When Evolutionary Multiobjective Optimization Meets Large-Scale Decision Variables: Challenges and Solutions

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Evolving Machine Intelligence (EMI) Group

◆ ***Members:***

Currently, there are two PDRAs （博后）, four RAs（研究助理）, and one master student in EMI.

Research Interests:

The Evolving Machine Intelligence (EMI) Lab focuses on evolvable intelligent systems. We are motivated to understand how evolution generates complexity, diversity and intelligence by computational simulations.

Future Work:

A major focus is on neuro-evolution, where large-scale deep neural networks are evolved for structure optimization. We are also interested in combining deep learning and evolutionary computation to develop optimization methods.

常年招收优秀研究助理、博士后，硕士生，博士生

Dr. Ran CHENG

◆ ***Background***:

I received the Ph.D. degree from the University of Surrey, Guildford, U.K., in 2016. My PhD study was financed by the Honda Research Institute Europe (HRI-EU).

Before joining the SUSTech as an Assistant Professor, I was a Research Fellow at the University of Birmingham.

◆ ***Research interests***:

Computational intelligence, deep Learning, evolutionary computation, large-scale optimization.

Outline

- ➢ Background
- Test problem for large-scale multiobjective optimization
- Real-world large-scale multiobjective optimization problems
- Solving large-scale many-objective optimization problems
- Accelerating large-scale multiobjective optimization
- Future Challenges

Background

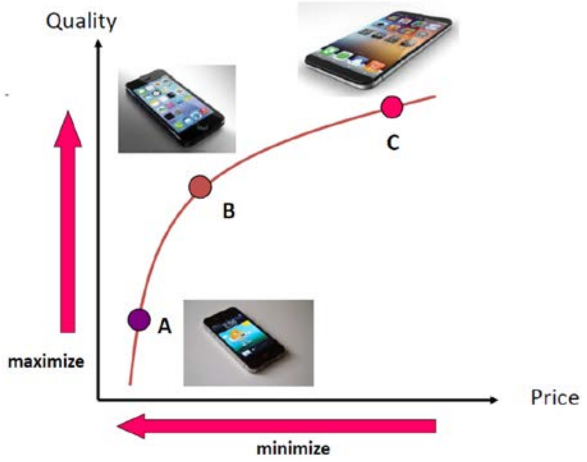
Large-scale multiobjective optimization

- Formulation of multiobjective optimization problem (MOP):

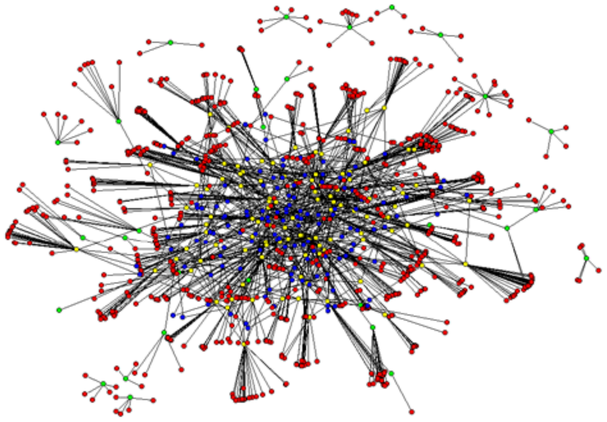
$$\min_{\mathbf{x}} f_i(\mathbf{x}) \quad i = 1,2,\dots,M$$
$$s.t. \quad \mathbf{x} \in [\mathbf{a},\mathbf{b}] \text{ and } M > 1,$$

where $\mathbf{x} = (x_1, x_2, \dots, x_D)$ is the **decision vector** which consists a large number of D decision variables, $f_i(\mathbf{x})$ are the **optimization objectives**, $\mathbf{a}$ and $\mathbf{b}$ are the **box constraints**.

- Difficulties in large-scale MOP (LSMOP)
  - ✓ Huge volume of search space
  - ✓ Complex fitness landscape
  - ✓ Multiple interactions:
    - Interaction between the variables
    - Interaction between the variables and objectives



Multi-objective optimization problem


An example of large-scale multi-objective optimization – Community detection in complex networks

Some definitions

- Variable interaction:

  $\mathbf{x}_i$ and $\mathbf{x}_j$ are interacting *iff* there exist a_1, a_2, b_1, b_2 statisfying

$$f(\mathbf{x})|_{x_i=a_2,x_j=b_1} \quad < \quad f(\mathbf{x})|_{x_i=a_1,x_j=b_1} \wedge$$
$$f(\mathbf{x})|_{x_i=a_2,x_j=b_2} \quad > \quad f(\mathbf{x})|_{x_i=a_1,x_j=b_2},$$

where

$$f(\mathbf{x})|_{x_i=a_2,x_j=b_1} \triangleq f(x_1, ..., x_{i-1}, a_2, ..., x_{j-1}, b_1, ..., x_D).$$

- Partially separable:

  Function $f_i(\boldsymbol{x})$ is called a partially separable function with k components *iff*

$$\arg\min_{\mathbf{x}} f(\mathbf{x}) = (\arg\min_{\mathbf{x_1}} f(\mathbf{x_1}, ...), ..., \arg\min_{\mathbf{x_k}} f(..., \mathbf{x_k})),$$

- Variable interaction in MOPs:

  Convergence-related

  Diversity-related

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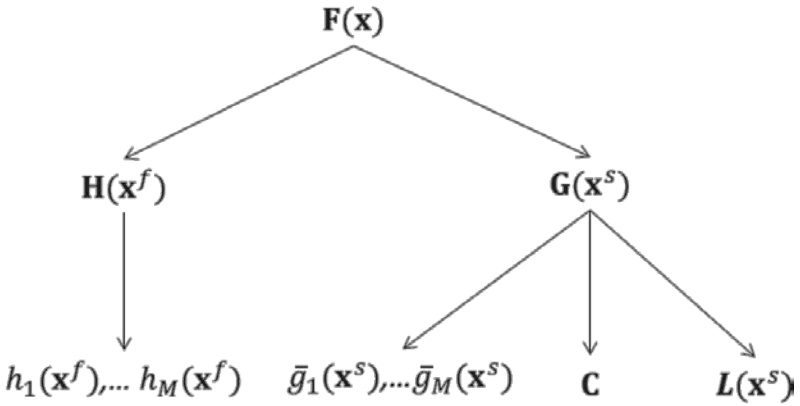
Test problem for large-scale multiobjective optimization

Motivation

- No existing benchmark test suite for LSMOPs
- Promote the research in large-scale multi-/many-objective optimization

Properties of the proposed LSMOPs:

- Uniform design formulation.
- Scalable to number of objectives
- Scalable to number of decision variables
- Exact shapes and locations of the PFs
  - ✓ $\mathbf{F}(\mathbf{x})$: Test function
  - ✓ $\mathbf{H}(\mathbf{x}^f)$: Shape matrix
  - ✓ $\mathbf{G}(\mathbf{x}^s)$: Landscape matrix
  - ✓ $h_1(\mathbf{x}^f),\dots h_M(\mathbf{x}^f)$: Shape functions
  - ✓ $\bar{g}_1(\mathbf{x}^s),\dots\bar{g}_M(\mathbf{x}^s)$: Landscape functions
  - ✓ $\mathbf{C}$: Correlation matrix
  - ✓ $L(\mathbf{x}^s)$: Linkage function



The structure of the designed test problems.

$$\mathbf{F}(\mathbf{x}) = \mathbf{H}\left(\mathbf{x}^f\right)\left(\mathbf{I} + \mathbf{G}\left(\mathbf{x}^s\right)\right)$$

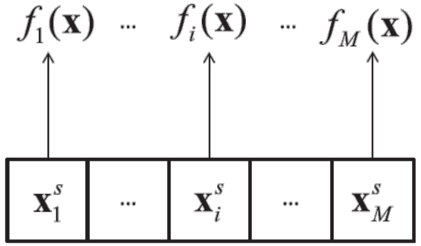
$$\begin{cases} f_1(\mathbf{x}) = h_1\left(\mathbf{x}^f\right)\left(1 + g_1(\mathbf{x}^s)\right) \\ f_2(\mathbf{x}) = h_2\left(\mathbf{x}^f\right)\left(1 + g_2(\mathbf{x}^s)\right) \\ \quad\quad\quad \cdots \\ f_M(\mathbf{x}) = h_M\left(\mathbf{x}^f\right)\left(1 + g_M(\mathbf{x}^s)\right) \end{cases}$$

The formulation of the designed test problems.

Cheng R, Jin Y, Olhofer M. Test problems for large-scale multiobjective and many-objective optimization. IEEE Transactions on Cybernetics, 2017, 47(12): 4108-4121.

