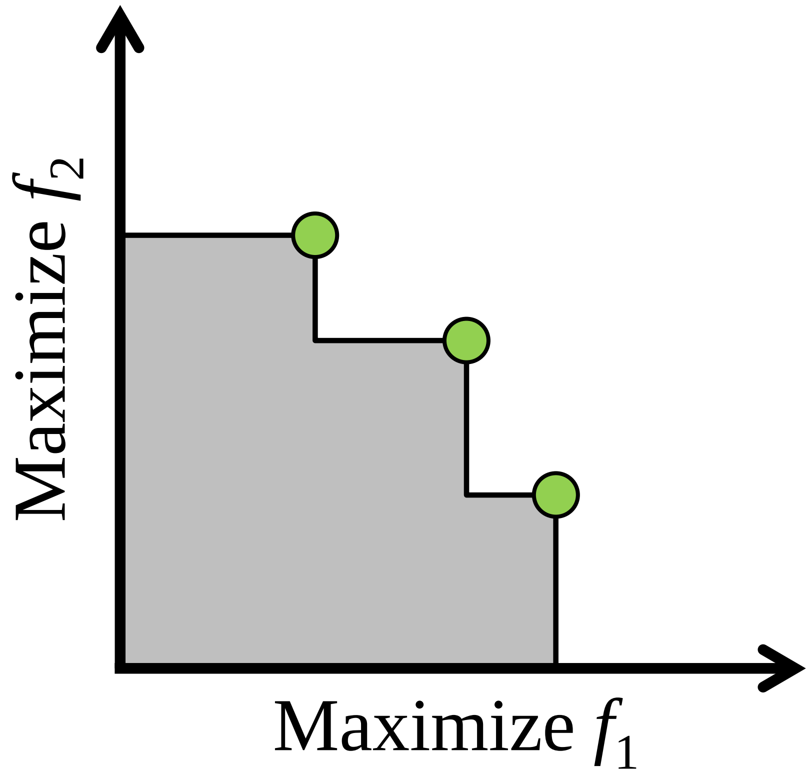
2. IGD (Inverted Generational Distance) Indicator

Property of These Indicators:

By increasing the number of solutions, the evaluation of a solution set by these indicators can be improved.

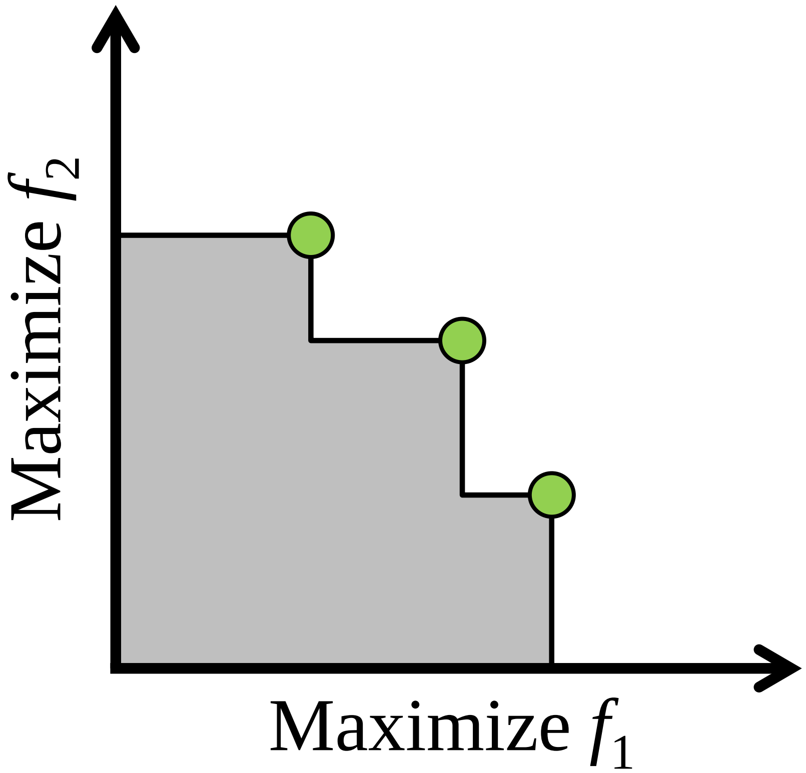
Hypervolume

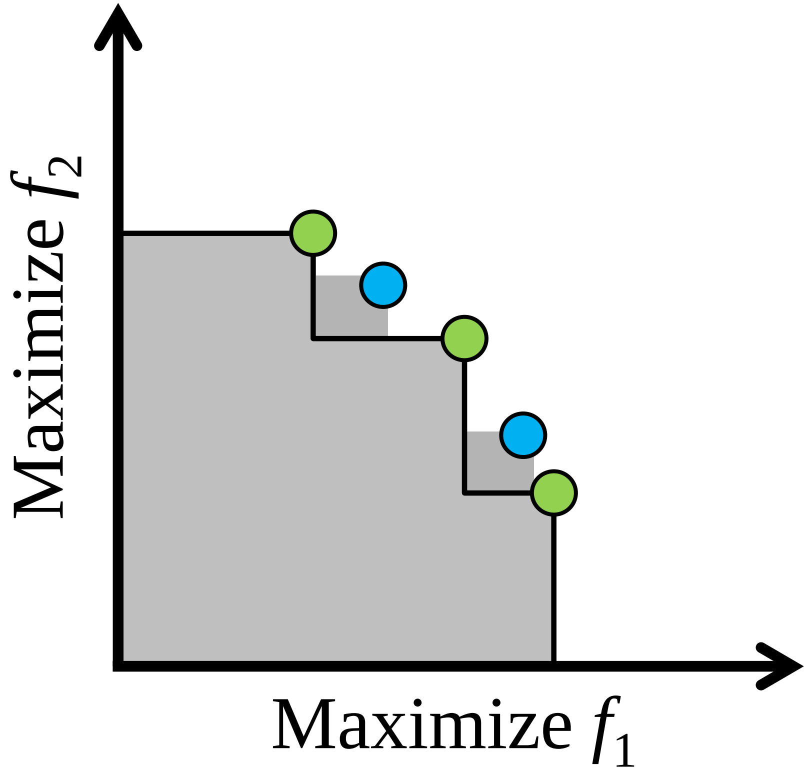
Hypervolume (HV) is the volume of the dominated region by the obtained solutions.



Hypervolume

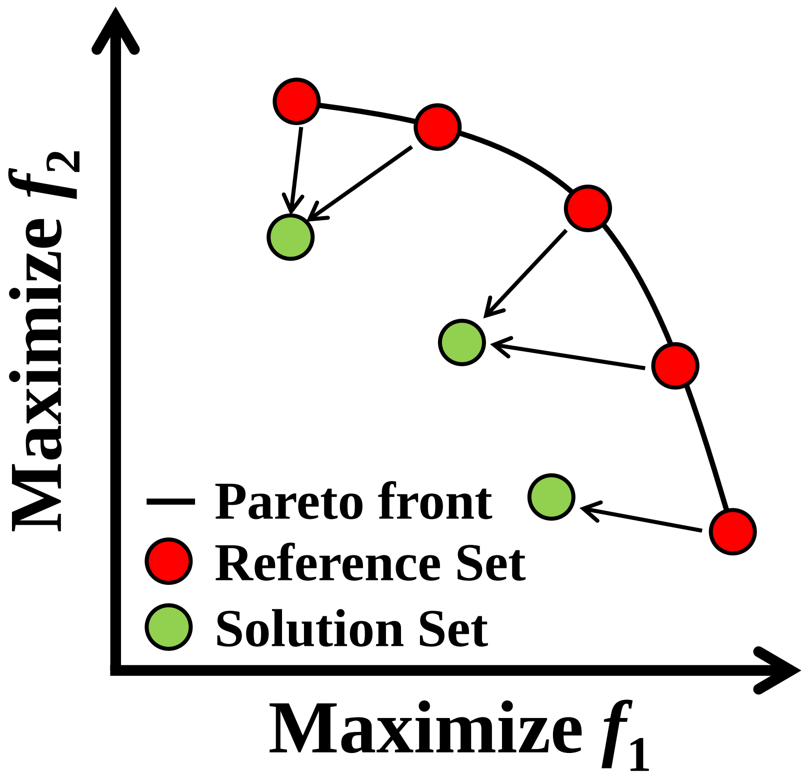
Hypervolume (HV) is the volume of the dominated region by the obtained solutions. **The HV value can can be improved by adding new solutions.**





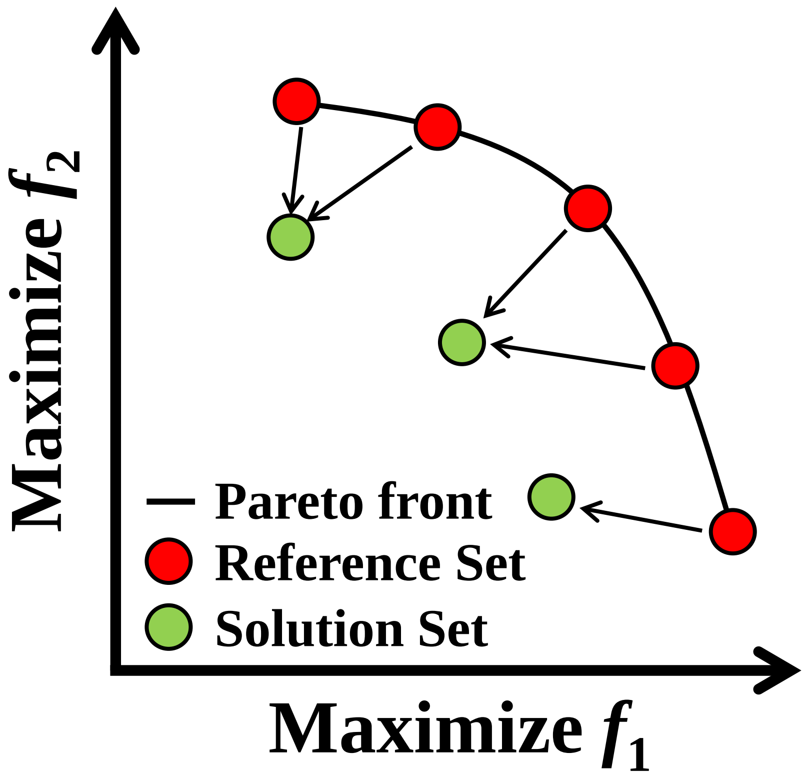
IGD: Inverted Generational Distance

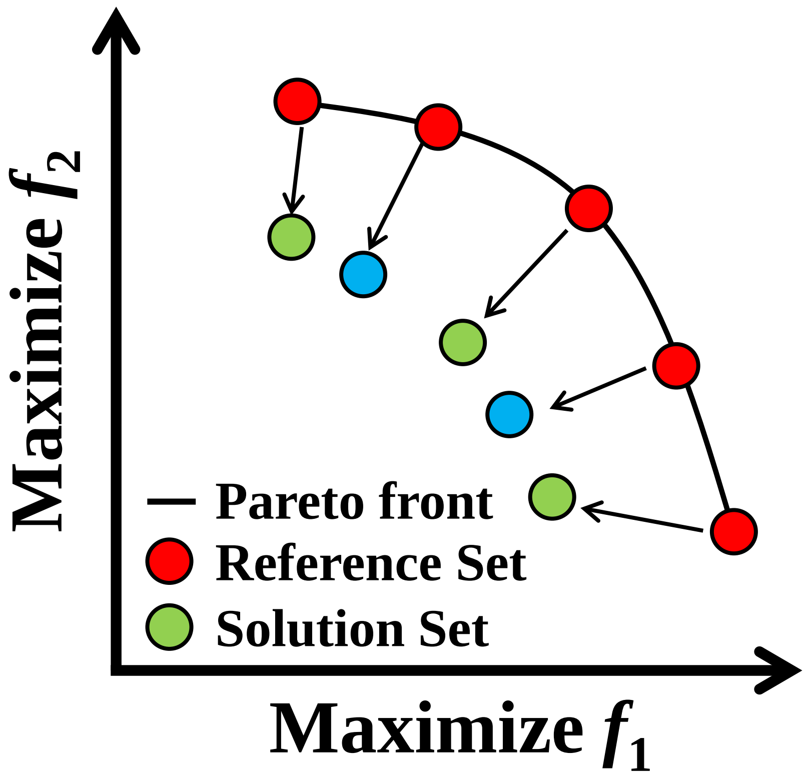
Average distance from each reference point on the Pareto front to the nearest solution.



IGD: Inverted Generational Distance

Average distance from each reference point on the Pareto front to the nearest solution. The IGD value can be improved by adding new solutions.





Specification of Population Size

How about the following settings?

Algorithm A:

Crossover probability: 1.0

Mutation probability: 1/*n* (*n*: string length)

Population size: 5,000

Algorithm B:

Crossover probability: 0.2

Mutation probability: 5/*n* (*n*: string length)

Population size: 50

Specification of Population Size

How about the following settings?

Algorithm A:

Crossover probability: 1.0

Mutation probability: 1/*n* (*n*: string length)

Population size: 5,000

Algorithm B:

Crossover probability: 0.2

Mutation probability: 5/*n*   (*n*: string length)

Population size: 50

Comparison under these settings may be OK for single-objective optimization.

Specification of Population Size

How about the following settings?

Algorithm A:

Crossover probability: 1.0

Mutation probability: 1/*n* (*n*: string length)

Population size: 5,000

Algorithm B:

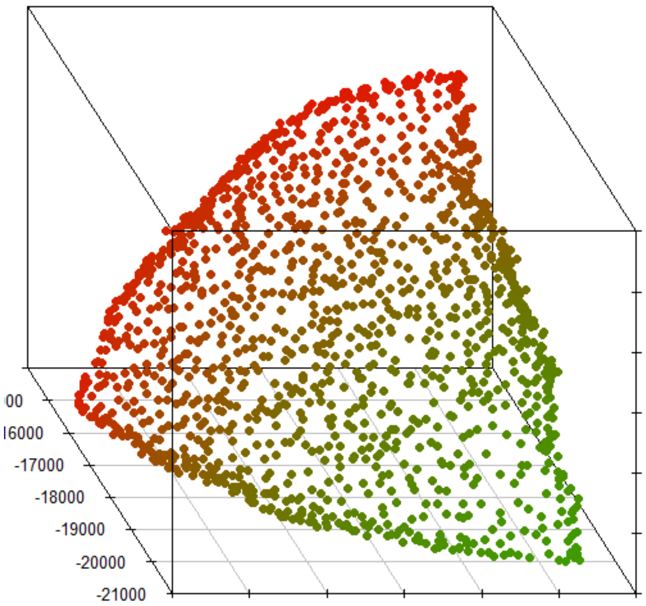
Crossover probability: 0.2

Mutation probability: 5/*n* (*n*: string length)

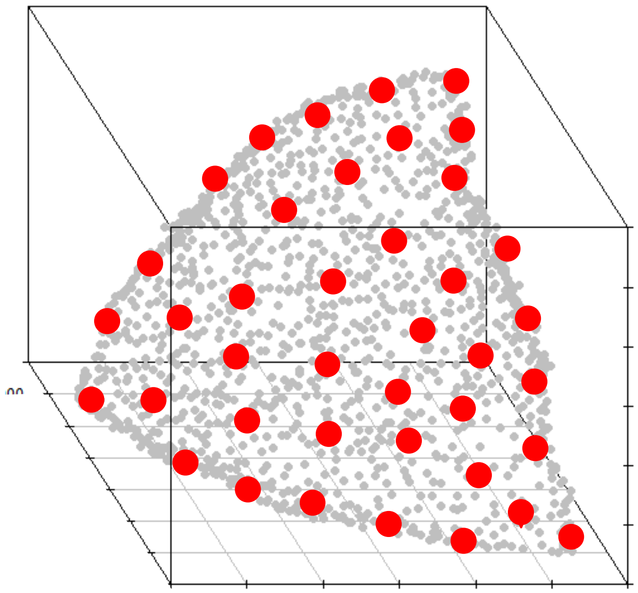
Population size: 50

Comparison under these settings may be OK for single-objective optimization. However, for multi-objective optimization, ...

Obtained Solution Sets



Algorithm A



Algorithm B

Specification of Population Size

How about the following settings?

Algorithm A:

Crossover probability: 1.0

Mutation probability: 1/n (n: string length)

Population size: 5,000

Algorithm B:

Crossover probability: 0.2

Mutation probability: 5/n (n: string length)

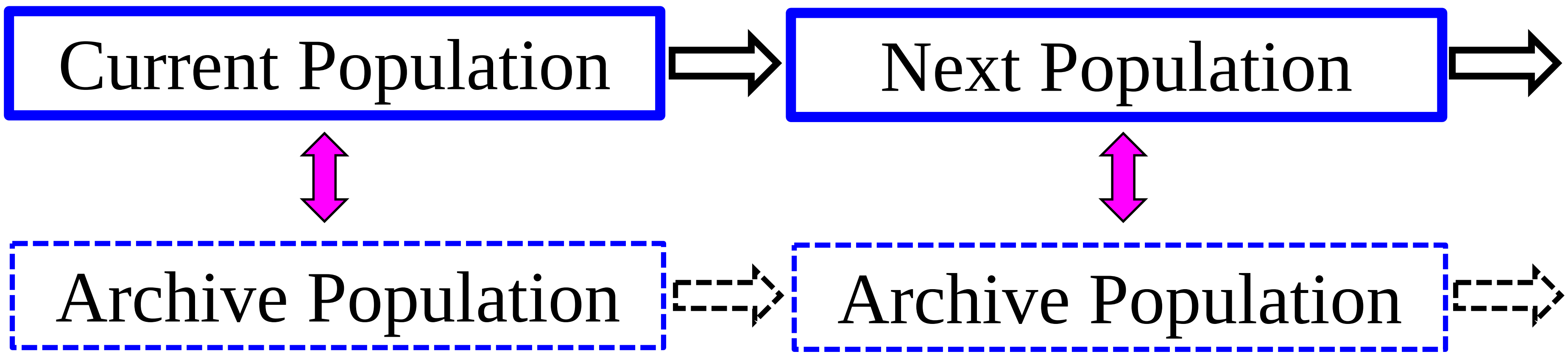
Population size: 50

The comparison may be unfair.