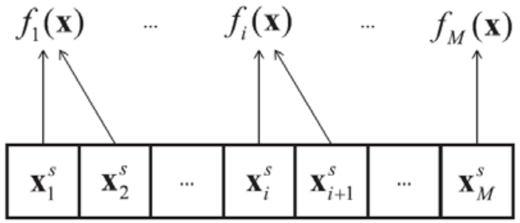
Correlation in LSMOPs

- Separable correlation



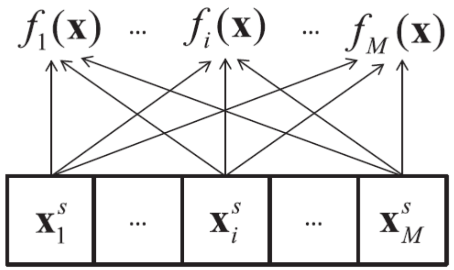
$$\mathbf{C}_1 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

- Overlapped correlation



$$\mathbf{C}_2 = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & 0 & \vdots \\ \vdots & 0 & \ddots & \ddots & 0 \\ \vdots & 0 & 0 & 1 & 1 \\ 0 & \cdots & \cdots & 0 & 1 \end{pmatrix}$$

- Full correlation



$$\mathbf{C}_3 = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

Test problem for large-scale multiobjective optimization

Problem characteristics

- The decision variables are nonuniformly divided into a number of groups.

$$\mathbf{x}^s = (\mathbf{x}_1^s, \ldots, \mathbf{x}_M^s) \Rightarrow \mathbf{x}_i^s = (\mathbf{x}_{i,1}^s, \ldots, \mathbf{x}_{i,n_k}^s)$$

- Different groups of decision variables are correlated with different objectives.

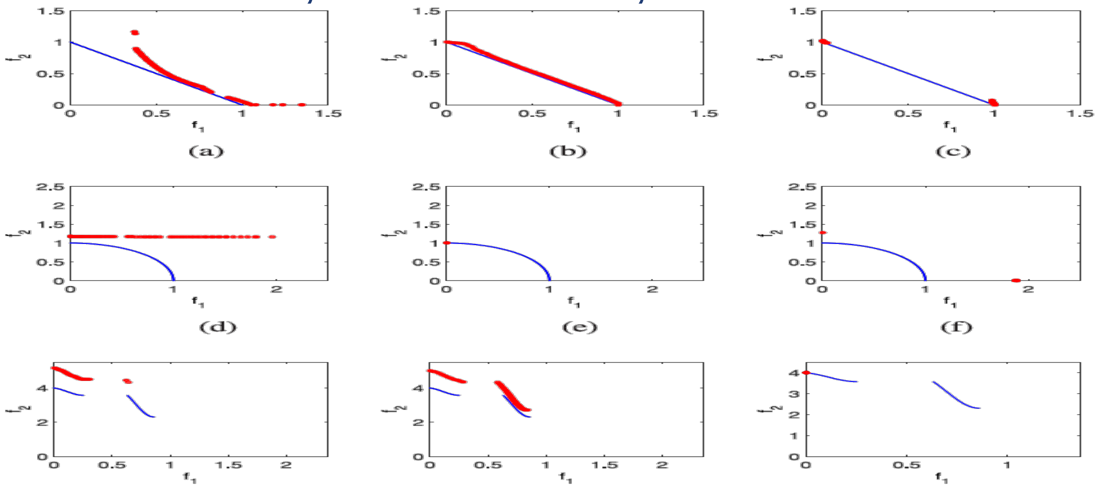
$$\mathbf{C} = \begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,M} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M,1} & c_{M,2} & \cdots & c_{M,M} \end{pmatrix} \Rightarrow \begin{cases} f_1(\mathbf{x}) = h_1(\mathbf{x}^f)\left(1 + \sum_{j=1}^{M} c_{1,j} \times \bar{g}_1\left(\mathbf{x}_j^s\right)\right) \\ \cdots \\ f_i(\mathbf{x}) = h_i(\mathbf{x}^f)\left(1 + \sum_{j=1}^{M} c_{i,j} \times \bar{g}_i\left(\mathbf{x}_j^s\right)\right) \\ \cdots \\ f_M(\mathbf{x}) = h_M(\mathbf{x}^f)\left(1 + \sum_{j=1}^{M} c_{M,j} \times \bar{g}_M\left(\mathbf{x}_j^s\right)\right) \end{cases}$$

- The decision variables have mixed seperability.
- The decision variables have linkages on the PSs.

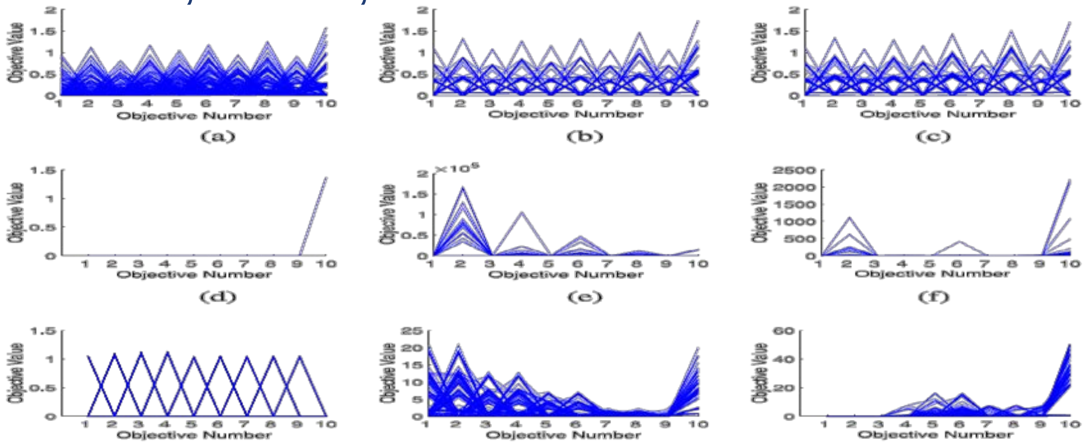
Test problem for large-scale multiobjective optimization

Performance of existing MOEAs on LSMOP

- IM-MOEA, MOEA/D-DE, NSGA-II



- IBEA, RVEA, NSGA-III



Problem	M	IM-MOEA	MOEA/D-DE	NSGA-II
LSMOP1	2	1.10E-01	1.90E-02	3.05E-01
		1.32E-01	2.13E-02	3.13E-01
		1.82E-01	7.08E-02	3.23E-01
	3	4.60E-01	1.01E+00	8.07E-01
		8.03E-01	1.13E+00	9.27E-01
		9.75E-01	1.30E+00	9.98E-01
LSMOP2	2	6.27E-02	8.38E-02	8.39E-02
		6.39E-02	8.61E-02	8.90E-02
		6.80E-02	9.07E-02	9.42E-02
	3	8.27E-02	8.48E-02	9.61E-02
		8.54E-02	8.56E-02	9.93E-02
		9.61E-02	8.73E-02	1.00E-01
LSMOP3	2	1.45E+00	5.02E-01	1.35E+00
		1.72E+00	7.08E-01	1.42E+00
		2.49E+00	7.08E-01	1.75E+00
	3	3.26E+00	5.46E+00	3.71E+00
		4.40E+00	7.41E+00	4.33E+00
		7.99E+00	7.88E+00	4.91E+00
LSMOP4	2	7.12E-02	3.12E-02	1.28E-01
		7.14E-02	6.33E-02	1.28E-01
		7.62E-02	9.72E-02	1.33E-01
	3	2.06E-01	2.21E-01	2.54E-01
		2.12E-01	2.22E-01	2.56E-01
		2.20E-01	2.27E-01	2.71E-01
LSMOP5	2	2.17E-01	1.40E-02	3.41E-01
		2.77E-01	1.61E-02	3.42E-01
		4.66E-01	1.82E-02	3.44E-01
	3	6.91E-01	5.97E-01	1.48E+00
		9.85E-01	7.03E-01	1.64E+00
		1.40E+00	1.20E+00	1.82E+00
LSMOP6	2	5.44E-01	7.44E-01	7.18E-01
		6.17E-01	7.44E-01	7.74E-01
		7.76E-01	7.44E-01	8.72E-01
	3	2.86E+00	1.20E+00	1.80E+00
		1.06E+01	1.74E+00	2.45E+00
		1.43E+01	2.01E+00	2.62E+00
LSMOP7	2	2.88E+00	1.12E+00	1.71E+00
		4.35E+00	1.93E+00	2.20E+00
		5.67E+00	2.97E+00	2.41E+00
	3	1.21E+00	9.48E-01	1.49E+00
		1.25E+00	9.48E-01	1.50E+00
		1.36E+00	9.48E-01	1.54E+00
LSMOP8	2	1.11E-01	4.81E-02	3.46E-01
		1.91E-01	4.97E-02	3.47E-01
		2.19E-01	5.21E-02	3.55E-01
	3	3.70E-01	5.60E-01	3.15E-01
		4.02E-01	5.69E-01	3.28E-01
		4.26E-01	5.85E-01	3.74E-01
LSMOP9	2	6.85E-01	3.20E-01	8.11E-01
		9.95E-01	3.36E-01	8.11E-01
		1.28E+00	3.42E-01	8.11E-01
	3	1.39E+00	4.16E-01	1.63E+00
		2.40E+00	4.80E-01	2.53E+00
		2.43E+00	4.90E-01	2.60E+00