Question

How to compare EMO algorithms with/without an archive population?

Some algorithms have an archive population whereas others do not have.



How to Compare Different Algorithms

Algorithm A:

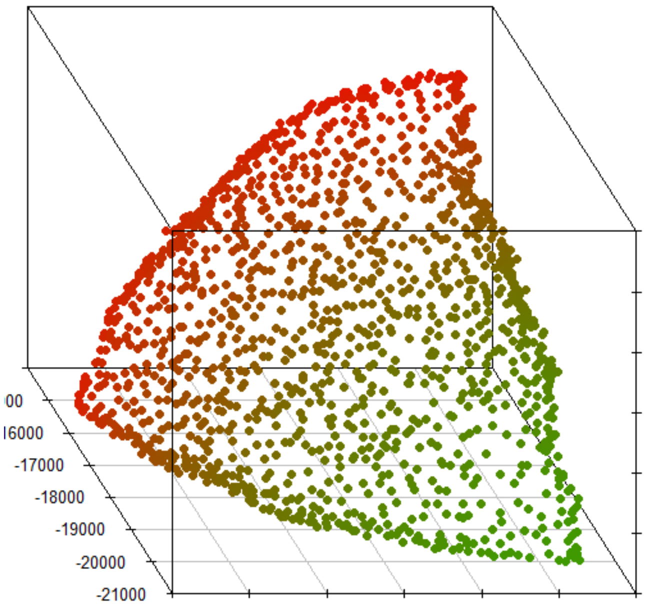
- Crossover probability: 1.0
- Mutation probability: 1/n (n: string length)
- Population size: 100
- Size of Archive Population: 1,000

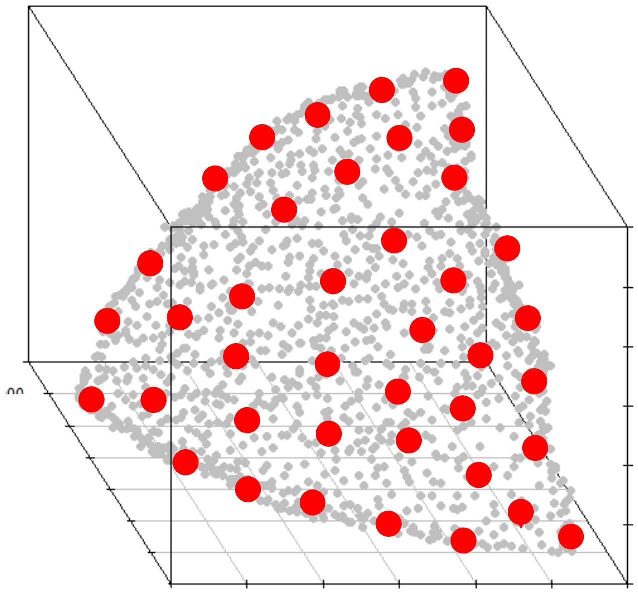
Algorithm B:

- Crossover probability: 0.2
- Mutation probability: 5/n   (n: string length)
- Population size: 100
- No Archive Population

The comparison may be unfair.

Obtained Solution Sets





Algorithm A

Algorithm B

Our Idea (CEC 2016): Solution selection from all the examined solutions

Algorithm A:

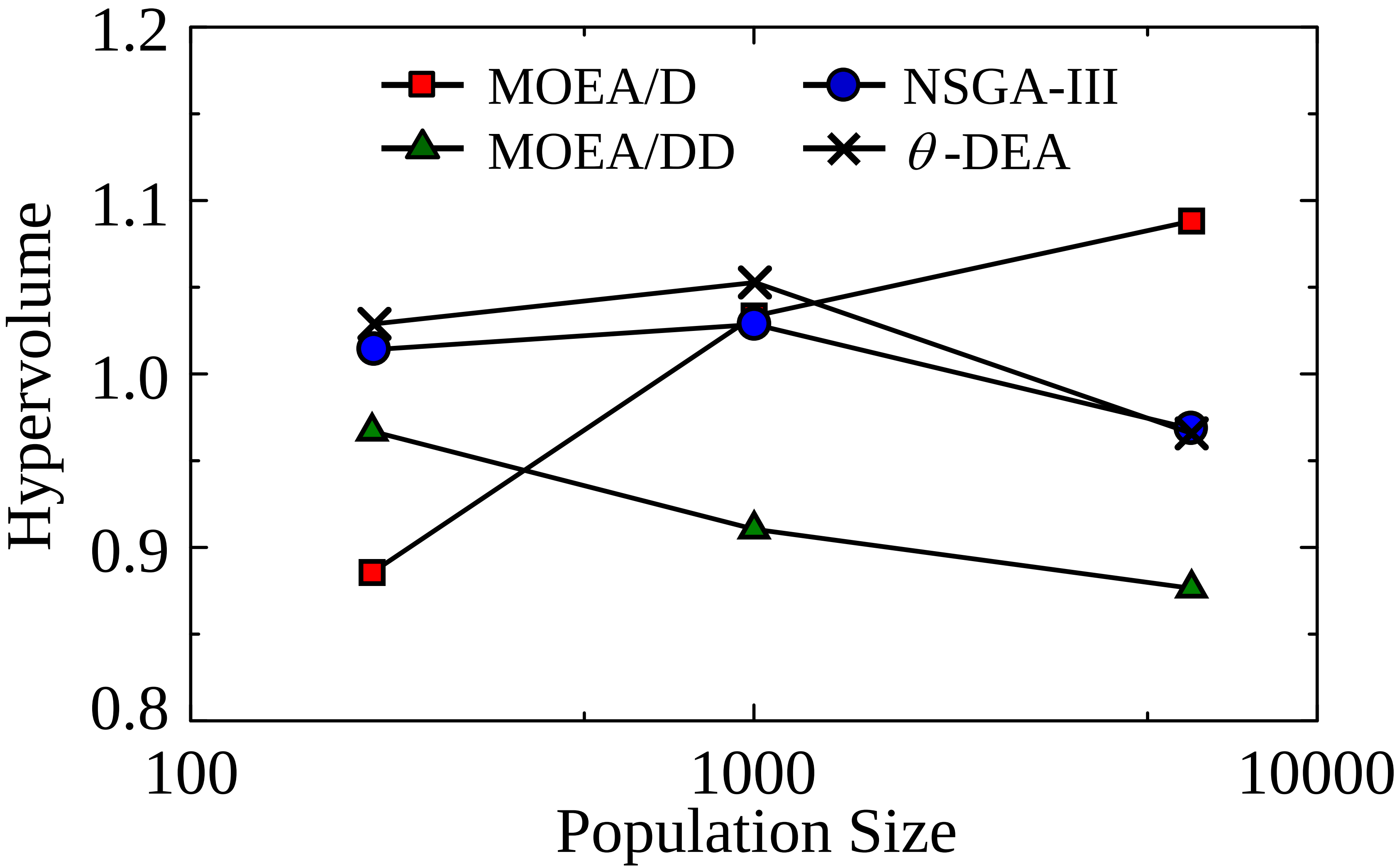
- Crossover probability: 1.0
- Mutation probability: 1/n (n: string length)
- Population size: 100
- Size of Archive Population: 1,000

Algorithm B:

- Crossover probability: 0.2
- Mutation probability: 5/n (n: string length)
- Population size: 100
- No Archive Population

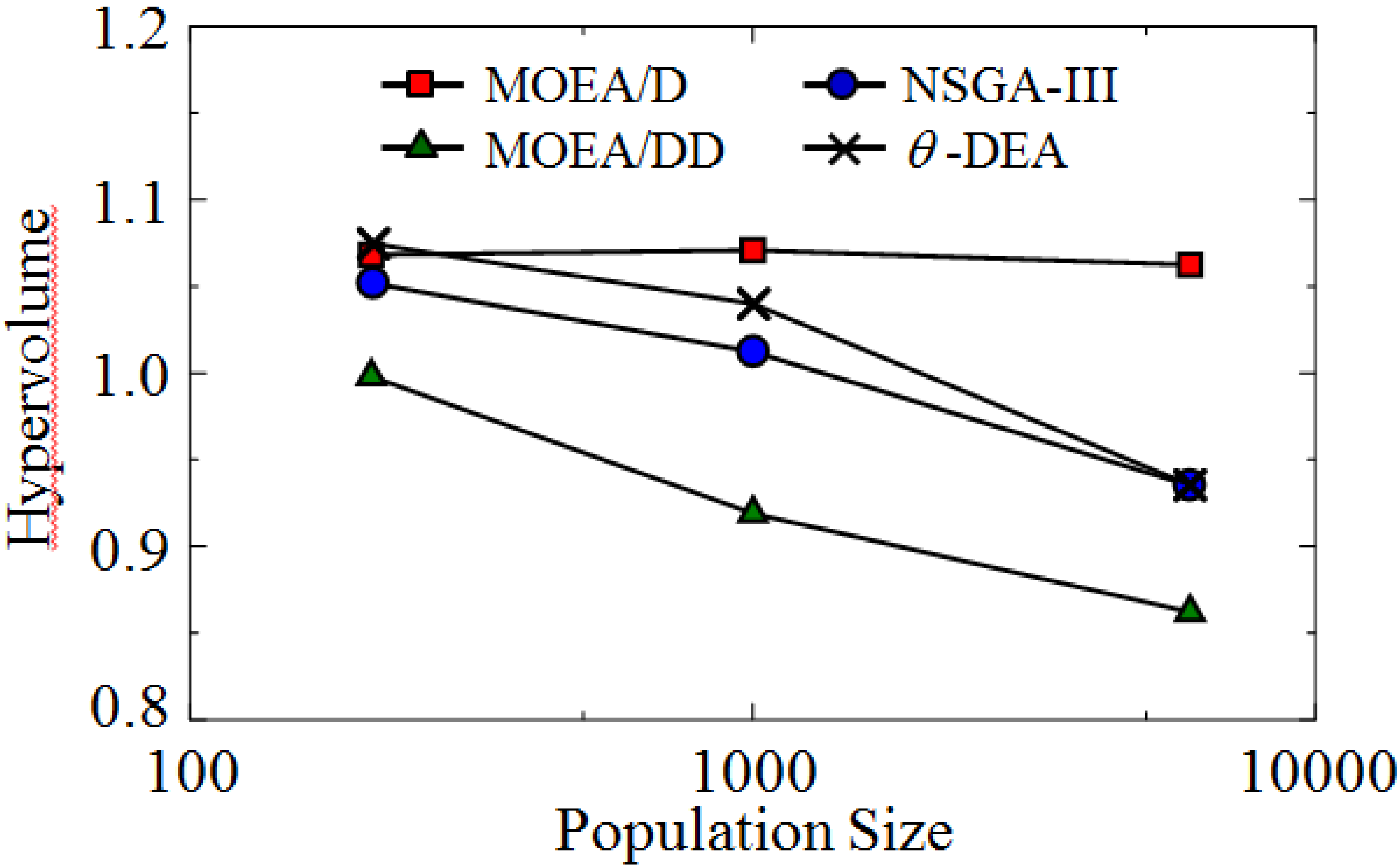
The comparison may be unfair ==> Solution selection from all the examined solutions.

Performance of the Final Population Five-Objective WFG3



Performance of the final population of different size

Selection of 50 Solutions
from all the Examined Solutions



Performance of the selected 50 solutions

Performance Comparison using Solution Selection Methods

R. Tanabe, H. Ishibuchi, and A. Oyama, "Benchmarking multi- and many-objective evolutionary algorithms under two optimization scenarios," IEEE Access, Dec 2017.

Two Optimization Scenarios:

(i) Use of the final population

(ii) Use of selected solutions from the examined solutions

Observation: Performance comparison results are different between the two optimization scenarios.

Difficulties in Performance Evaluation

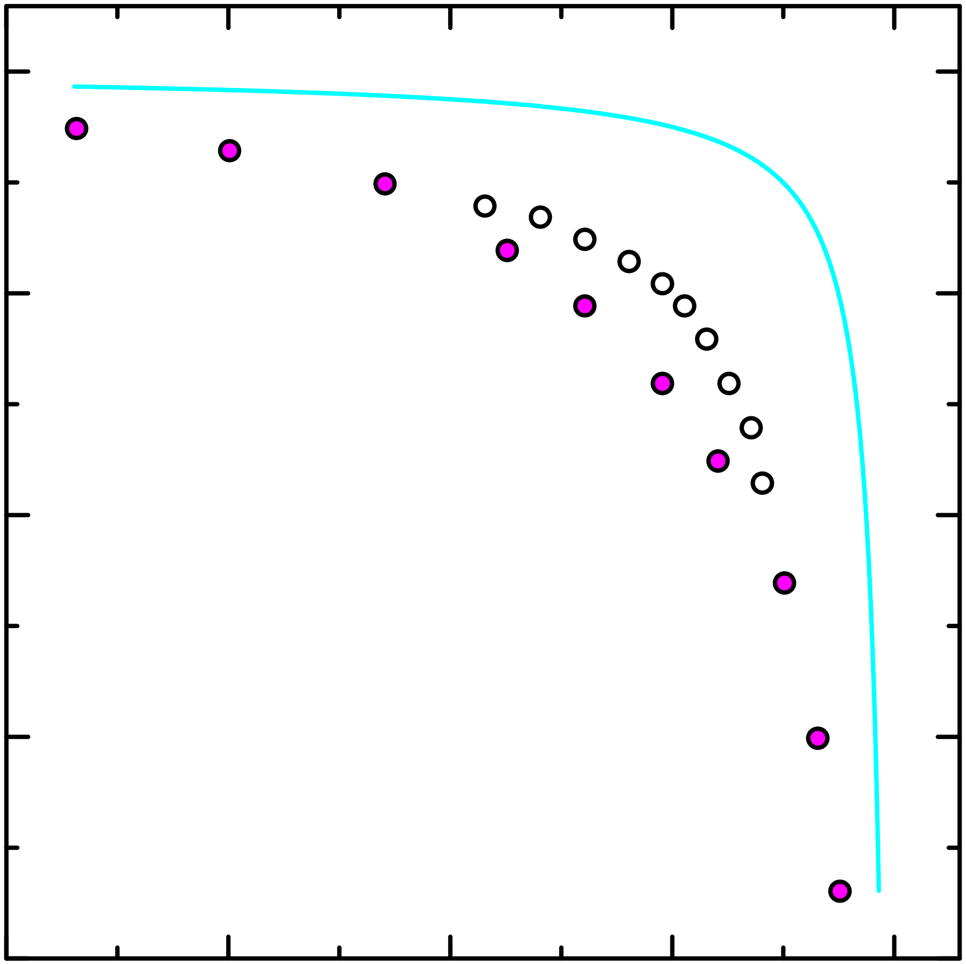
1. **How to Specify the Population Size**
2. **How to Specify the Reference Point for HV**
3. **How to Specify the Reference Points for IGD**

[1] H. Ishibuchi et al., Reference point specification in hypervolume calculation for fair comparison and efficient search, *Proc. of GECCO 2017*, pp. 585-592. (Proposal of the Basic Idea)

[2] H. Ishibuchi et al., How to specify a reference point in hypervolume calculation for fair performance comparison," *Evolutionary Computation* (2018). (Extended Journal Version)

Two Solution Sets:

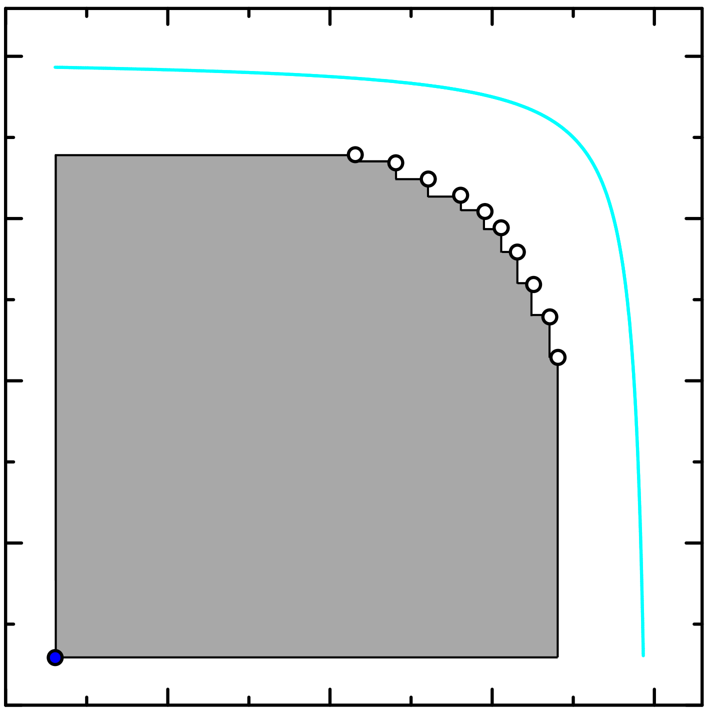
Which has the larger hypervolume?



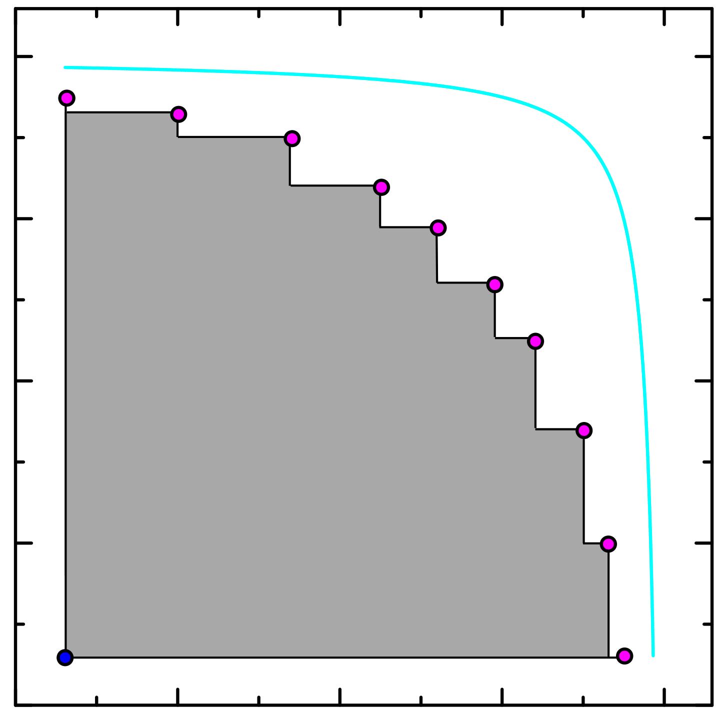
Hypervolume (HV)

Comparison results depends on the reference point

When the reference point is close to the Pareto front:



>

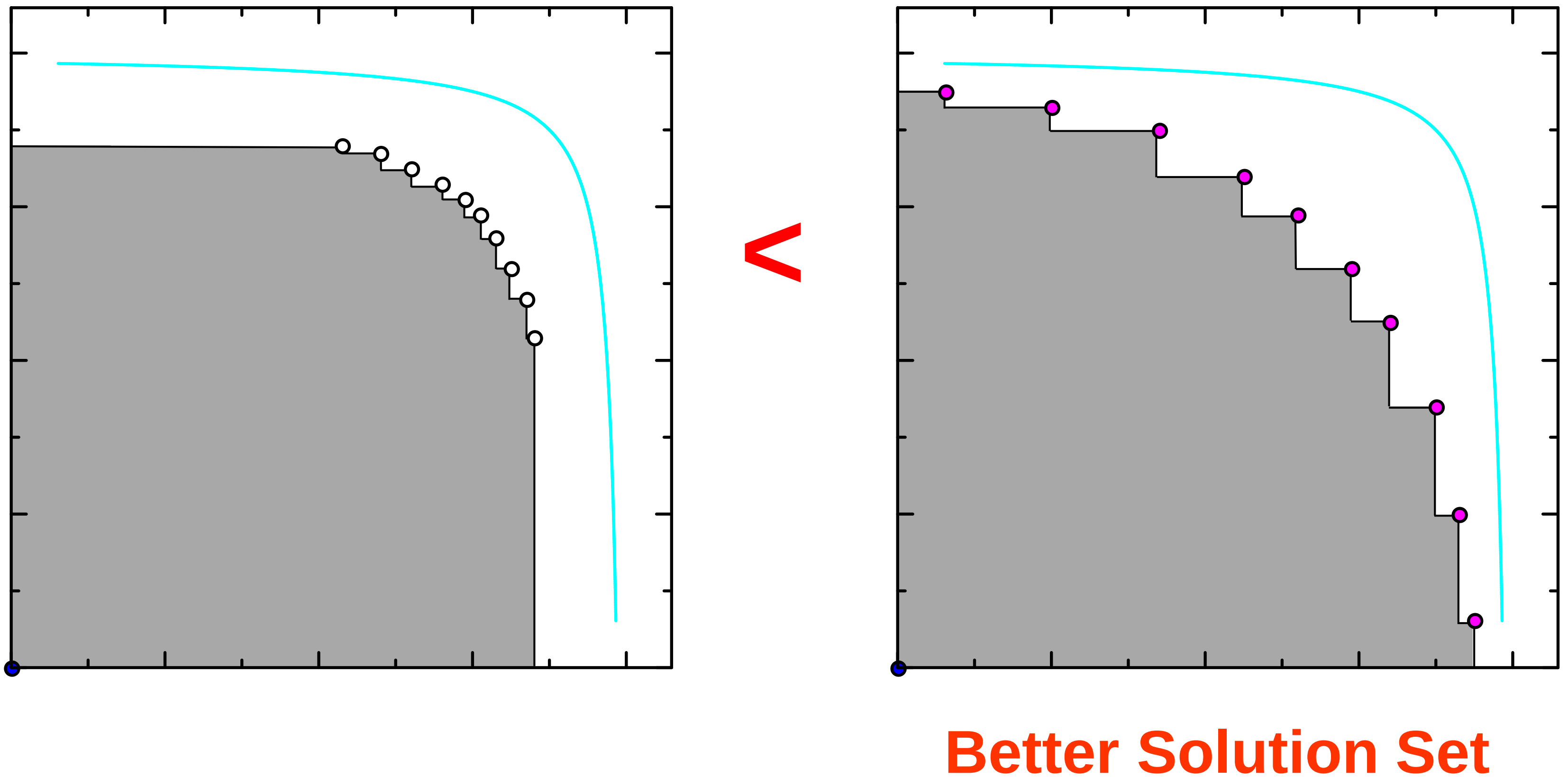


Better Solution Set

Hypervolume (HV)

Comparison results depends on the reference point

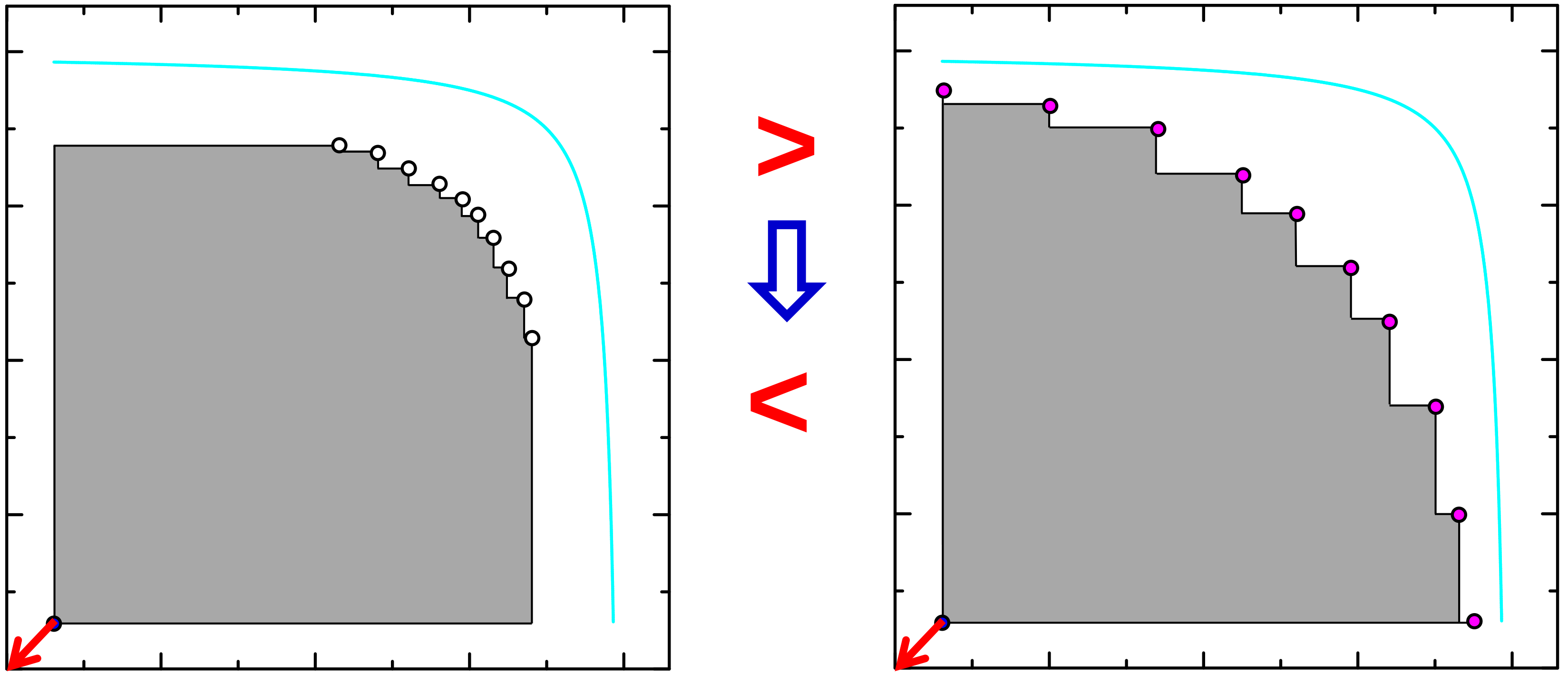
When the reference point is far from the Pareto front:



Hypervolume (HV)

Comparison results depends on the reference point

A small move can change the comparison result.

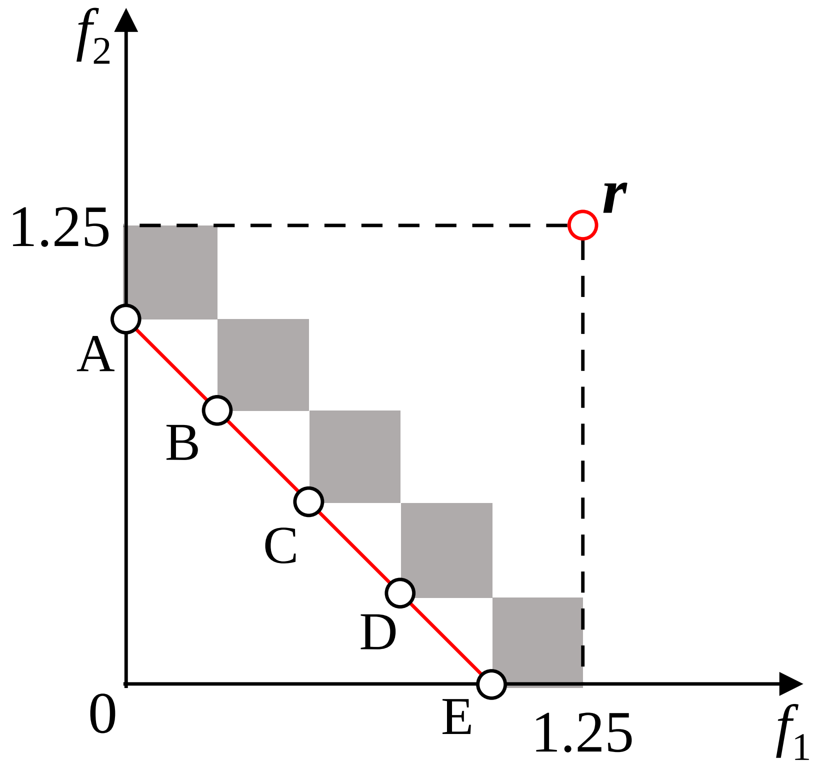


Better Solution Set

How to Specify the Reference Point?
Ishibuchi et al. GECCO 2017, EC Journal

To specify a reference point so that no solution in a uniformly obtained solution set has a dominant effect.

➔ All solutions have the same (similar) HV contribution.

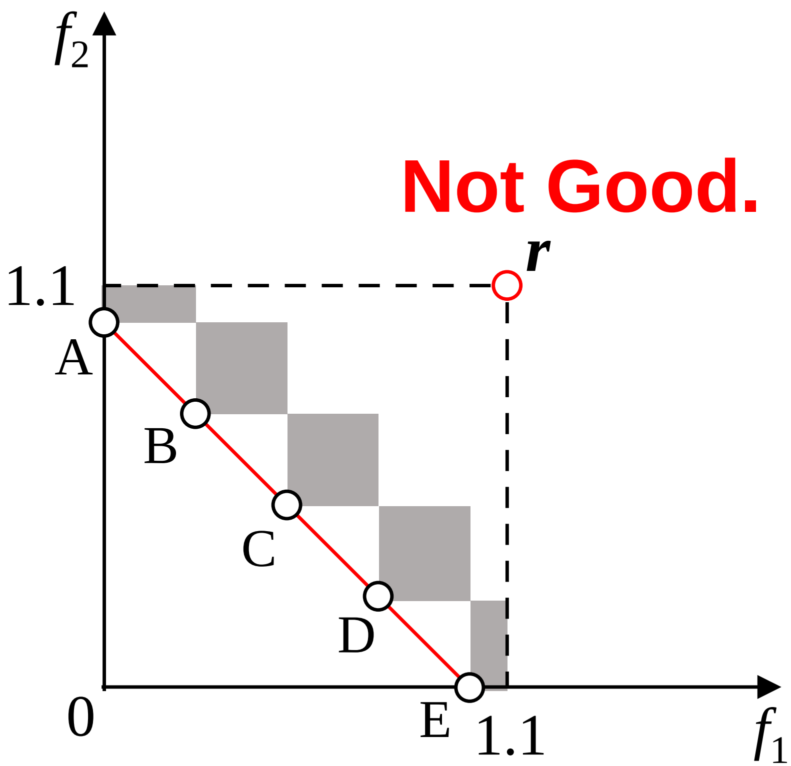


Proposed Specification

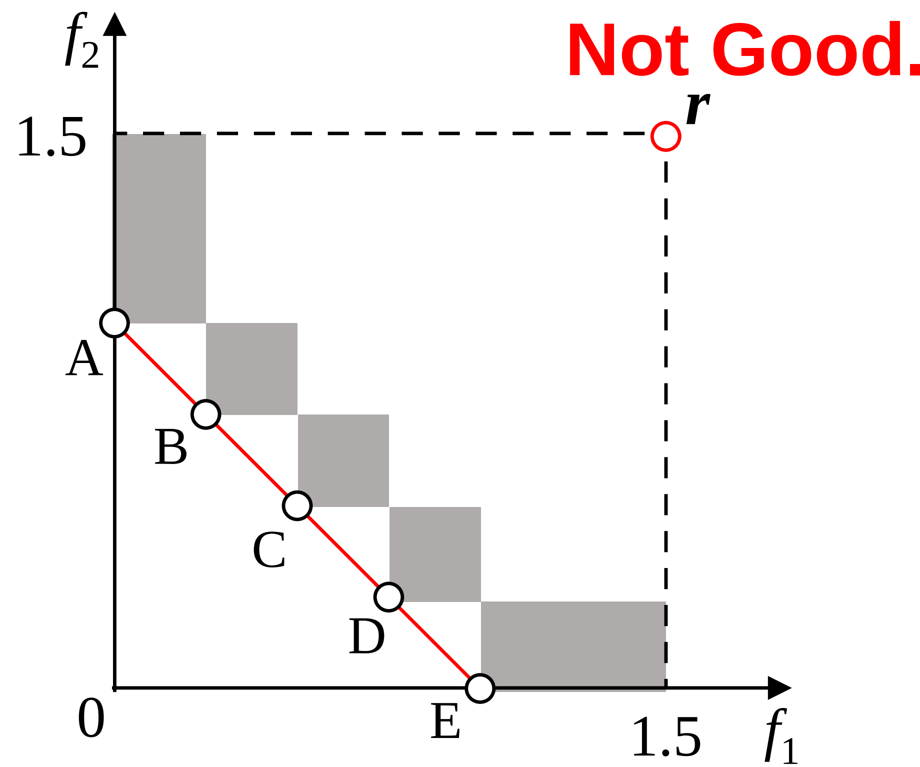
Proposed Idea: Basic Idea

To specify a reference point so that no solution in a uniformly obtained solution set has a dominant effect.

➔ All solutions have the same (similar) HV contribution.