Problem	M	IBEA	RVEA	NSGA-III
LSMOP1	6	6.41E-01	6.83E-01	2.27E+00
		9.13E-01	7.06E-01	3.47E+00
		9.34E-01	1.09E+00	4.18E+00
	10	7.11E-01	7.01E-01	5.05E+00
		9.15E-01	7.67E-01	5.62E+00
		1.06E+00	8.51E-01	6.33E+00
LSMOP2	6	1.66E-01	2.19E-01	2.22E-01
		1.76E-01	2.21E-01	2.23E-01
		1.92E-01	2.23E-01	2.24E-01
	10	2.24E-01	2.39E-01	2.51E-01
		2.36E-01	2.44E-01	2.52E-01
		2.45E-01	2.46E-01	2.53E-01
LSMOP3	6	7.42E+00	8.76E-01	1.14E+01
		1.63E+01	1.03E+00	1.83E+01
		2.05E+01	1.04E+00	2.27E+01
	10	1.06E+00	1.02E+00	7.73E-01
		2.27E+00	1.07E+00	4.27E+00
		3.34E+00	1.21E+00	1.16E+01
LSMOP4	6	1.81E-01	2.87E-01	2.79E-01
		1.82E-01	2.95E-01	2.82E-01
		1.90E-01	3.04E-01	2.84E-01
	10	2.32E-01	2.75E-01	2.93E-01
		2.36E-01	2.77E-01	2.94E-01
		2.45E-01	2.81E-01	2.96E-01
LSMOP5	6	4.33E-01	8.77E-01	5.37E+00
		1.00E+00	8.83E-01	6.39E+00
		1.22E+00	9.24E-01	7.79E+00
	10	7.54E-01	1.25E+00	4.23E+00
		7.55E-01	1.25E+00	4.70E+00
		1.26E+00	1.25E+00	1.56E+01
LSMOP6	6	1.54E+00	1.23E+00	1.42E+00
		1.78E+00	1.28E+00	1.43E+00
		1.89E+00	1.30E+00	2.10E+00
	10	1.68E+00	1.13E+00	1.95E+00
		1.75E+00	1.34E+00	2.17E+00
		2.01E+00	1.36E+00	3.67E+02
LSMOP7	6	1.97E+00	2.29E+00	6.10E+01
		2.21E+00	3.16E+00	6.83E+02
		2.24E+00	6.41E+00	2.88E+03
	10	1.44E+00	2.07E+00	2.23E+00
		1.58E+00	2.59E+00	4.93E+00
		2.19E+00	3.53E+00	4.69E+02
LSMOP8	6	6.69E-01	8.44E-01	2.16E+00
		6.88E-01	8.55E-01	3.06E+00
		8.07E-01	9.00E-01	3.45E+00
	10	7.48E-01	9.65E-01	9.59E-01
		7.55E-01	1.01E+00	9.81E-01
		8.04E-01	1.03E+00	4.52E+00
LSMOP9	6	7.87E+00	1.05E+01	7.84E+00
		8.74E+00	1.34E+01	8.63E+00
		9.14E+00	1.18E+02	9.13E+00
	10	1.19E+01	5.51E+01	3.74E+01
		1.31E+01	8.35E+01	3.81E+01
		1.35E+01	3.10E+02	4.36E+01

The IGD results achieved by the compared algorithms.

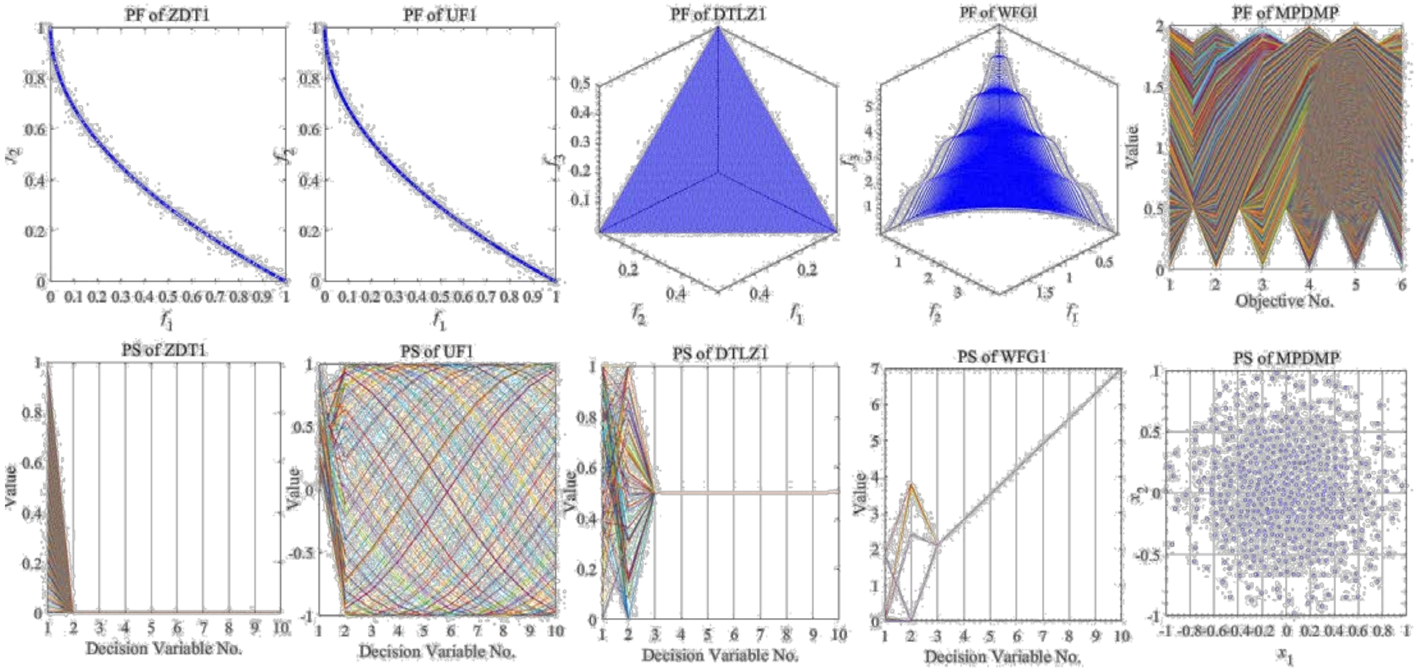
Outline

- Introduction to EMI Group
- Test problem for large-scale multiobjective optimization
- ➢ Real-world large-scale multiobjective optimization problems
- Solving large-scale many-objective optimization problems
- Accelerating large-scale multiobjective optimization
- Future Challenges

Real-world large-scale multiobjective optimization problems

The benchmark LSMOPs are regular in terms of their formulations, Pareto optimal fronts (PF), Pareto optimal sets (PS), etc.

- The multiplication/addition-based form
  - ✓ $f(\mathbf{x}) = (1 + g(\mathbf{x})) \cdot h(\mathbf{x})$
  - ✓ $f(\mathbf{x}) = g(\mathbf{x}) + h(\mathbf{x}),$
- Properties of existing test problems
  - ✓ Regular PF
  - ✓ Regular PS
  - ✓ Regular variable interaction



The PSs and PFs of some popular test problems.

Cheng He, Ran Cheng, Ye Tian, Chuanji Zhang, Hongbin Li. Formulating Time-Varying Ratio Error Estimation for Large-Scale Multiobjective Optimization, manuscript, 2019.

Real-world large-scale multiobjective optimization problems

The time-varying ratio error estimation (TREE) task in the power delivery system

- Problem descriptions

$$\boldsymbol{x} = (x_{A1},\cdots,x_{An},x_{B1},\cdots,x_{Bn},x_{C1},\cdots,x_{Cn})$$

(The decision variables of the TREE, the real voltage values)

$$\boldsymbol{d_1} = (d_{A11},\cdots,d_{An1},d_{B11},\cdots,d_{Bn1},d_{C11},\cdots,d_{Cn1})$$

$$\cdots$$

$$\boldsymbol{d_k} = (d_{A1k},\cdots,d_{Ank},d_{B1k},\cdots,d_{Bnk},d_{C1k},\cdots,d_{Cnk})$$

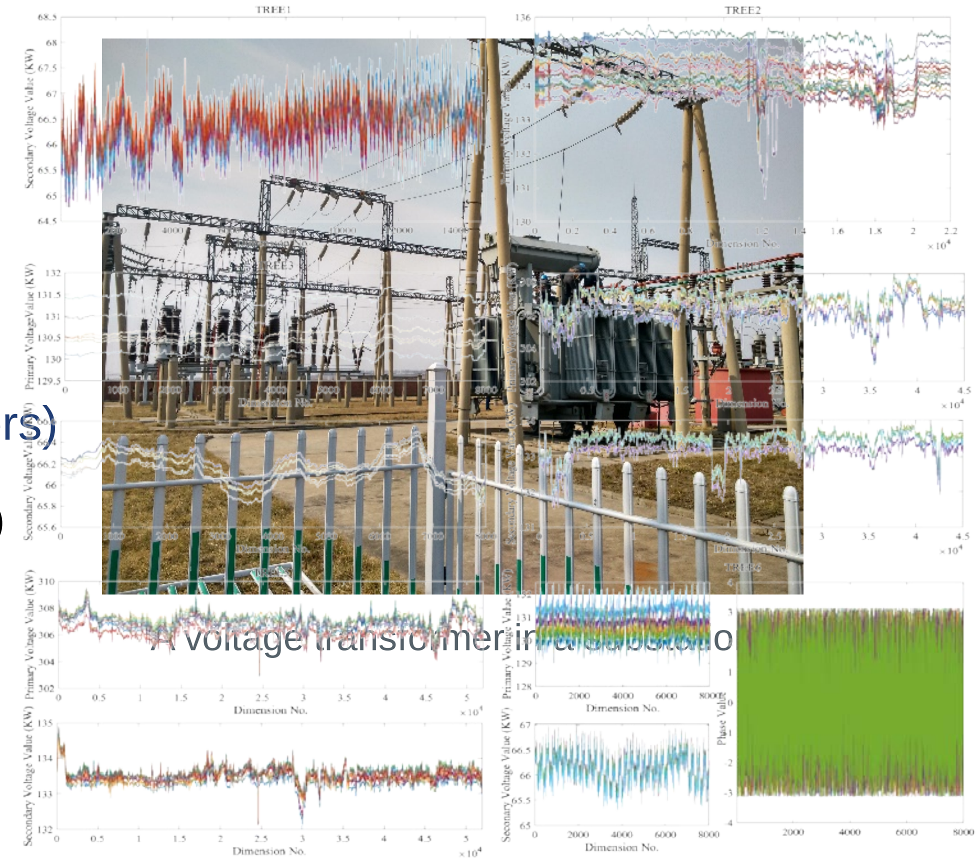
(The voltage values measured by k three-phase voltage transformers)