Too close to the Pareto front

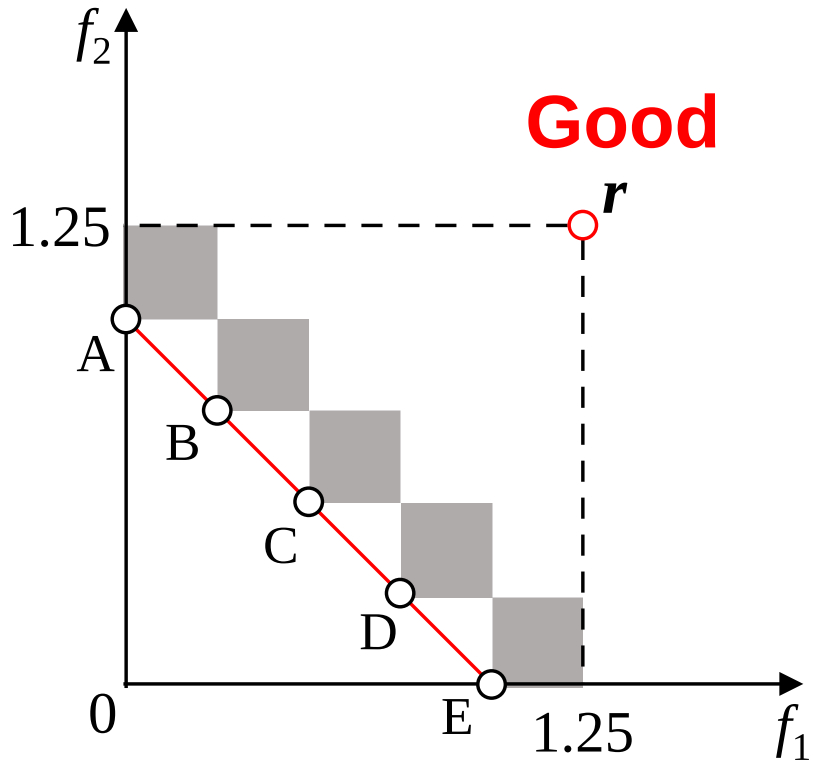


Too far from the Pareto front

Proposed Idea: Basic Idea

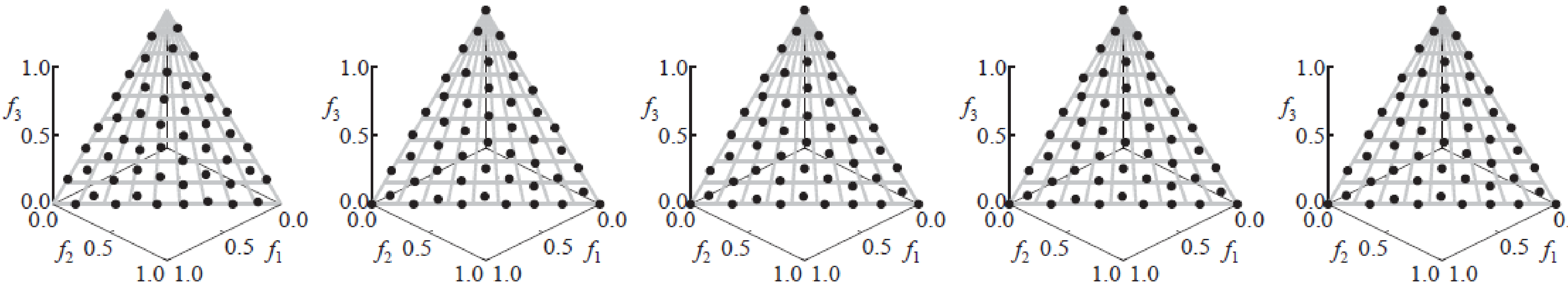
To specify a reference point so that no solution in a uniformly obtained solution set has a dominant effect.

➔ All solutions have the same (similar) HV contribution.



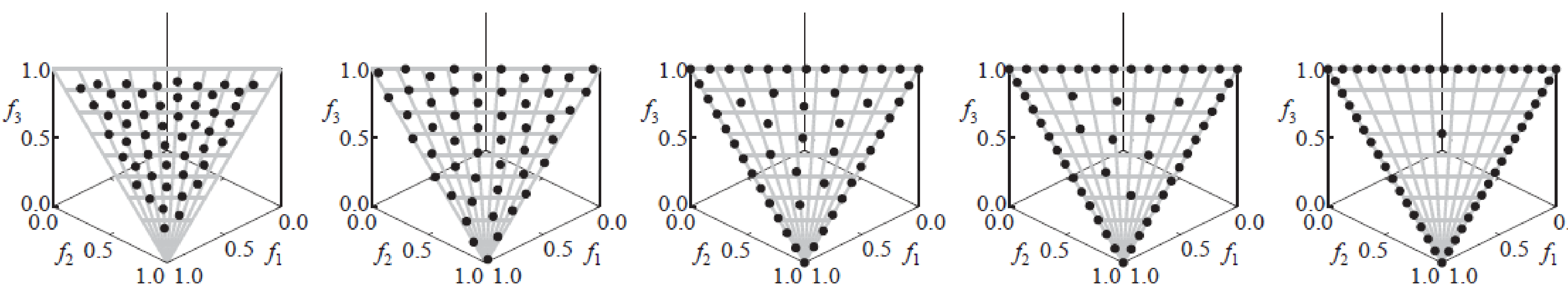
Proposed Specification

Dependency of Optimal Distribution of Solutions on the Shape of the Pareto Front



(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

Figure 1: Obtained solution sets for the three-objective normalized DTLZ1.

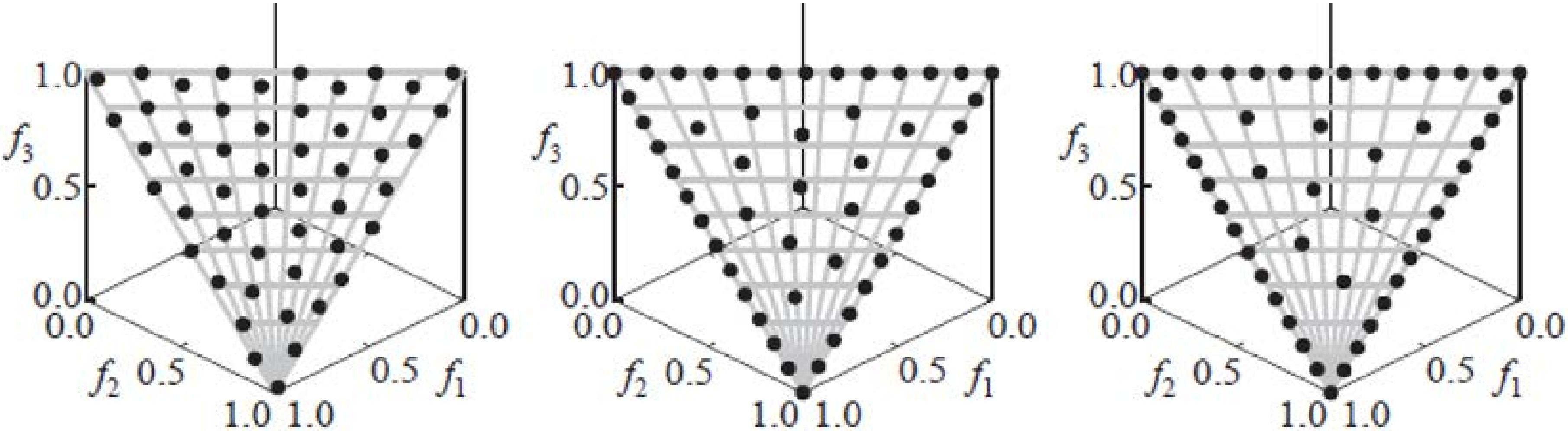


(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

Figure 2: Obtained solution sets for the three-objective normalized Minus-DTLZ1.

Optimal Distribution of Solutions depends on the reference point specification

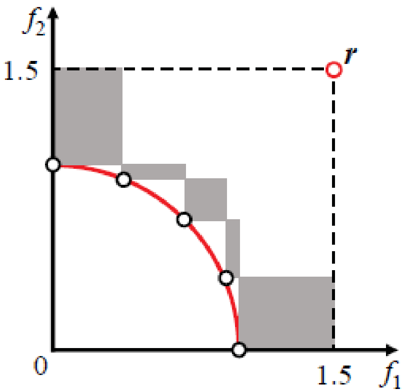
==> This means that the best weight (reference) vector specification in MOEA/D, NSGA-III, MOEA/DD etc. depends on the reference point specification.



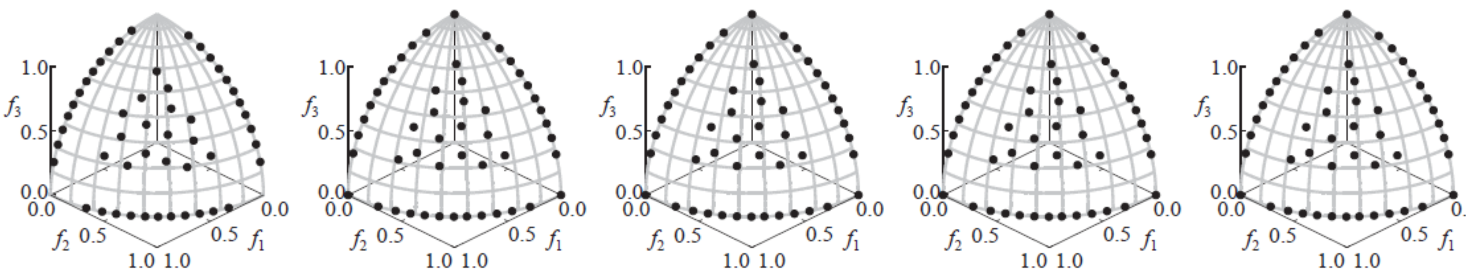
(b) $r = 1.1.$ (c) $r = 1.5.$ (d) $r = 2.0.$

More boundary vectors are needed.

Dependency of Optimal Distribution of Solutions on the Shape of the Pareto Front

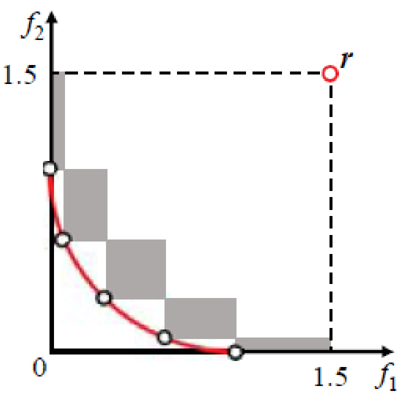


(a) Two-objective DTLZ2.

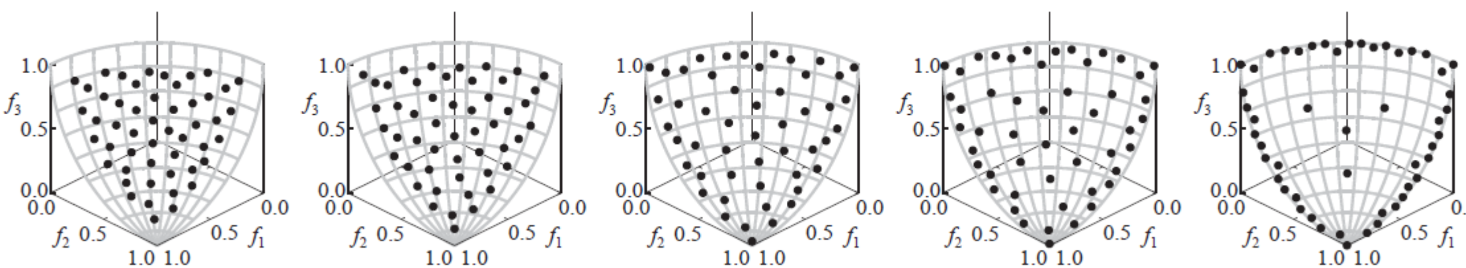


(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

Figure 8: Obtained solution sets for the three-objective DTLZ2 problem.



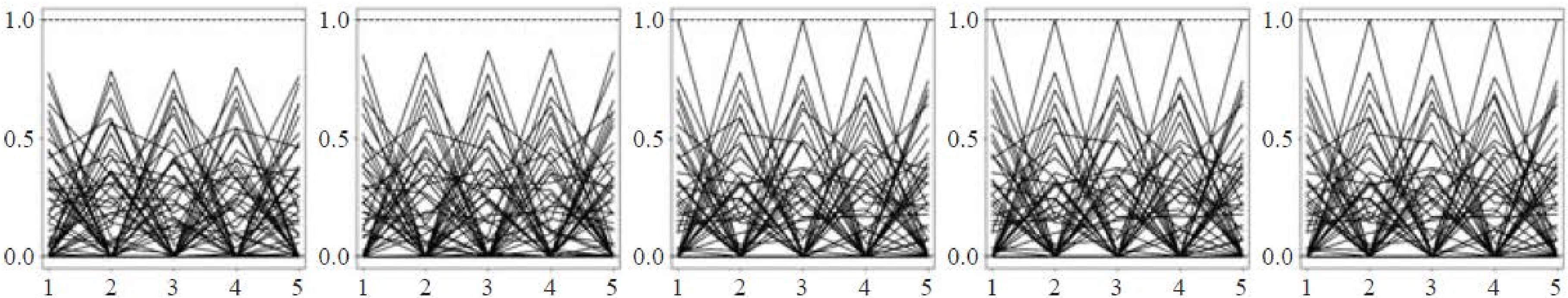
(b) Two-objective Minus-DTLZ2.



(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

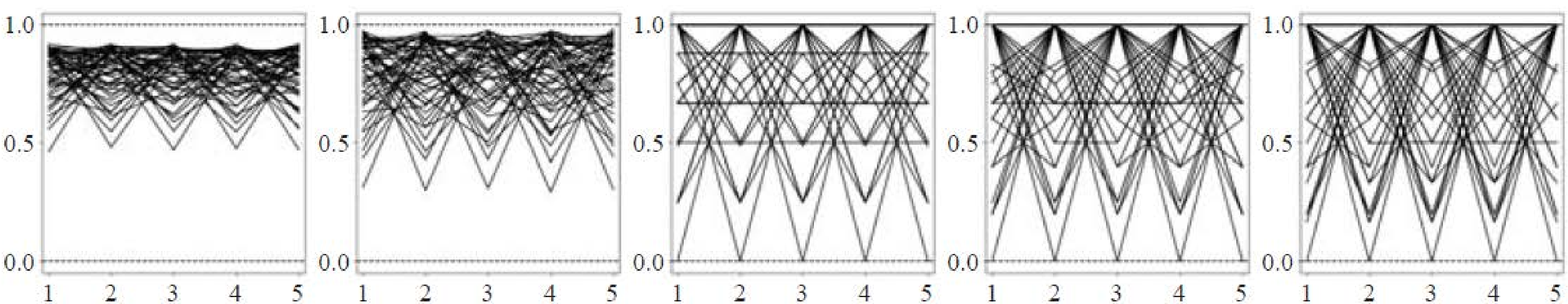
Figure 9: Obtained solution sets for the three-objective Minus-DTLZ2 problem.

Dependency of Optimal Distribution of Solutions on the Shape of the Pareto Front



(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

Figure 6: Obtained solution sets for the five-objective normalized DTLZ1 problem.



(a) $r = 1.0$. (b) $r = 1.1$. (c) $r = 1.5$. (d) $r = 2.0$. (e) $r = 10$.

Figure 7: Obtained solution sets for the five-objective Minus-DTLZ1 problem.

Difficulties in Performance Evaluation

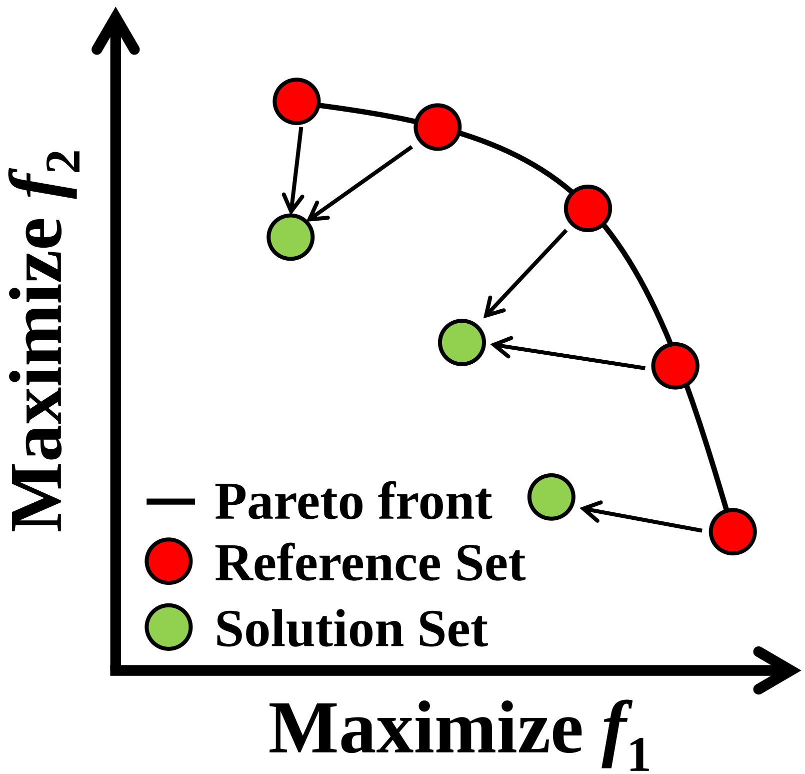
1. **How to Specify the Population Size**
2. **How to Specify the Reference Point for HV**
3. **How to Specify the Reference Points for IGD**

[1] **H. Ishibuchi et al., Reference point specification in inverted generational distance for triangular linear Pareto front, *IEEE Trans. on Evolutionary Computation* (2018). (Reference Point Specification)**

[2] **H. Ishibuchi, H. Masuda, Y. Nojima, A study on performance evaluation ability of a modified inverted generational distance indicator," *Proc. of GECCO 2015*, pp. 695-702. (Modification of the IGD Indicator)**

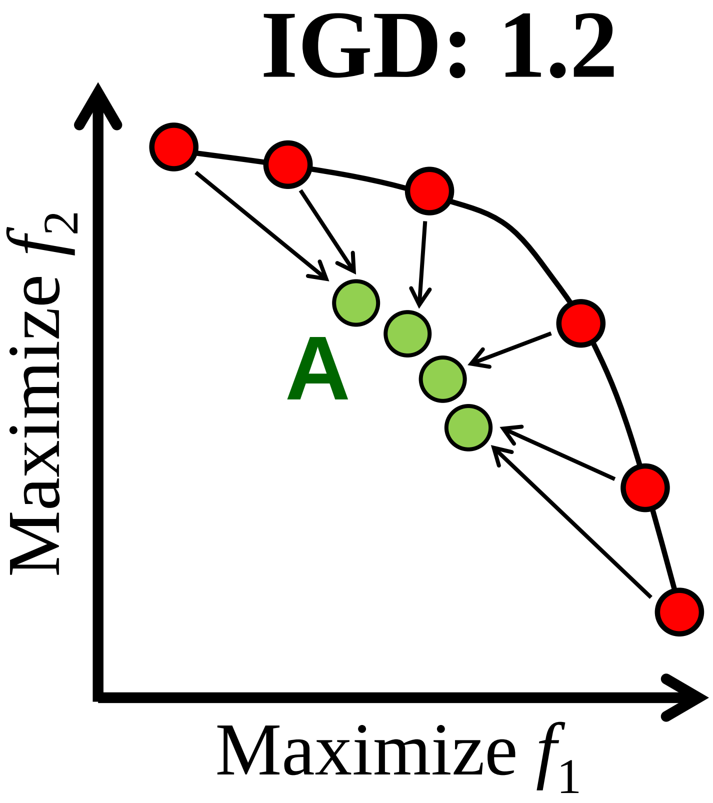
IGD-based performance comparison results depends on the reference point specifications

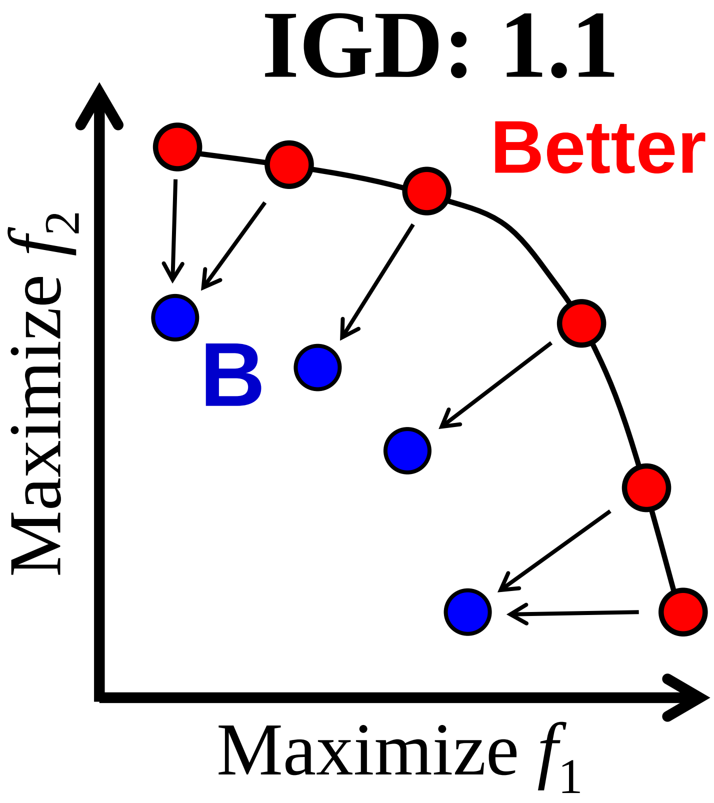
Average distance from each reference point on the Pareto front to the nearest solution.



IGD-based performance comparison results depends on the reference point specifications

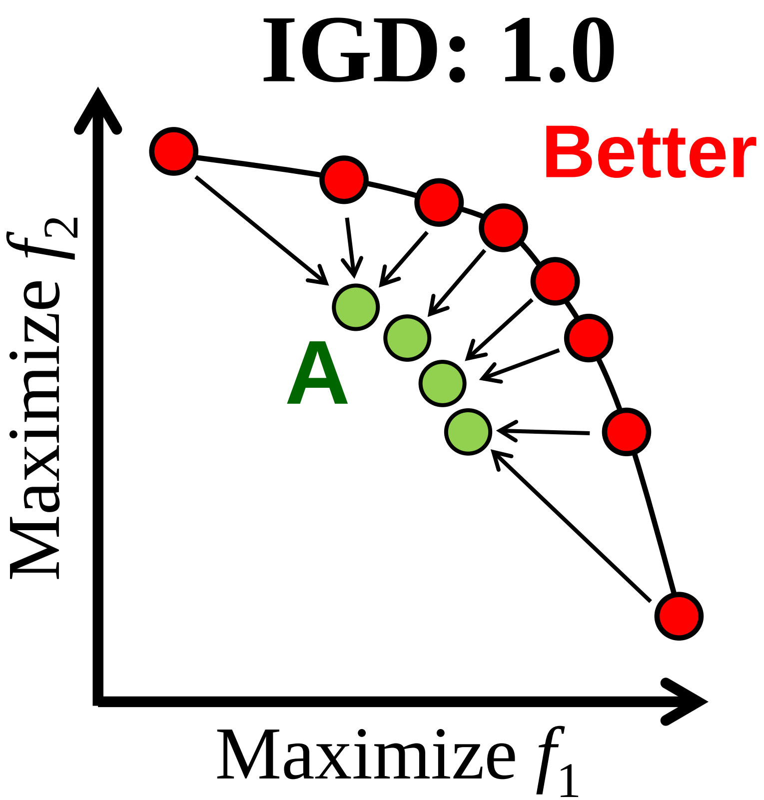
Specification of reference points is important.

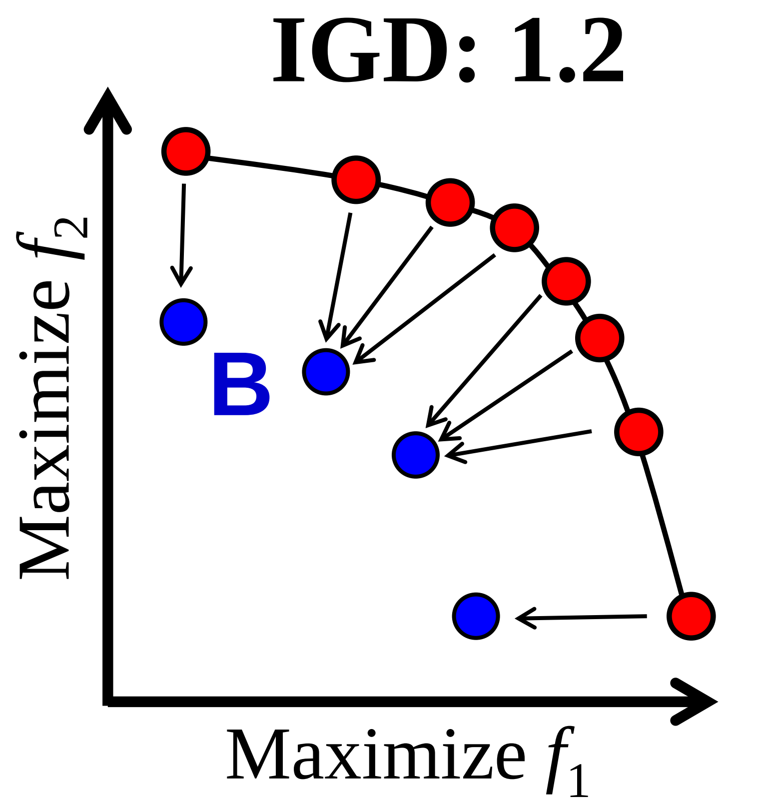




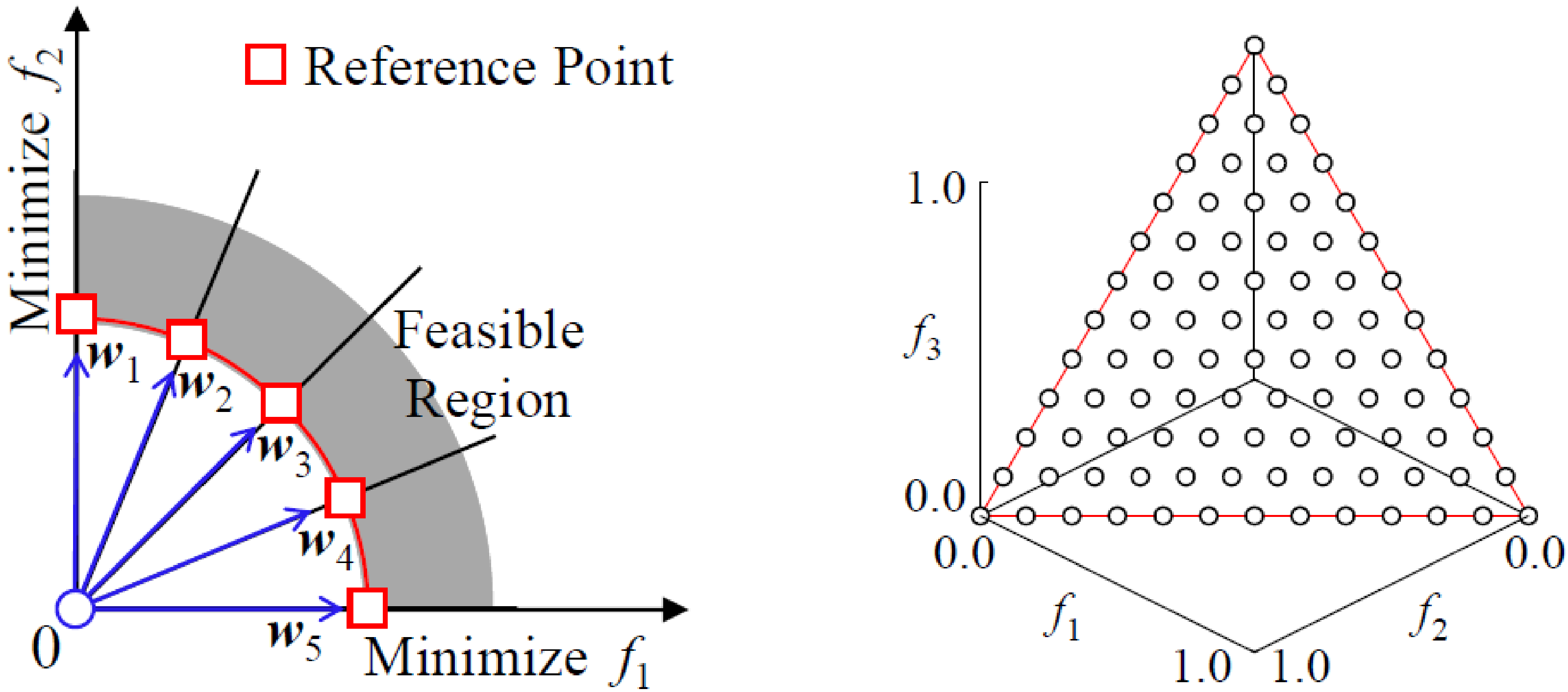
IGD-based performance comparison results depends on the reference point specifications

Specification of reference points is important.





Use of uniformly distributed solutions



(a) Reference point specification.

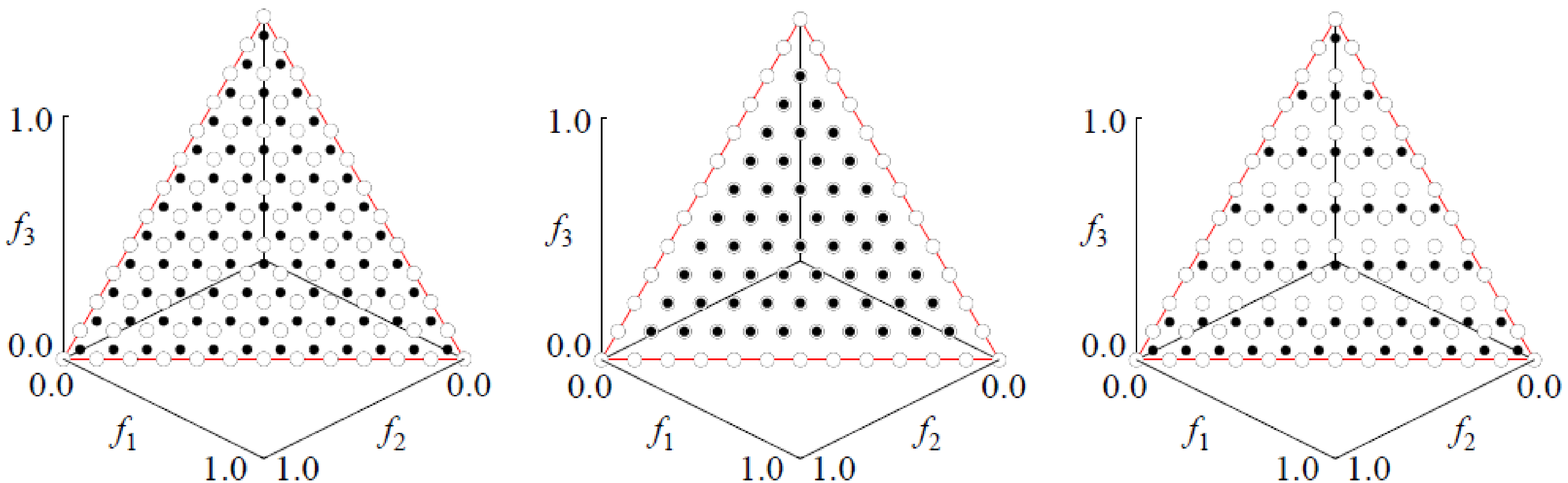
(b) Reference points ($H = 12$).

Fig. 5. Illustration of the reference point specification in (a), and an example of specified reference points for the three-objective normalized DTLZ1 in (b).

Another Approach:
Use of uniformly distributed solutions

Counter-Intuitive Example:

Comparison of the three solution sets (small closed circles) in (a)-(c) using the reference point set (open circles).



(a) IGD = 0.0680.

(b) IGD = 0.0495.

(c) IGD = 0.0680.

Fig. 6. Three solution sets to be compared (solutions: small closed circles, reference points: open circles) and their IGD values.

How to specify a set of reference points

Current Standard:

Use of a large number of uniformly distributed solutions.

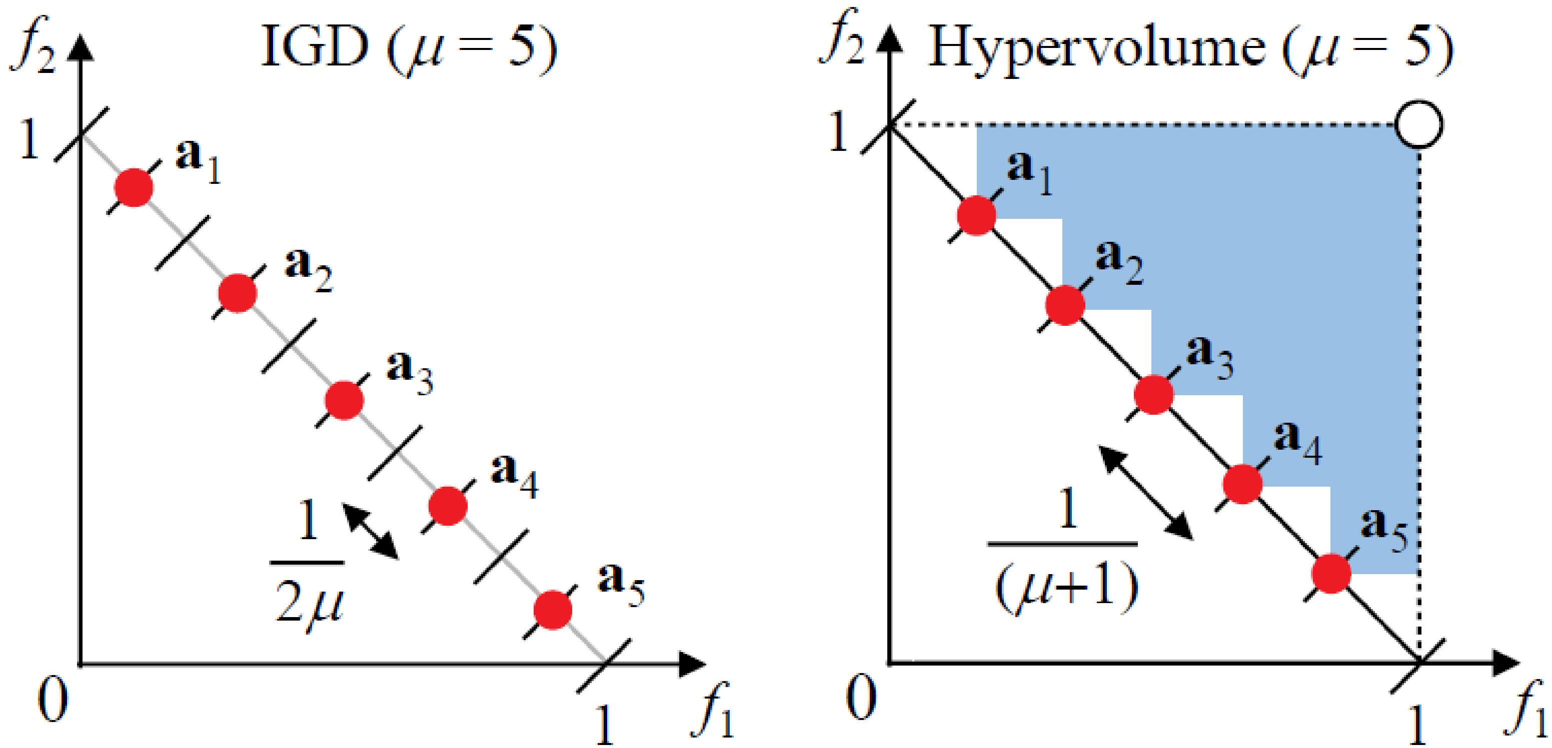
How to specify a set of reference points

Current Standard:

Use of a large number of uniformly distributed solutions.