$$\boldsymbol{e^i}=\left(\frac{d_{A1i}-x_{A1}}{x_{A1}},\cdots,\frac{d_{Ani}-x_{An}}{x_{An}},\frac{d_{B1i}-x_{B1}}{x_{B1}},\ldots,\frac{d_{Bni}-x_{Bn}}{x_{Bn}},\frac{d_{C1i}-x_{C1}}{x_{C1}},\ldots,\frac{d_{Cni}-x_{Cn}}{x_{Cn}}\right)$$

(Ratio error between the measured value and the real value)

$$\boldsymbol{D^i}=(e_2^i-e_1^i,\cdots,e_{n+1}^i-e_n^i)$$

(Time-varying relationship of the ratio errors)



The data collected from different voltage transformers

Real-world large-scale multiobjective optimization problems

The time-varying ratio error estimation (TREE) task in the power delivery system

- Formulation (physical and statistic rules extracted by experts), including 2~3 objectives, 3~6 constraints, and hundreds to millions of decision variables.

✓ $$f_1(\boldsymbol{x}, \boldsymbol{d_1}, \dots, \boldsymbol{d_k}) = \sum_{i=1}^{n} \sqrt{\sum_j^k (\boldsymbol{e}^j)^2}$$

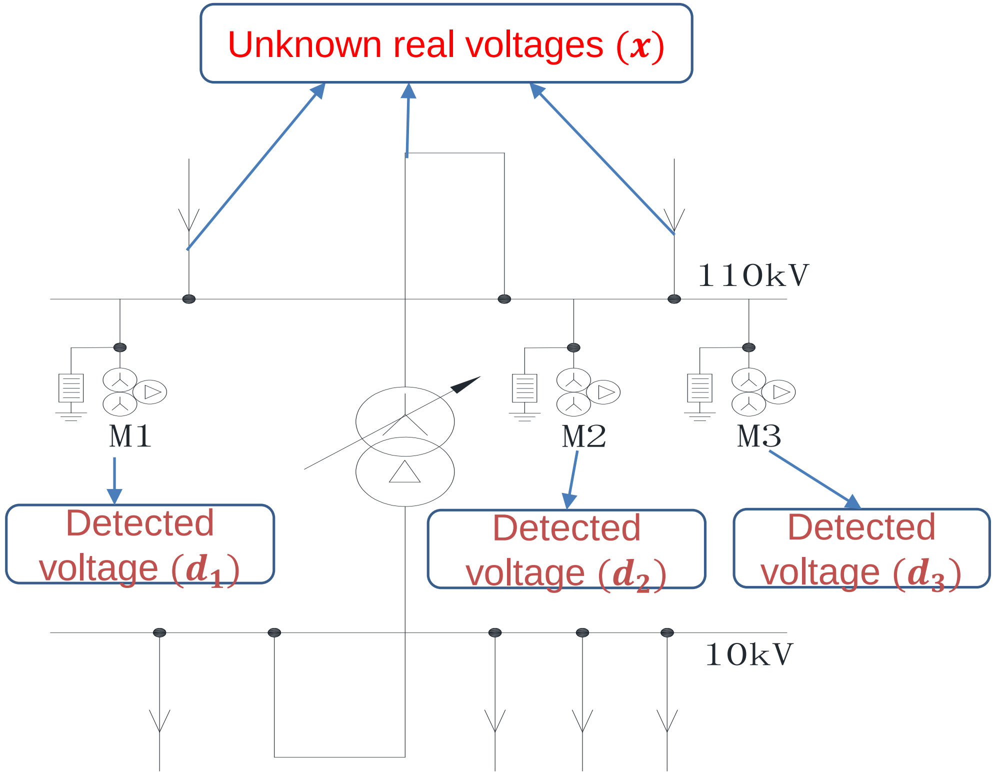
✓ $$f_2(\boldsymbol{x}, \boldsymbol{d_1}, \dots, \boldsymbol{d_k}) = \sum_{i=1}^{k} \sqrt{std(\boldsymbol{D^i})}$$

✓ $$|e_A^i(n)| < C_1$$

✓ $$\frac{\max\{|A_0(n)-\mu(n)|, |B_0(n)-\mu(n)|, |C_0(n)-\mu(n)|\}}{\mu(n)} < C_2$$

✓ $$\left|\frac{1}{N-1}\sum_{k=1}^{N-1} dA_0(k)\right| < C_3$$

✓ $$\sqrt{D(dA_0)} < C_4$$



A smaller $(f_1(x), f_2(x))$ indicates a better prediction accuracy.

Real-world large-scale multiobjective optimization problems

Variable interaction in TREE

- Variable interaction with one function (by DG2)
- Variable interaction with multiple functions (by LMEA and MOEA/DVA).

Problem	Part	f_1	f_2	f_3	g_1	g_2	g_3	g_4	g_5	g_6
TREE1	part 1	0	200:200:200	–	600	600	600	–	–	–
	part 2	600	0	–	0	0	0	–	–	–
TREE2	part 1	0	400:400:400	–	1200	1200	1200	–	–	–
	part 2	1200	0	–	0	0	0	–	–	–
TREE3	part 1	0	200:200:200	–	600	600	600	–	–	–
	part 2	600	0	–	0	0	0	–	–	–
TREE4	part 1	0	400:400:400	–	1200	1200	1200	1200	–	–
	part 2	1200	0	–	0	0	0	0	–	–
TREE5	part 1	0	400:400:400	–	1200	1200	1200	1200	–	–
	part 2	1200	0	–	0	0	0	0	–	–
TREE6	part 1	0	300:150:150	300:150:150	1200	1200	1200	1200	600	600:600
	part 2	1200	600	600	0	0	0	0	600	0

‘part 1’ is the number of decision variables in each group and ‘part 2’ is the number of groups with one decision variable.

Differential groupings associated with each objective/ constraint function.

Problem	MOEA/DVA		LMEA	
	Objectives	Constraints	Objectives	Constraints
TREE1	600:0	598:2	600:0	337:263
TREE2	1199:1	1200:0	1200:0	341:859
TREE3	597:3	600:0	600:0	14:586
TREE4	1197:3	1200:0	1200:0	126:1074
TREE5	599:1	1200:0	1200:0	199:1001
TREE6	1193:1207	2380:20	1200:1200	2346:54

Convergence-/diversity-related variable analysis associated with all the objectives/constraints.

- ✓ Fully separable/non-separable and partially separable interactions are involved.
- ✓ Convergence-/diversity-related variables are involved in both the objectives and constraints.
- ✓ It is interesting to observe the different analysis results obtained by LMEA and MOEA/DVA.