This is not always a good method as shown in the following slides.

Analysis of IGD from a Viewpoint of Optimal Distribution of Solutions



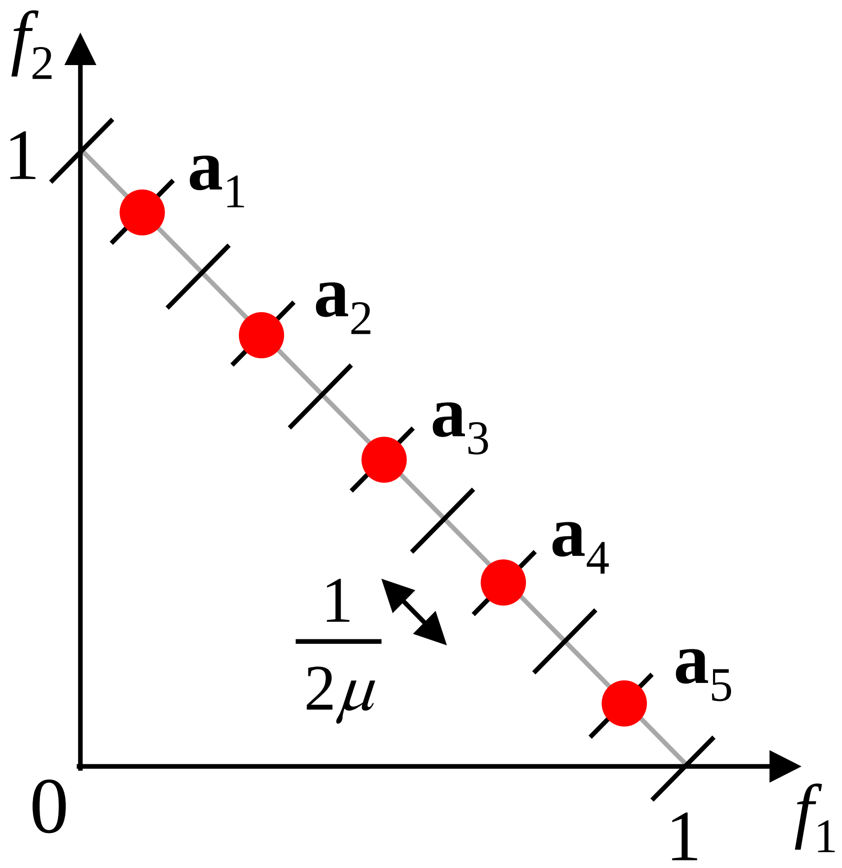
IEEE Trans. on Evolutionary Computation (2018)

Reference Point Specification in Inverted Generational Distance for Triangular Linear Pareto Front

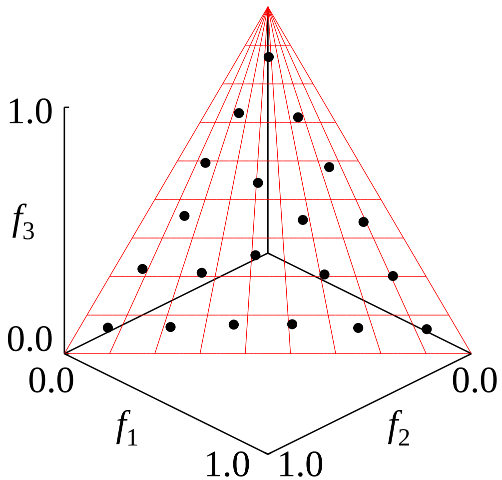
Hisao Ishibuchi, Ryo Imada, Yu Setoguchi, and Yusuke Nojima

Optimal Distribution of Solutions for IGD

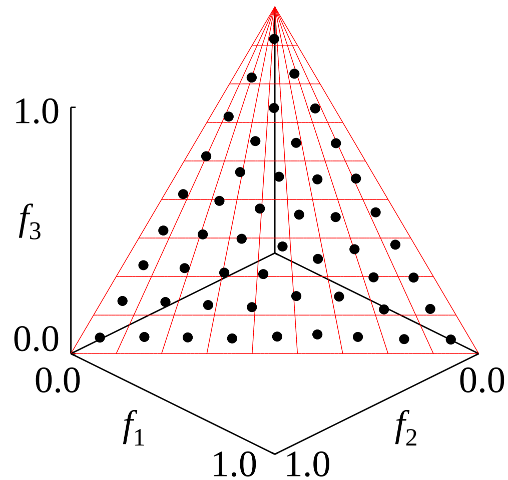
When an infinitely large number of uniformly distributed reference points on the Pareto front are used, the best distribution of solution is as follows (μ: population size)



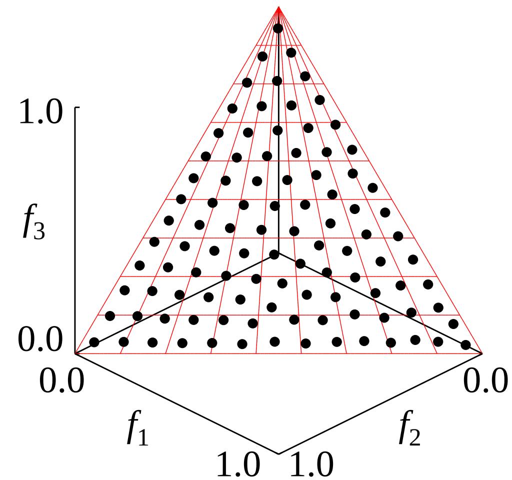
Optimal Distributions of Solutions for IGD are not always intuitive



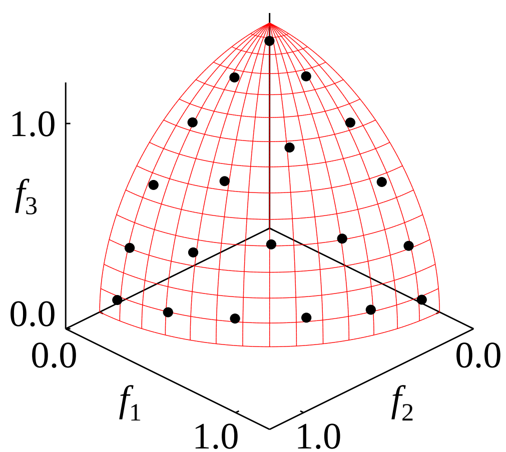
Population size 20



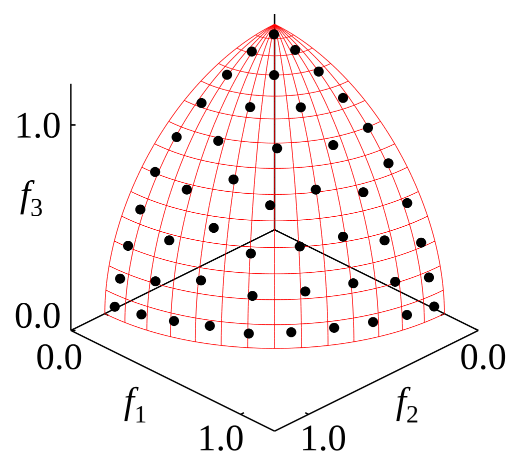
Population size 50



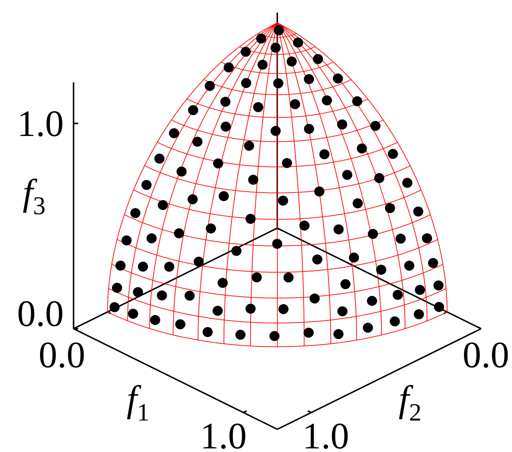
Population size 100



Population size 20

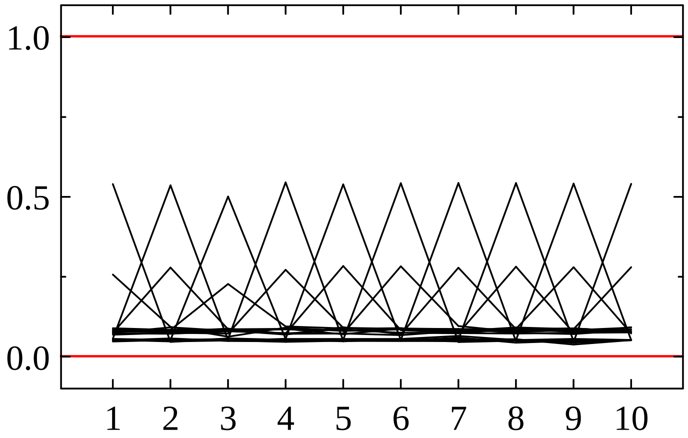


Population size 50

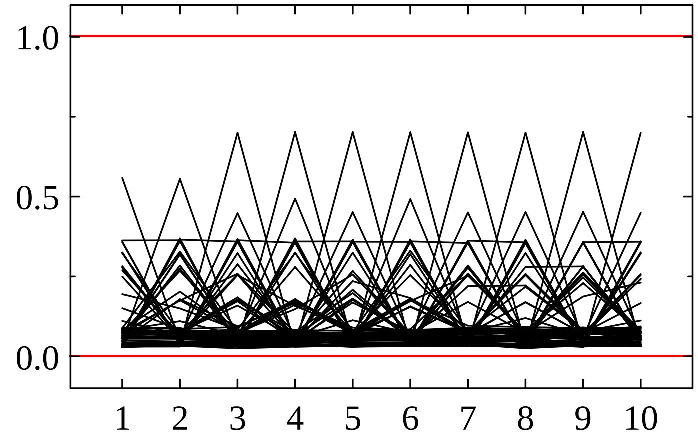


Population size 100

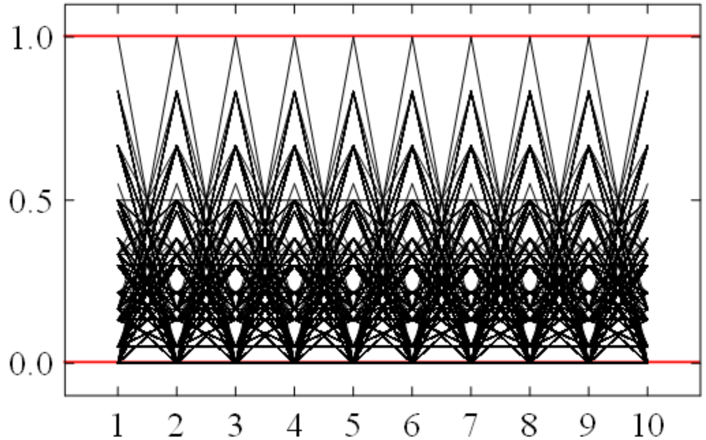
Optimal Distributions of Solutions for IGD are not always intuitive



Population size 20



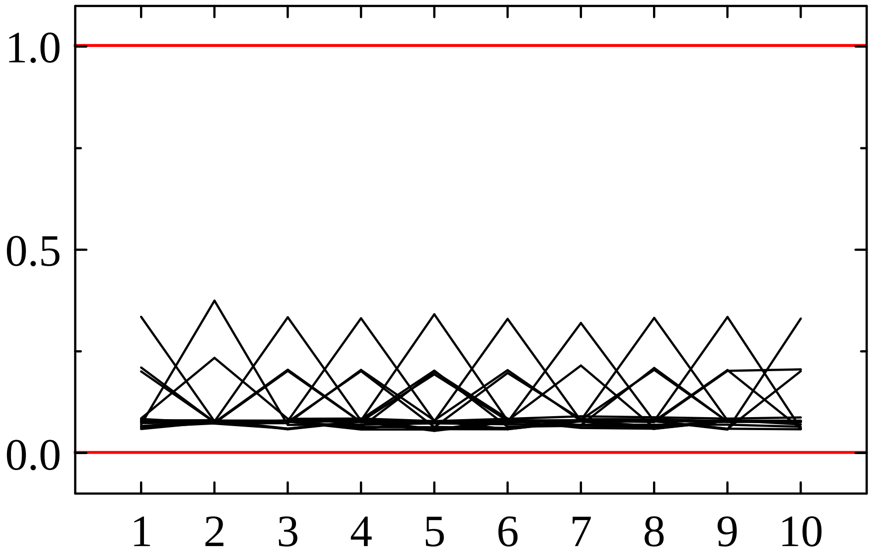
Population size 100



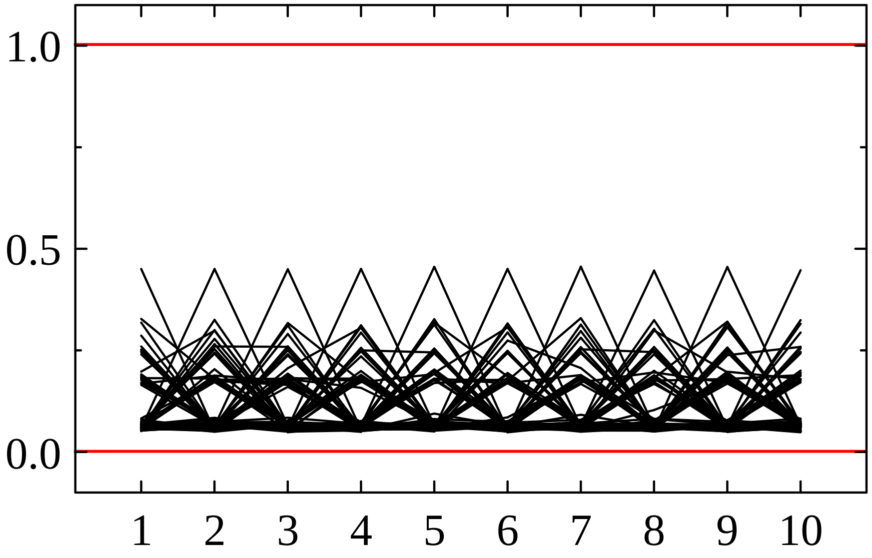
Reference points: 10,010 points

Optimal Distributions of Solutions for IGD are not always intuitive

When we randomly generate 100,000 reference points, the optimal distributions of solutions are as follows:



Population size 20



Population size 100

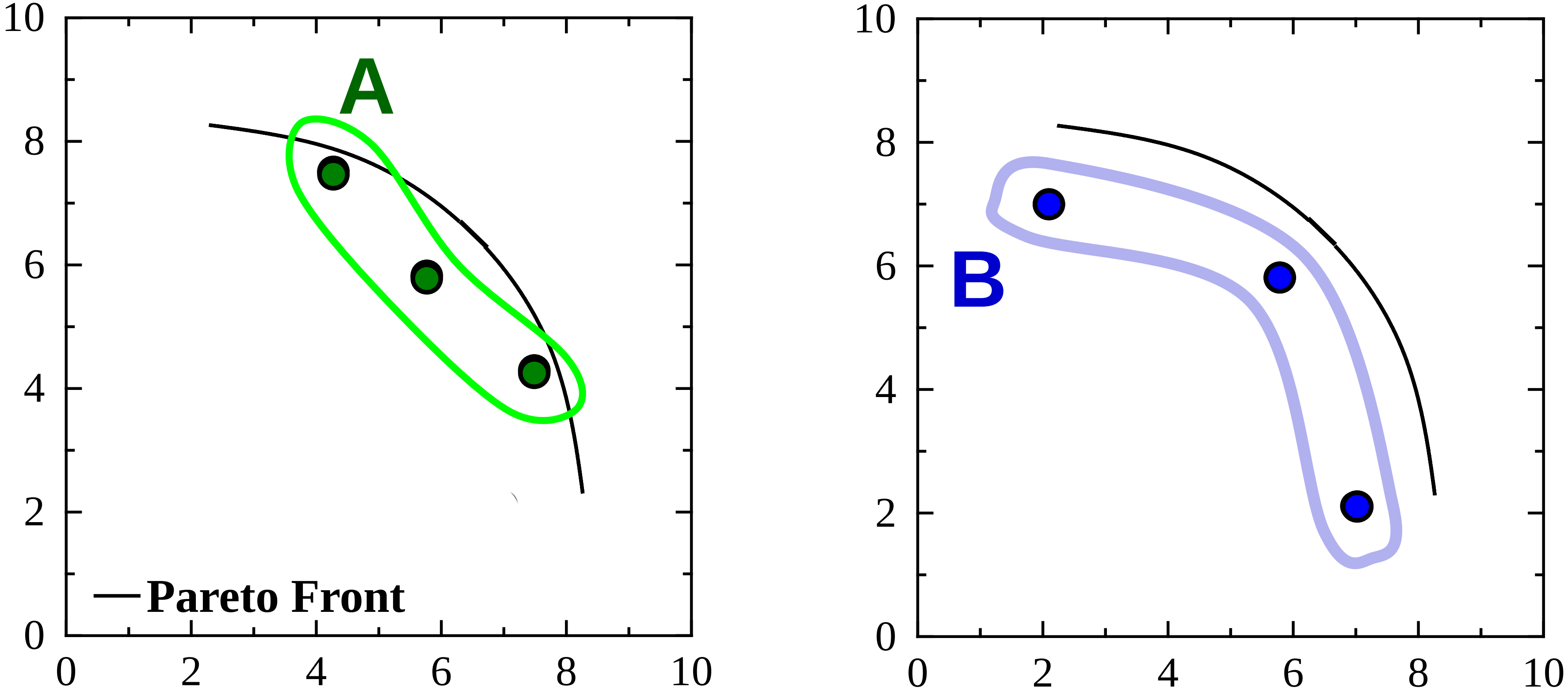
Pareto Compliance of IGD

[1] H. Ishibuchi et al., Reference point specification in inverted generational distance for triangular linear Pareto front, *IEEE Trans. on Evolutionary Computation* (2018). (Reference Point Specification)