Real-world large-scale multiobjective optimization problems

Performance of existing MOEAs on TREE problems

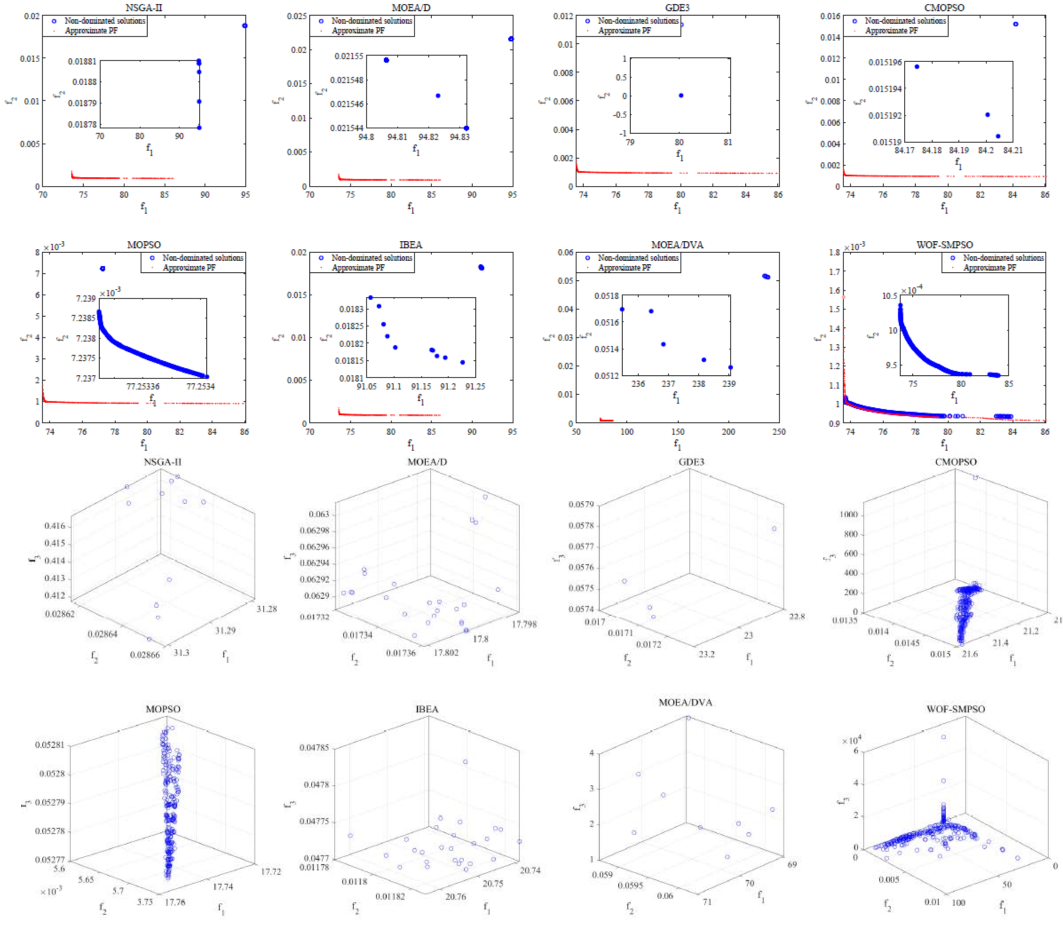
- NSGA-II, MOEA/D, GDE3, CMOPSO, MOPSO, IBEA, MOEA/DVA, WOF-SMPSO

Problem	Dim	NSGA-II	MOEA/D	GDE3	CMOPSO	MOPSO	IBEA	MOEA/DVA	WOF-SMPSO
TREE1	1	1.99E+01(5.61E-02)	2.00E+01(1.82E-02)	2.30E+01(4.28E-01)	2.21E+01(1.73E-01)	2.47E+01(9.22E-02)	2.02E+01(8.83E-02)	5.91E+01(2.38E-01)	1.39E+00(4.01E-01)
	2	2.58E+01(6.28E-02)	2.58E+01(8.64E-02)	2.88E+01(2.78E-01)	2.68E+01(2.17E-01)	3.05E+01(1.55E-01)	2.52E+01(5.01E-02)	1.30E+02(3.68E-01)	2.48E+01(1.17E+00)
	3	3.85E+01(2.70E-01)	3.87E+01(1.34E-01)	4.28E+01(5.00E-01)	3.90E+01(6.36E-01)	4.44E+01(1.89E-01)	3.73E+01(5.16E-02)	1.96E+02(2.95E-01)	3.56E+01(7.90E-01)
	4	5.26E+01(3.46E-01)	5.25E+01(2.33E-01)	6.04E+01(4.71E-01)	5.36E+01(1.23E+00)	6.16E+01(1.96E-01)	5.09E+01(7.49E-02)	2.60E+02(5.99E-01)	4.49E+01(1.22E+00)
	5	6.64E+01(2.85E-01)	6.60E+01(2.59E-01)	7.45E+01(4.77E-01)	6.64E+01(7.95E-01)	7.48E+01(2.87E-01)	6.39E+01(1.50E-01)	3.27E+02(4.74E-01)	5.59E+01(9.68E-01)
TREE2	1	5.06E+01(4.65E-01)	5.05E+01(2.72E-01)	5.40E+01(7.52E-01)	5.16E+01(2.75E-01)	5.63E+01(1.34E-01)	4.90E+01(6.07E-02)	2.57E+02(5.69E-01)	3.49E+01(1.53E+00)
	2	1.00E+02(6.52E-01)	9.90E+01(4.81E-01)	1.09E+02(6.22E-01)	9.81E+01(2.42E+00)	1.10E+02(2.21E-01)	9.57E+01(6.25E-02)	5.19E+02(9.28E-01)	6.95E+01(2.08E+00)
	3	1.56E+02(9.61E-01)	1.53E+02(5.84E-01)	1.69E+02(2.16E-01)	1.54E+02(3.10E+00)	1.70E+02(2.73E-01)	1.49E+02(3.36E-01)	7.76E+02(9.57E-01)	1.11E+02(1.56E+01)
	4	1.96E+02(2.52E-01)	1.92E+02(1.52E-02)	2.12E+02(1.34E-01)	1.97E+02(2.95E-01)	2.12E+02(7.87E-02)	1.88E+02(6.09E-02)	1.05E+03(6.07E-01)	1.25E+02(2.40E+00)
	5	2.53E+02(0.00E+00)	2.47E+02(0.00E+00)	2.81E+02(5.99E-14)	2.48E+02(0.00E+00)	2.79E+02(5.99E-14)	2.44E+02(0.00E+00)	1.30E+03(0.00E+00)	1.75E+02(3.00E-14)
TREE3	1	2.20E+02(1.93E-01)	3.58E+02(3.44E+00)	2.20E+02(5.57E-02)	1.13E+02(3.36E+00)	2.15E+02(1.33E+00)	2.02E+02(1.07E+00)	2.92E+02(6.34E+00)	6.78E+02(4.00E-03)
	2	4.34E+02(3.77E-01)	7.43E+02(2.77E+00)	4.33E+02(9.86E-02)	2.56E+02(5.72E+00)	4.27E+02(7.88E-01)	4.04E+02(1.56E+00)	6.05E+02(4.48E+00)	1.36E+03(2.91E-03)
	3	8.04E+02(4.10E-01)	1.27E+03(8.98E+00)	8.00E+02(1.21E-01)	5.44E+02(3.40E+00)	7.96E+02(1.49E+00)	7.60E+02(1.24E+00)	1.17E+03(6.12E+00)	2.80E+03(2.08E-02)
	4	1.09E+03(3.17E-01)	1.69E+03(6.43E+00)	1.08E+03(5.38E-02)	7.47E+02(6.34E+00)	1.08E+03(6.31E-01)	1.03E+03(1.68E+00)	1.69E+03(4.87E+00)	4.28E+03(9.48E-03)
	5	1.33E+03(4.23E-01)	2.24E+03(8.61E+00)	1.32E+03(2.33E-01)	9.59E+02(4.15E+00)	1.32E+03(2.04E+00)	1.27E+03(2.61E+00)	2.10E+03(1.44E+01)	5.89E+03(1.87E+00)
TREE4	1	2.31E+01(1.03E-02)	2.09E+01(0.00E+00)	2.18E+01(0.00E+00)	1.96E+01(3.74E-15)	1.99E+01(0.00E+00)	2.07E+01(3.74E-15)	9.32E+01(0.00E+00)	2.28E+01(0.00E+00)
	2	5.32E+01(1.50E-14)	4.27E+01(7.49E-15)	4.45E+01(7.49E-15)	4.07E+01(7.49E-15)	4.07E+01(7.49E-15)	4.64E+01(7.49E-15)	1.88E+02(0.00E+00)	4.38E+01(7.49E-15)
	3	–	–	–	–	–	–	–	–
	4	8.42E+00(1.77E+01)	4.13E+01(8.39E-05)	–	–	–	–	–	–
	5	–	–	–	–	–	–	–	–
TREE5	1	1.50E+01(4.76E+01)	1.53E+01(4.83E+01)	2.16E+01(6.84E+01)	1.80E+01(5.70E+01)	2.33E+01(7.37E+01)	1.49E+01(4.73E+01)	5.97E+01(1.89E+02)	2.39E+01(7.57E+01)
	2	1.12E+02(1.50E-14)	1.19E+02(1.50E-14)	2.60E+02(5.99E-14)	1.37E+02(3.00E-14)	–	–	–	–
	3	1.93E+02(6.37E-01)	1.99E+02(5.38E-01)	4.01E+02(4.41E+00)	2.48E+02(1.46E+00)	4.14E+02(1.78E-01)	1.88E+02(2.37E-01)	–	–
	4	2.49E+02(1.14E+00)	2.54E+02(1.03E+00)	5.57E+02(5.07E+00)	3.27E+02(1.78E+00)	1.12E+02(2.37E+02)	2.43E+02(1.77E-01)	–	–
	5	9.60E+01(1.55E+02)	3.26E+02(1.25E+00)	1.44E+02(3.03E+02)	9.02E+01(1.90E+02)	1.36E+02(2.86E+02)	–	–	–
TREE6	1	2.07E+03(3.91E-01)	2.07E+03(7.90E-02)	2.07E+03(7.15E-02)	2.07E+03(1.86E+00)	2.07E+03(2.19E-02)	2.07E+03(3.10E-02)	2.10E+03(9.98E-01)	1.83E+03(6.33E+02)
	2	6.78E+07(8.24E-02)	6.78E+07(4.83E-02)	6.78E+07(4.58E-03)	6.78E+07(1.57E-08)	6.78E+07(0.00E+00)	6.78E+07(1.57E-08)	6.78E+07(0.00E+00)	3.41E+05(6.14E-11)
	3	–	–	–	–	–	–	–	–
	4	–	–	–	–	–	–	–	–
	5	1.59E+01(2.55E+01)	3.96E+01(1.15E-01)	–	–	–	–	–	–

'–' indicate that the compared algorithm fail to obtain any feasible solution.