[2] H. Ishibuchi, H. Masuda, Y. Nojima, A study on performance evaluation ability of a modified inverted generational distance indicator," *Proc. of GECCO 2015*, pp. 695-702. (Modification of the IGD Indicator)

Example of Two Solution Sets A and B

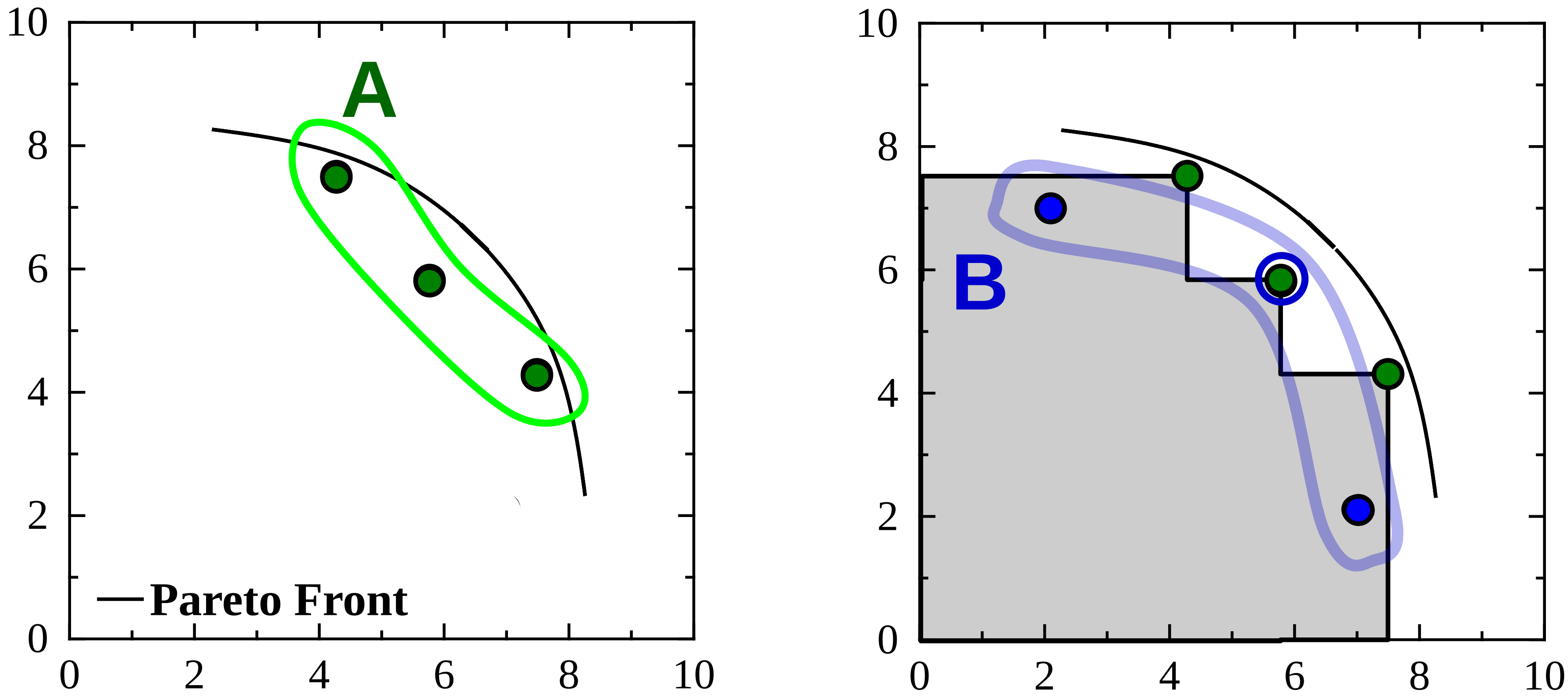
Solution set A dominates B



A dominates B (A is better than B).

Example of Two Solution Sets A and B

Solution set A dominates B

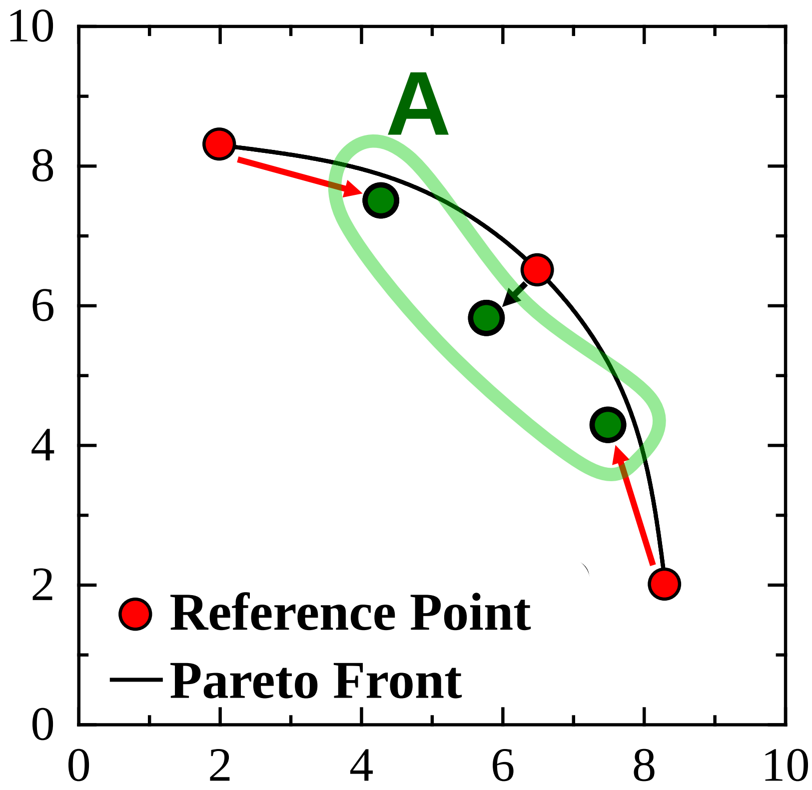


A dominates B (A is better than B).

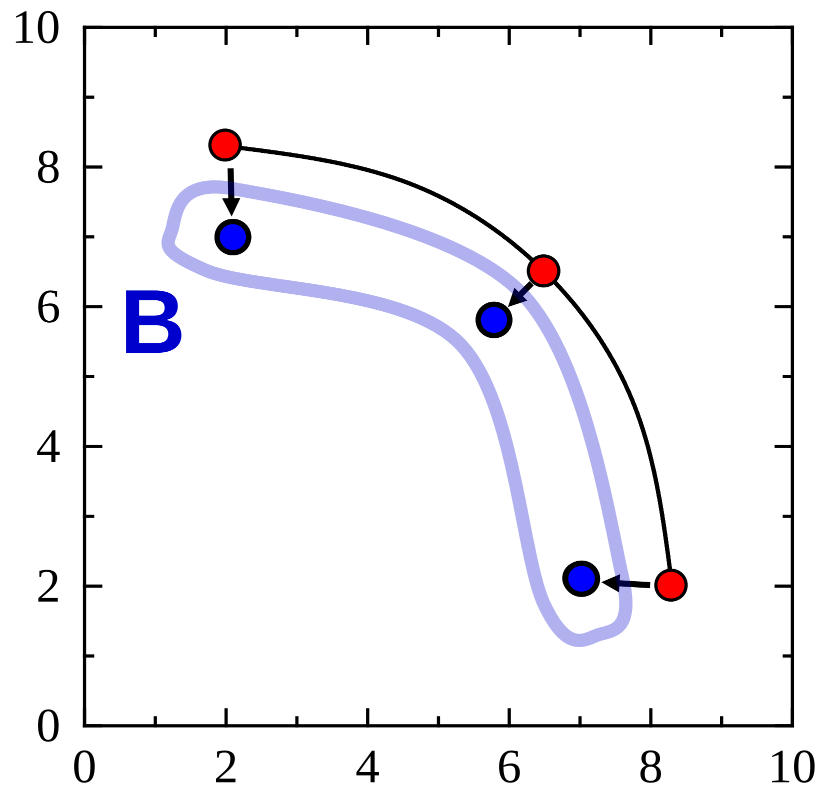
Use of IGD ==> Inconsistent Results

Solution set B is evaluated as being better than A

IGD is larger



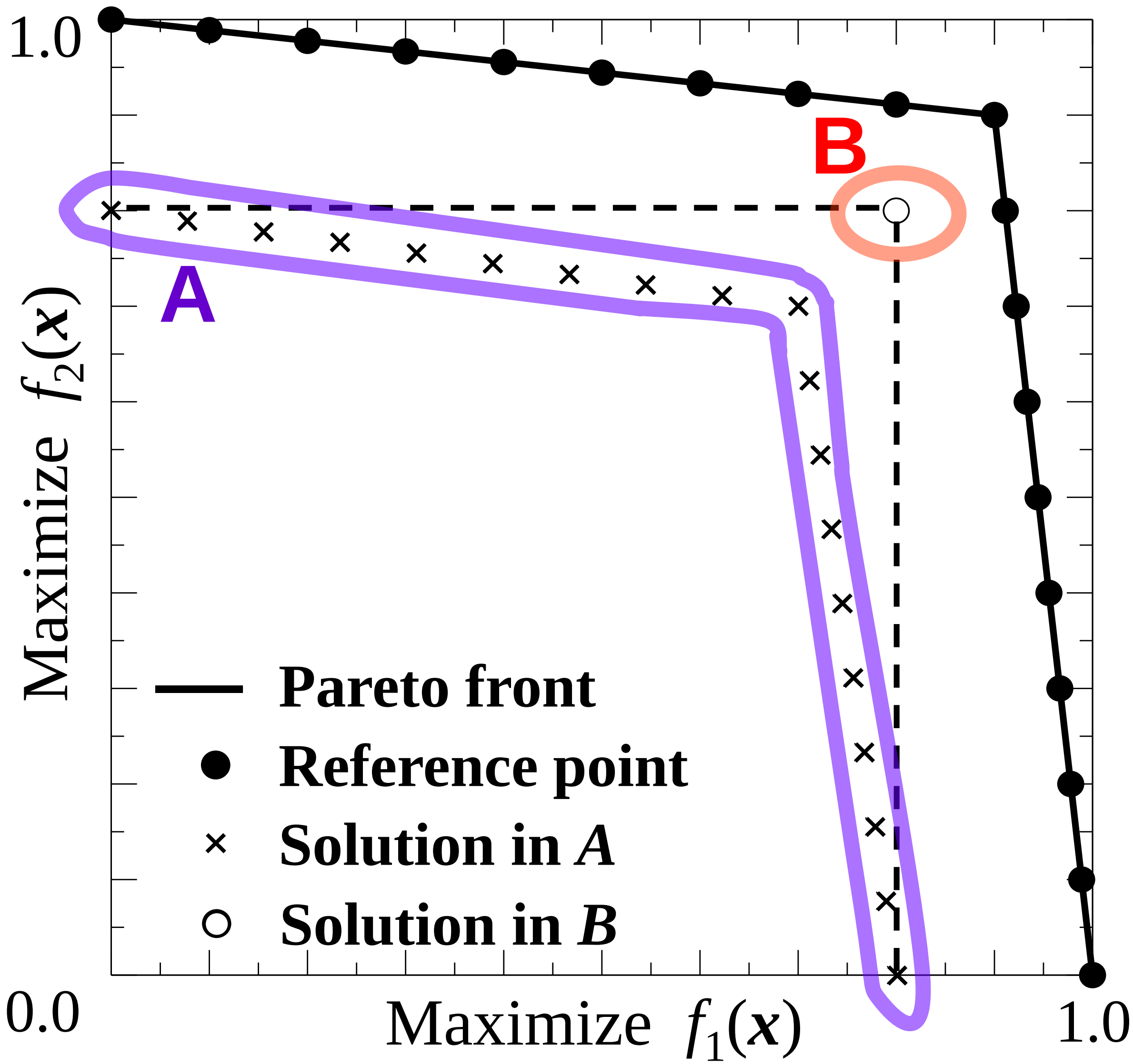
IGD is smaller



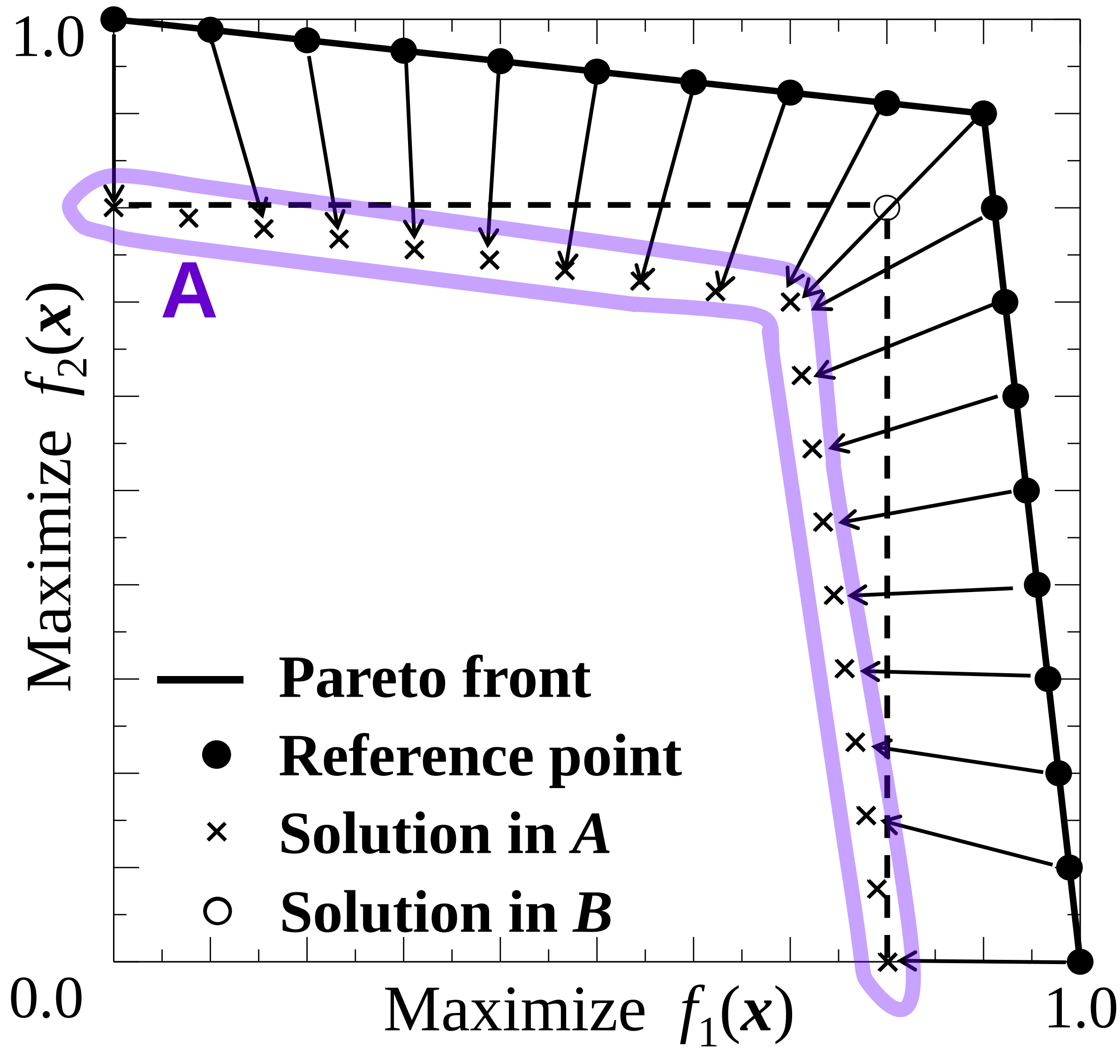
A dominates B (A is better than B).
However, B is evaluated as being better than A.

Another Example

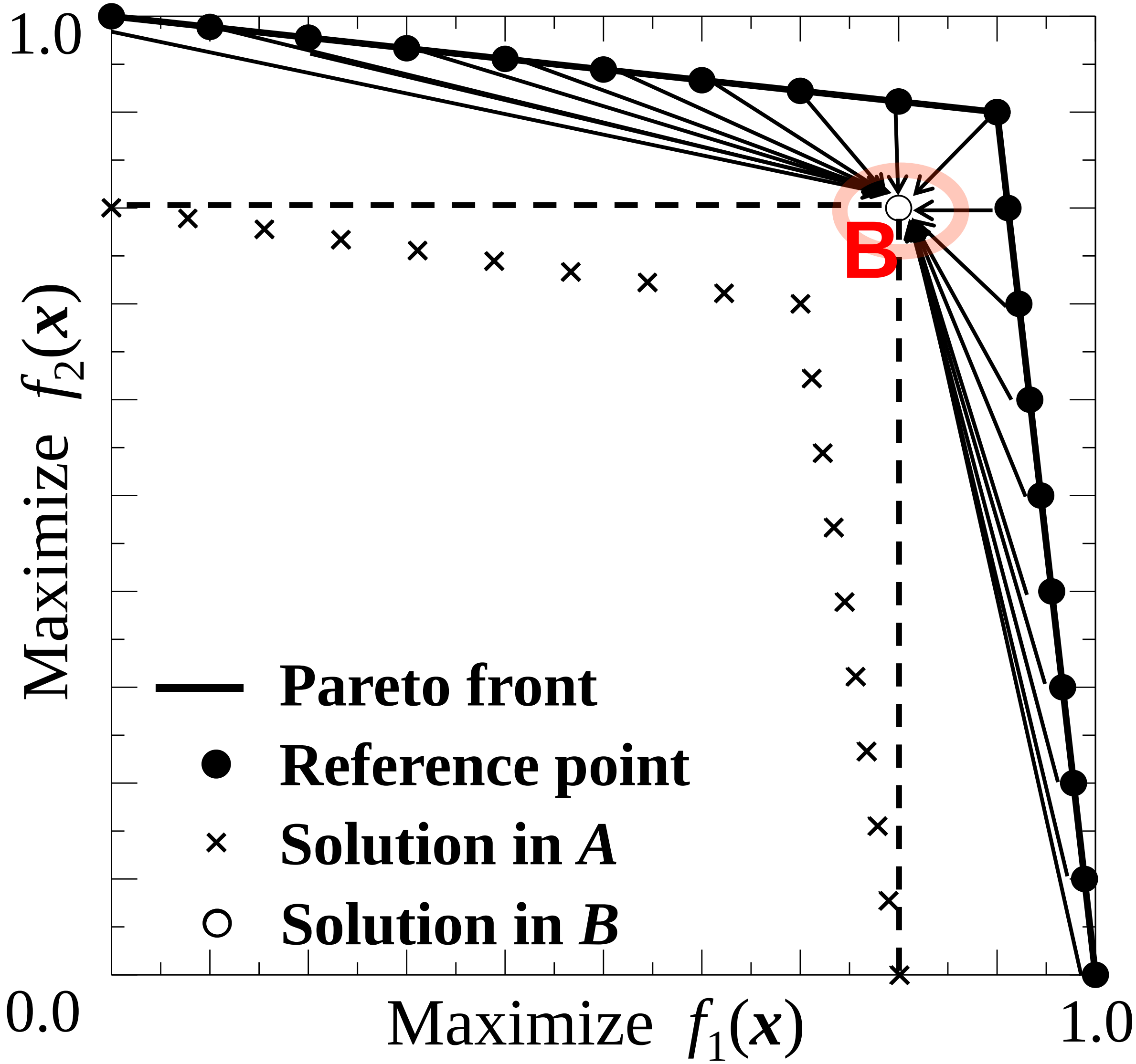
Solution Set B (o) dominates A (many x)



IGD Calculation for Solution Set A

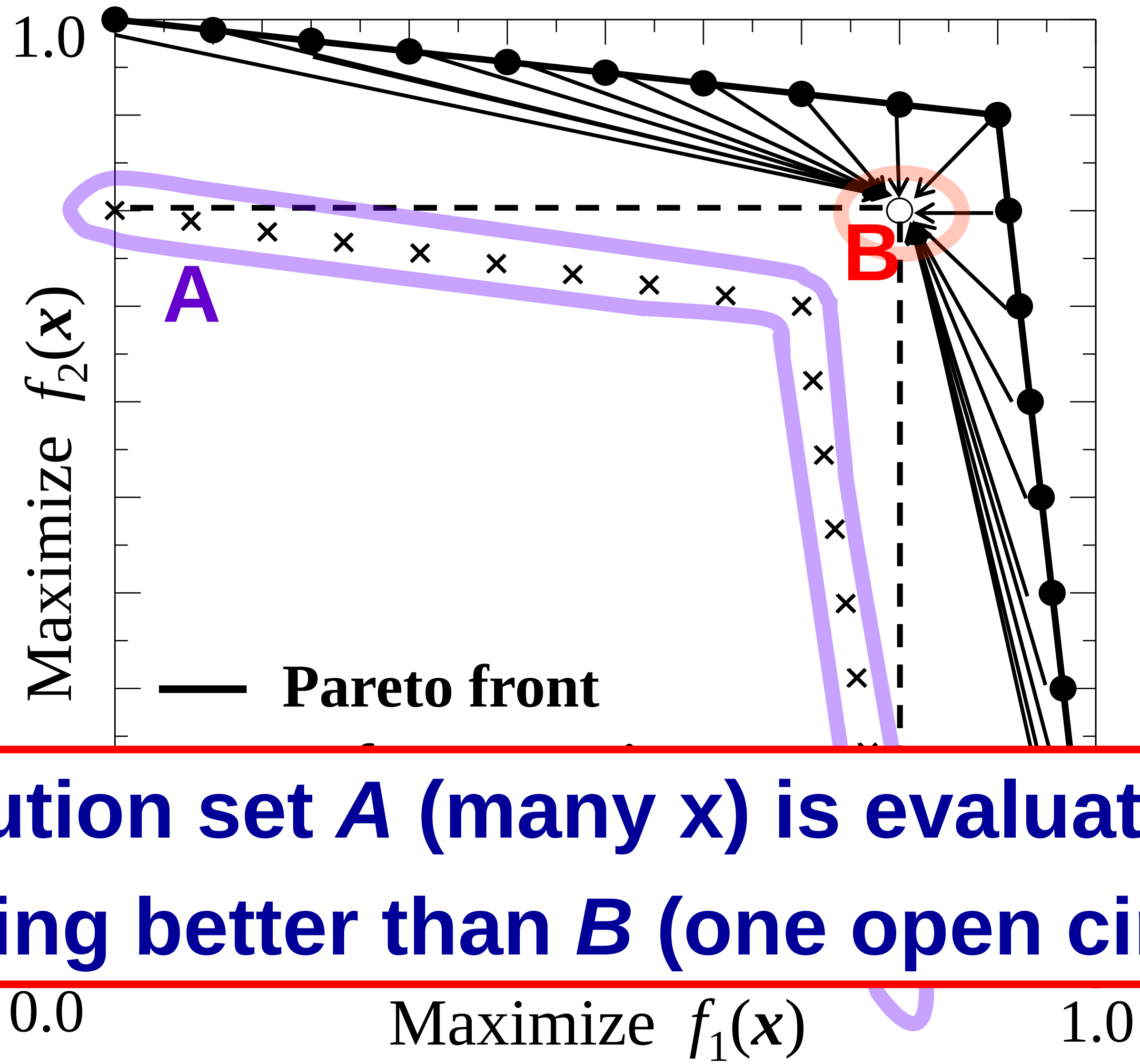


IGD Calculation for Solution Set B



Inconsistent Evaluation Results

IGD(A) is smaller than IGD(B)

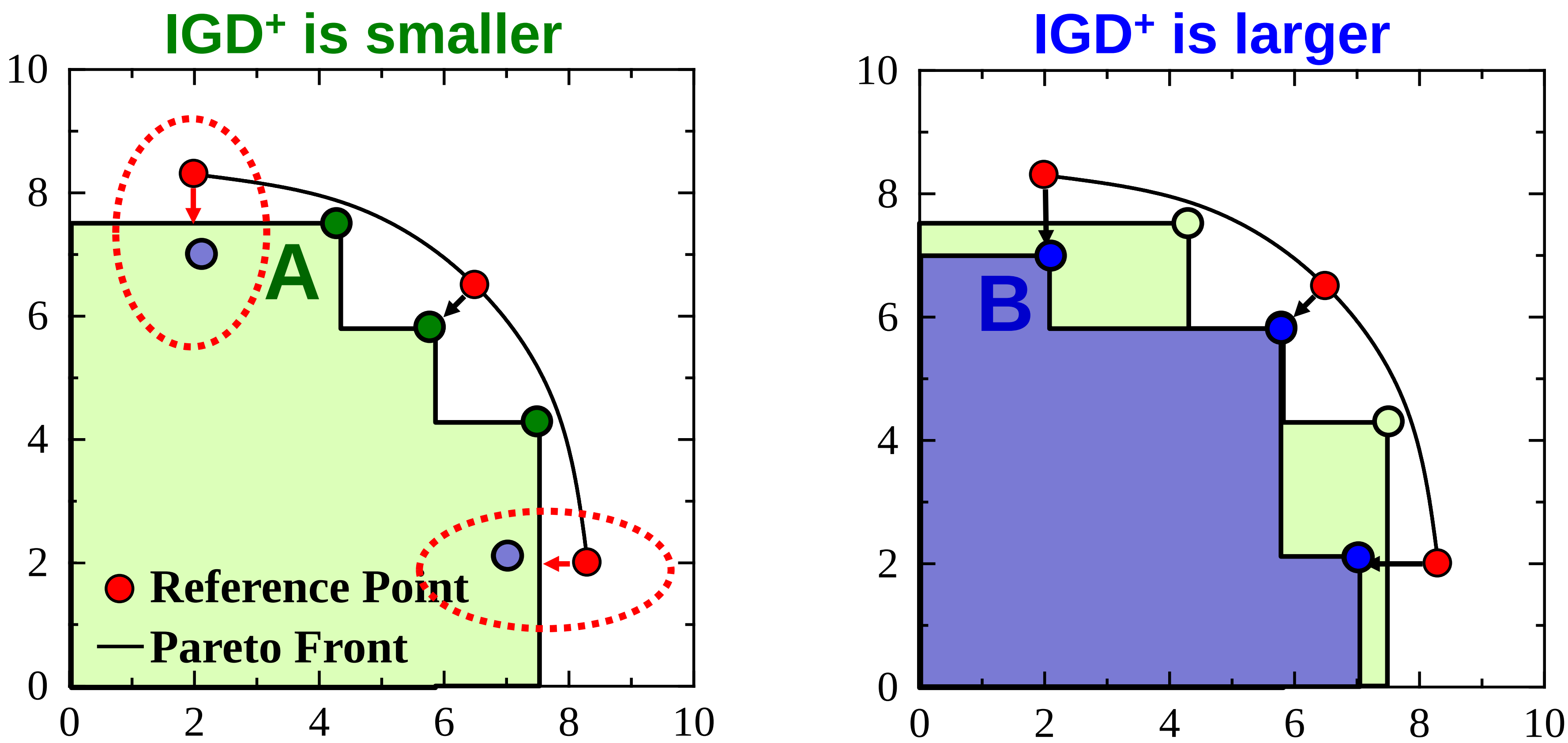


Solution set *A* (many x) is evaluated as being better than *B* (one open circle).

IGD$^+$ Calculation

(Ishibuchi et al., EMO 2015, GECCO 2015)

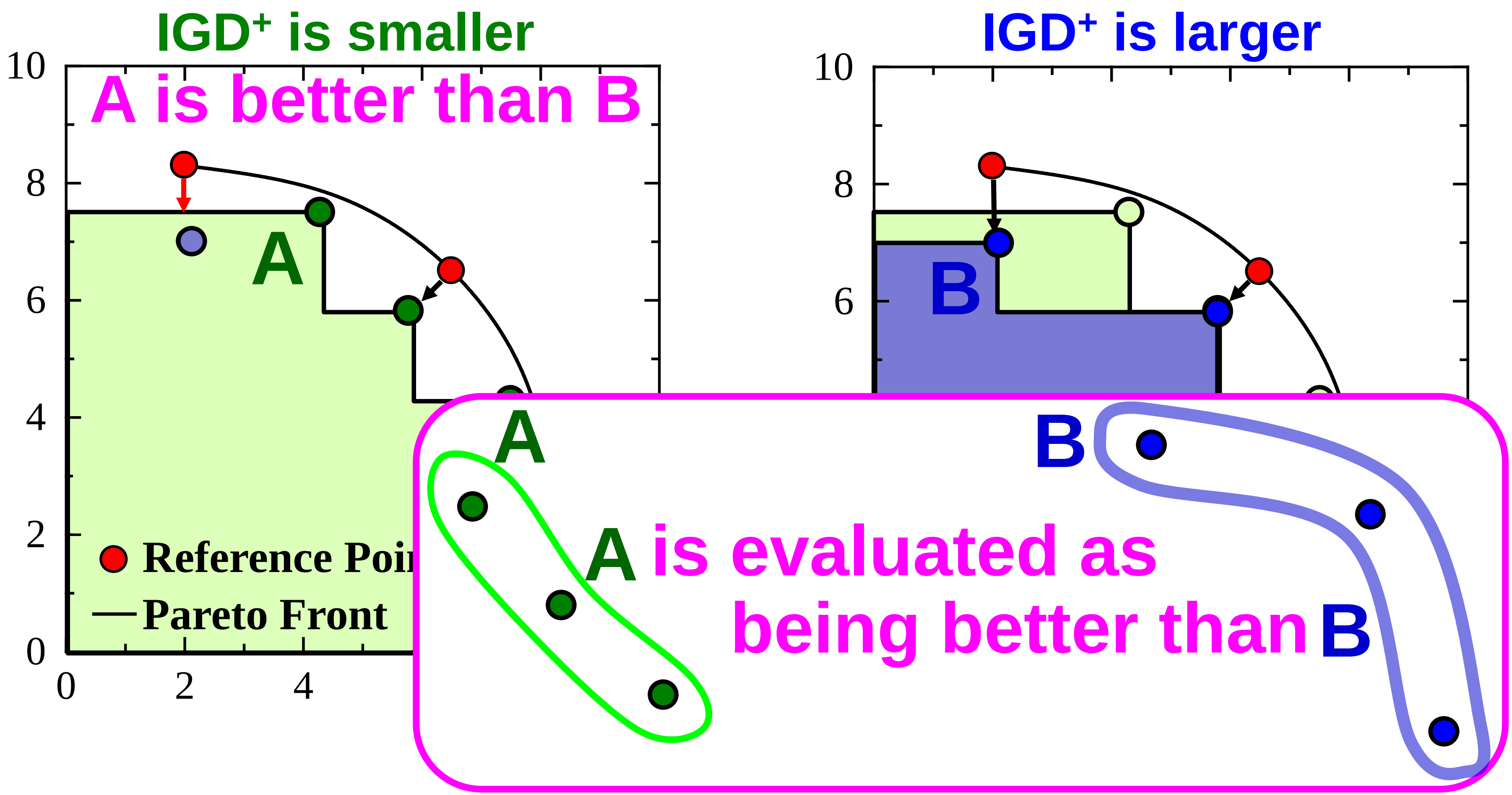
The calculation is from each reference point to the dominated region by the obtained solution set.



Use of IGD+ ==> Consistent Results

Solution set A is evaluated as being better than B

IGD+ is not inconsistent with the Pareto dominance relation between solution sets.



Performance Indicators in jMetal 5 Web Site

Welcome to the **jMetal 5** Web Site

jMetal is ...

jMetal stands for **Metaheuristic Algorithms in Java**, and it is an object-oriented Java-based framework for multi-objective optimization with metaheuristics.

Summary of features

- Multi-objective algoritms: NSGA-II, SPEA2, PAES, PESA-II, OMOPSO, MOCell, AbYSS, MOEA/D, GDE3, IBEA, SMPSO, SMPSOhv, SMS-EMOA, MOEA/D-STM, MOCHC, MOMBI, MOMBI-II, NSGA-III, WASF-GA, GWASF-GA

- Quality indicators: hypervolume, spread, generational distance, inverted generational distance, inverted generational distance plus, additive epsilon.

Conclusion

1. New EMO algorithms may be needed for many-objective problems.
2. A wide variety of many-objective test problems with various characteristic features are needed for healthy algorithm development. Analysis of real-world problems seems to be very important.
3. How to evaluate many-objective algorithms (with no information from the decision maker) may need a lot of further discussions.

Conclusion

4. **Use of test problems with inverted triangular Pareto fronts makes various issues clear:**

- **Strong dependency of the performance of MOEA/D on the shape of the Pareto front.**
- **Necessity of weight (reference) vector adaptation.**
- **Strong dependency of the optimal distribution of solutions for HV maximization on the reference point specification.**

Other Topics

1. **Solution Selection:** To choose a small number of non-dominated solutions as candidate solutions, which are presented to the decision maker.
2. **Objective Selection:** (i) to improve the efficiency of many-objective search by decreasing the number of objectives, (ii) to support the solution selection by decreasing the number of non-dominated solutions.
3. **Normalization:** Objective space normalization is included in many EMO algorithms. Its necessity is clear. It also have some potential negative effects.

Other Topics

4. **Scalability:** Problems with

- a large number of objectives (many-objective)
- a large number of variables (large-scale)
- a large number of constraints
- high percentage of infeasible solutions
- a large number of overlapping Pareto optimal solutions in the objective space (multi-modal).
- a large number of local Pareto fronts.
- expensive fitness evaluation
- search for a large number of non-dominated solution for knowledge extraction