IGD results achieved by different algorithms

- ✓ Not well converged or distributed
- ✓ Small number of feasible solutions
- ✓ Fail to solve problems with complex objectives or large-scale decision variables



Non-dominated solutions obtained by each algorithm on TREE1 and TREE6

Outline

- Introduction to EMI Group
- Test problem for large-scale multiobjective optimization
- Real-world large-scale multiobjective optimization problems
- ➢ Solving large-scale many-objective optimization problems
- Accelerating large-scale multiobjective optimization
- Future Challenges

Solving large-scale many-objective optimization problems

LSMOPs are challenging: too many decision variables to optimize – no way to optimize all together.

- Solve in a divide-and-conquer manner
  - ✓ Cooperative coevolution based MOEA (CCGDE3, 2013)
  - ✓ Dimension reduction based method (DR_NSGA-II_KN, 2014)
  - ✓ Decision Variable clustering based MOEA (LMEA, 2018)

Use of Cooperative Coevolution for Solving Large Scale Multiobjective Optimization Problems

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A Memetic Optimization Strategy Based on Dimension Reduction in Decision Space

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A Decision Variable Clustering-Based Evolutionary Algorithm for Large-Scale Many-Objective Optimization

Xingyi Zhang, Ye Tian, Ran Cheng, and Yaochu Jin, *Fellow, IEEE*

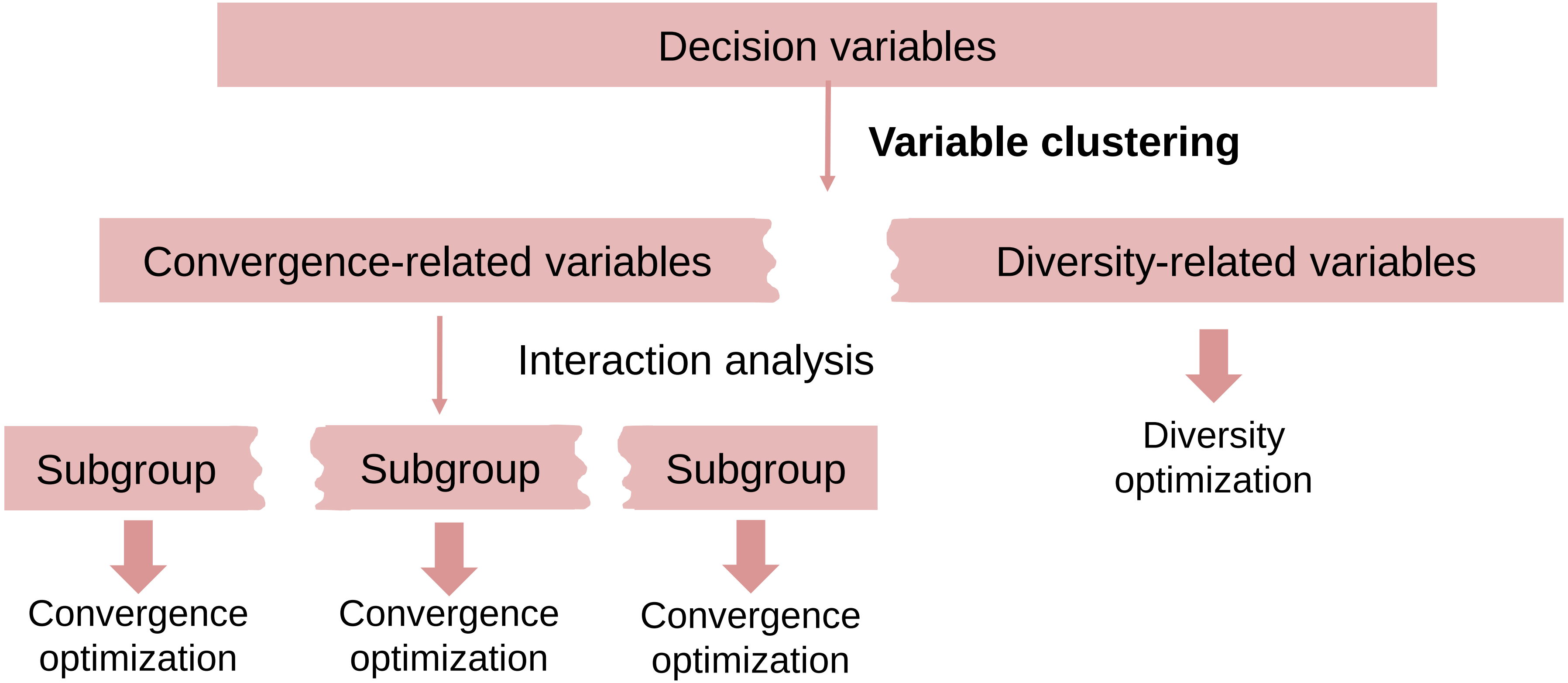
- How about large-scale many-objective optimization problems?
  - ✓ Huge decision space as well as objective space
  - ✓ Difficulty in balancing convergence and diversity

Zhang X, Tian Y, Cheng R*, and Jin Y. A decision variable clustering-based evolutionary algorithm for large-scale many-objective optimization. IEEE Transactions on Evolutionary Computation, 2018, 22(1): 97-112.

Solving large-scale many-objective optimization problems

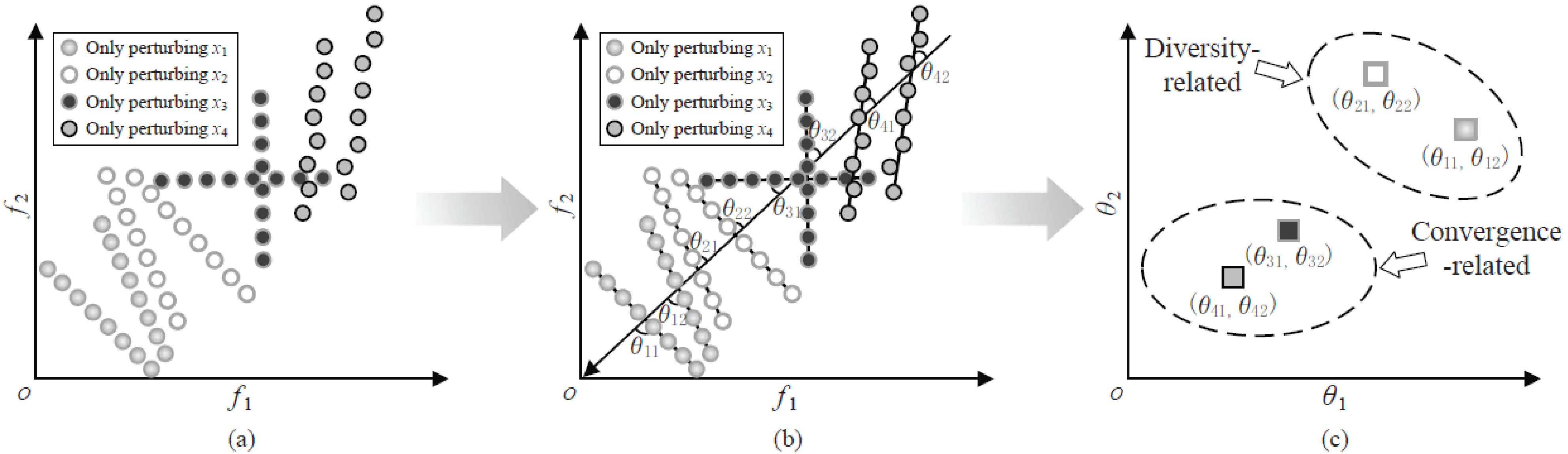
LMEA: Large-scale many-objective evolutionary algorithm

- Solve in a divide and conquer manner
  - ✓ Cluster the decision variables into two groups (*DV* and *CV*)
  - ✓ Further divided the *CV* into several subgroups (*subCVs*)
  - ✓ Iteratively optimize *subCV* and *DV*



Variable clustering in LMEA

- Perturbation and clustering
  - ✓ Perturb each variable of one solution several times
  - ✓ Generate a line to fit the solutions obtained by perturbing each variable
  - ✓ Use K-means to divide all variables into two clusters

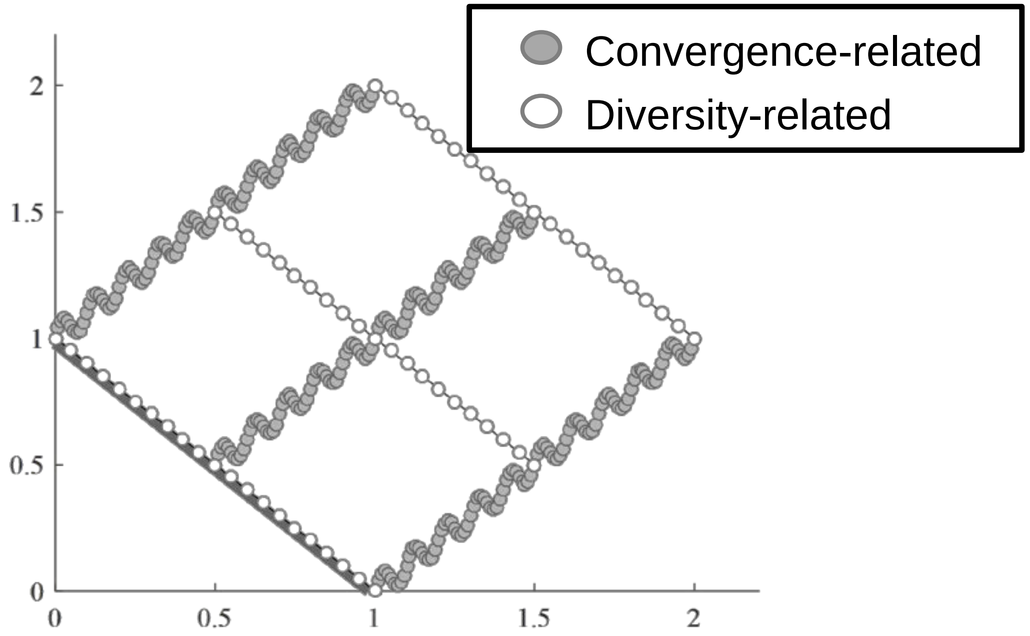


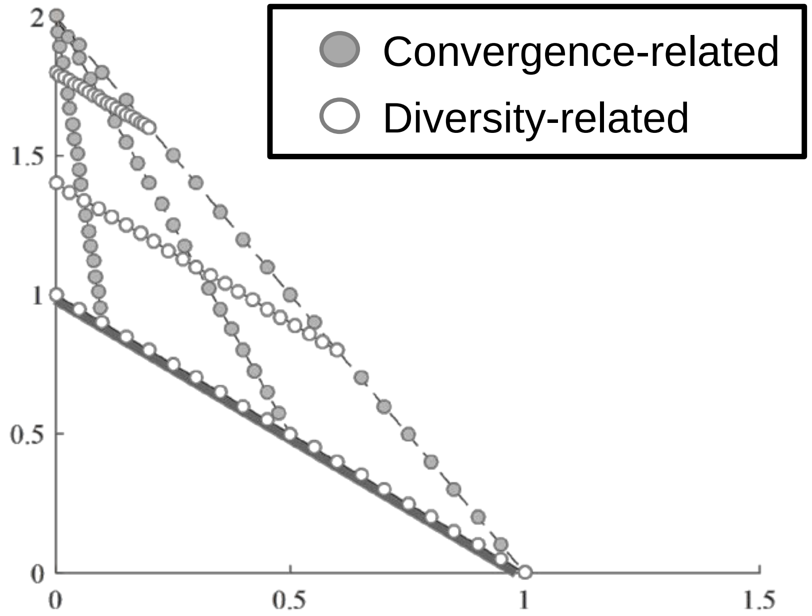
An example to illustrate the clustering method in LMEA, which divides the decision variables into two groups.

Solving large-scale many-objective optimization problems

Variable clustering in LMEA

- Convergence-related and diversity-related variables
  - ✓ Convergence-related variables: can make better convergence with little diversity change
  - ✓ Diversity-related variables: change the distribution of the solutions but contribute little to convergence





Solving large-scale many-objective optimization problems

Decision variable grouping accuracy in comparison with MOEA/DVA

Problem	Obj.	MOEA/DVA			LMEA		Remark
		Diversity	Convergence	Both	Diversity	Convergence	
DTLZ1	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	$\emptyset$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	Same results
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	$\emptyset$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ2	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	$\emptyset$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	$\emptyset$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ3	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	$\emptyset$	$\{x_1, x_2, x_3, , x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	$\emptyset$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ4	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	$\emptyset$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	$\emptyset$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ7	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	$\emptyset$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	$\emptyset$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ5	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_7, x_9, x_{10}, x_{11}, x_{13}, x_{14}, x_{15}\}$	$\{x_5, x_6, x_8, x_{12}, x_{16}\}$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	The variables related to both convergence and diversity are labeled as convergence related variables
	10	$\{x_1, \ldots, x_9\}$	$\{x_{11}\}$	$\{x_{10}, x_{12}, \ldots, x_{15}\}$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
DTLZ6	5	$\{x_1, x_2, x_3, , x_4\}$	$\{x_5, \ldots, x_{11}, x_{13}, x_{15}, x_{16}\}$	$\{x_{12}, x_{14}\}$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{12}, x_{13}\}$	$\{x_{10}, x_{11}, x_{14}, x_{15}\}$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
WFG3	5	$\{x_1, x_2, x_3, x_4\}$	$\{x_8, \ldots, x_{13}, x_{15}, x_{16}\}$	$\{x_5, x_6, x_7, x_{14}\}$	$\{x_1, x_2, x_3, x_4\}$	$\{x_5, \ldots, x_{16}\}$	
	10	$\{x_1, \ldots, x_9\}$	$\{x_{14}, x_{15}\}$	$\{x_{10}, x_{11}, x_{12}, x_{13}\}$	$\{x_1, \ldots, x_9\}$	$\{x_{10}, \ldots, x_{15}\}$	
UF9	3	$\emptyset$	$\{x_3, \ldots, x_{16}\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{16}\}$	Labeled as diversity related
UF10	3	$\emptyset$	$\{x_3, \ldots, x_{16}\}$	$\{x_1, x_2\}$	$\{x_1, x_2\}$	$\{x_3, \ldots, x_{16}\}$	

Variable grouping results on some test instances.

- ✓ Mixed variables are treated as convergence-related variables
- ✓ Similar grouping results

Solving large-scale many-objective optimization problems

Performance on large-scale many-objective optimization

Problem	Obj.	Dec.	MOEA/D	NSGA-III	KnEA	MOEA/DVA	LMEA
DTLZ1	5	100	1.1859e-1(1.63e-3)−	3.9340e+0(1.72e+0)−	6.6202e+0(1.83e+0)−	6.2932e-2(5.72e-5)−	5.9982e-2(4.22e-4)
		500	1.4648e-1(4.30e-2)−	5.2597e+1(5.63e+0)−	7.6436e+1(6.86e+0)−	6.3284e-2(1.63e-4)−	6.1124e-2(5.16e-4)
		1000	1.6833e-1(4.29e-2)−	1.1669e+2(5.08e+0)−	1.3392e+2(1.28e+1)−	6.3442e-2(1.26e-4)−	6.0423e-2(4.09e-4)
	10	100	2.2187e+0(4.72e+0)−	1.1944e+2(2.72e+1)−	5.5749e+0(1.53e+0)−	1.4356e-1(1.79e-2)≈	1.6302e-1(4.87e-3)
		500	1.6987e+2(1.01e+2)−	3.9829e+2(2.77e+1)−	5.0951e+1(1.25e+1)−	1.7047e-1(1.14e-2)≈	1.5995e-1(4.05e-3)
		1000	4.8922e+2(2.04e+2)−	7.9841e+2(4.16e+1)−	1.4841e+2(1.93e+1)−	1.3805e-1(2.18e-2)≈	1.6002e-1(5.24e-3)
DTLZ2	5	100	3.2006e-1(2.26e-8)−	1.9494e-1(7.98e-7)−	2.2045e-1(1.18e-2)−	1.9493e-1(9.26e-8)−	1.8825e-1(2.14e-3)
		500	3.2006e-1(5.31e-8)−	1.9494e-1(5.62e-8)−	2.3629e-1(7.63e-3)−	1.9494e-1(3.17e-8)−	1.8832e-1(2.29e-3)
		1000	3.2006e-1(1.12e-7)−	1.9494e-1(9.86e-7)−	2.2844e-1(1.00e-2)−	1.9494e-1(6.25e-8)−	1.8816e-1(2.53e-3)
	10	100	7.1528e-1(1.81e-2)−	4.2141e-1(1.58e-4)+	4.1230e-1(2.24e-3)+	4.5923e-1(5.13e-2)≈	5.0905e-1(1.48e-2)
		500	7.2369e-1(8.63e-3)−	4.2152e-1(4.79e-5)+	4.1402e-1(5.07e-3)+	4.1976e-1(3.53e-4)+	5.0617e-1(1.78e-2)
		1000	7.3217e-1(1.90e-2)−	4.2172e-1(5.76e-5)+	4.1222e-1(1.85e-3)+	4.2065e-1(1.19e-4)+	5.0689e-1(2.84e-2)
DTLZ3	5	100	3.2877e-1(2.49e-3)−	2.2868e+1(6.14e+0)−	7.6962e-1(4.18e-1)−	1.9505e-1(5.92e-5)−	1.8985e-1(2.14e-3)
		500	4.1776e-1(1.97e-1)−	2.9421e+2(3.34e+1)−	6.1228e-1(1.26e-1)−	1.9536e-1(1.01e-4)≈	1.9035e-1(4.44e-3)
		1000	4.4681e-1(1.40e-2)−	6.8229e+2(5.50e+1)−	8.2477e-1(2.42e-1)−	1.9563e-1(3.70e-4)≈	1.8812e-1(4.10e-3)
	10	100	8.0019e-1(4.87e-2)−	8.2193e+2(1.37e+2)−	6.3652e+0(3.88e+0)−	5.0747e-1(3.77e-2)≈	5.5352e-1(3.56e-2)
		500	1.2941e+2(2.78e+2)−	2.2891e+3(1.35e+2)−	9.4498e+0(4.57e+0)−	5.2820e-1(9.88e-2)≈	5.5126e-1(1.66e-2)
		1000	2.5643e+2(3.27e+2)−	4.4071e+3(1.91e+2)−	4.4621e+0(2.34e+0)−	4.7728e-1(4.26e-2)+	5.4964e-1(1.85e-2)
DTLZ4	5	100	6.2588e-1(2.50e-1)−	2.7298e-1(1.31e-1)+	2.1434e-1(4.33e-3)+	2.6957e-1(1.29e-1)≈	2.6411e-1(1.55e-2)
		500	5.2727e-1(1.18e-1)−	1.9496e-1(2.64e-5)+	2.1571e-1(9.97e-3)+	3.4421e-1(1.29e-1)≈	2.7256e-1(2.46e-2)
		1000	4.3848e-1(1.80e-1)−	2.2829e-1(1.09e-1)≈	2.1380e-1(4.21e-3)+	3.4420e-1(1.29e-1)≈	2.7071e-1(2.36e-2)
	10	100	8.3550e-1(3.18e-2)−	4.2123e-1(1.95e-4)+	4.2764e-1(2.53e-2)+	4.3772e-1(3.33e-2)+	5.0820e-1(2.47e-2)
		500	8.3052e-1(2.93e-2)−	4.2154e-1(1.02e-4)+	4.0104e-1(4.57e-3)+	4.1970e-1(5.83e-5)+	5.2786e-1(3.98e-3)
		1000	8.2082e-1(2.40e-2)−	4.2173e-1(7.92e-5)+	4.0081e-1(2.61e-3)+	4.5695e-1(3.18e-2)+	5.2345e-1(9.26e-3)
DTLZ7	5	100	5.2987e-1(2.57e-2)−	5.2849e-1(1.56e-1)−	2.4790e-1(1.14e-2)+	5.2044e-1(2.51e-6)−	3.0913e-1(1.10e-2)
		500	5.1542e-1(3.73e-7)−	2.3928e+0(1.60e-1)−	2.3191e-1(7.58e-3)+	5.2043e-1(4.57e-7)−	3.2032e-1(8.53e-3)
		1000	5.2120e-1(1.78e-2)−	2.6633e+0(1.38e-1)−	2.3408e-1(1.44e-2)+	5.2043e-1(7.54e-7)−	3.1051e-1(7.59e-3)
	10	100	4.4020e+0(1.37e+0)−	4.6684e+0(4.21e-1)−	1.3854e+0(3.48e-2)−	1.6385e+0(7.84e-2)−	1.0749e+0(6.40e-3)
		500	5.3581e+0(6.94e-1)−	1.1875e+1(8.36e-1)−	1.3572e+0(1.65e-2)−	1.6320e+0(1.02e-1)−	1.0752e+0(3.29e-3)
		1000	5.7150e+0(2.69e-1)−	1.5027e+1(9.39e-1)−	1.3300e+0(9.50e-3)−	1.5249e+0(4.16e-2)−	1.0781e+0(4.58e-3)
+/ − / ≈			0/30/0	8/21/1	12/18/0	6/13/11	

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that of LMEA, respectively.

Problem	Obj.	Dec.	MOEA/D	NSGA-III	KnEA	MOEA/DVA	LMEA
DTLZ5	5	100	4.5161e-2(9.31e-7)−	1.4957e-1(2.58e-2)−	2.7185e-1(4.11e-2)−	2.0440e-1(5.06e-4)−	4.1162e-3(1.44e-4)
		500	4.5161e-2(1.04e-6)−	1.9413e-1(1.78e-2)−	3.1740e-1(6.11e-2)−	2.0469e-1(5.20e-8)−	4.0861e-3(1.48e-4)
		1000	4.5162e-2(3.32e-7)−	2.0606e-1(1.10e-2)−	3.8913e-1(6.77e-2)−	2.0461e-1(1.36e-4)−	4.0729e-3(9.90e-5)
	10	100	4.9994e-2(2.41e-4)−	3.1946e-1(2.03e-2)−	3.6432e-1(5.71e-2)−	1.8877e-1(1.87e-4)−	2.3954e-3(6.95e-5)
		500	5.0407e-2(4.16e-4)−	5.2642e-1(2.04e-2)−	3.6389e-1(5.43e-2)−	1.8866e-1(3.30e-4)−	2.2721e-3(4.47e-5)
		1000	5.0759e-2(2.12e-5)−	6.2093e-1(1.14e-2)−	4.1806e-1(5.07e-2)−	1.8880e-1(2.03e-4)−	2.0713e-3(6.98e-5)
DTLZ6	5	100	1.4970e-1(3.14e-2)−	2.5642e-1(2.29e-2)−	5.8811e-1(1.34e-1)−	1.8236e-1(2.43e-6)−	3.9943e-3(2.14e-4)
		500	1.3010e+0(1.04e-1)−	4.9939e-1(1.89e-2)−	7.2754e-1(1.43e-1)−	1.8236e-1(4.25e-7)−	4.5127e-3(1.22e-3)
		1000	2.7140e+0(1.97e-1)−	6.5774e-1(2.19e-2)−	1.5085e+0(4.53e-1)−	1.8236e-1(5.91e-7)−	3.9747e-3(2.29e-4)
	10	100	6.7510e-2(1.85e-2)−	7.2120e+0(1.35e+0)−	3.7560e+0(9.56e-1)−	1.6531e-1(4.09e-2)−	2.4477e-3(5.11e-4)
		500	1.1735e+0(2.52e-1)−	8.7171e+1(4.88e+0)−	6.3085e+0(2.18e+0)−	1.2750e-1(5.33e-2)−	3.0711e-3(7.20e-4)
		1000	2.6191e+0(5.73e-1)−	1.9202e+2(9.83e+0)−	4.8989e+0(2.54e+0)−	1.1844e-1(2.26e-2)−	3.7077e-3(1.66e-3)
WFG3	5	100	2.0705e+0(8.91e-2)−	7.7137e-1(3.83e-2)−	6.2696e-1(4.24e-1)−	2.3769e+0(7.55e-3)−	1.2581e-1(2.91e-2)
		500	2.2789e+0(6.03e-2)−	8.6982e-1(2.37e-2)−	2.2284e-1(2.56e-2)−	2.4699e+0(9.15e-3)−	1.1736e-1(3.58e-2)
		1000	2.3370e+0(8.10e-2)−	8.8753e-1(2.53e-2)−	4.6414e-1(1.49e-1)−	2.4410e+0(3.52e-2)−	1.2493e-1(2.41e-2)
	10	100	3.4569e+0(1.29e-1)−	3.0344e+0(5.71e-2)−	2.2907e+0(7.93e-1)−	3.4846e+0(2.45e-2)−	1.8542e-1(5.96e-2)
		500	3.8106e+0(8.24e-2)−	3.1112e+0(5.04e-2)−	1.6148e+0(5.53e-1)−	3.5264e+0(9.75e-2)−	4.8685e-1(5.49e-2)
		1000	3.9456e+0(7.23e-2)−	3.1454e+0(4.20e-2)−	1.9861e+0(1.27e+0)−	3.5070e+0(1.17e-1)−	6.9330e-1(1.16e-1)
UF9	3	100	2.9851e-1(1.58e-2)−	2.2030e-1(9.19e-2)−	5.3546e-1(1.39e-1)−	4.3517e-2(2.50e-6)+	5.7008e-2(8.91e-3)
		500	3.1975e-1(2.92e-2)−	3.1029e-1(7.27e-2)−	4.6017e-1(1.19e-1)−	4.3516e-2(9.76e-7)+	5.3626e-2(6.94e-3)
		1000	3.0557e-1(8.39e-2)−	3.7850e-1(4.21e-2)−	5.3607e-1(8.03e-2)−	4.3516e-2(7.00e-7)+	5.1231e-2(4.50e-3)
UF10	3	100	5.9354e-1(1.50e-1)−	3.3482e-1(8.13e-2)−	7.5510e-1(1.49e-1)−	1.1024e-1(2.92e-3)+	1.6632e-1(1.45e-2)
		500	6.3119e-1(1.92e-1)−	3.6779e-1(8.36e-2)−	1.3142e+0(8.69e-1)−	1.0158e-1(8.55e-4)+	1.5547e-1(4.99e-3)
		1000	5.6232e-1(2.48e-1)−	4.2148e-1(1.10e-1)−	9.1794e-1(1.35e-1)−	1.0277e-1(1.01e-3)+	1.6924e-1(9.48e-3)
LSMOP1			3.6215e-1(2.77e-2)−	2.0411e-1(3.05e-3)−	6.5295e-1(3.88e-1)−	1.7219e-1(7.47e-3)−	1.5151e-1(9.99e-3)
LSMOP2			2.4541e-1(8.98e-4)−	1.4727e-1(1.67e-3)−	2.3724e-1(7.06e-2)−	1.4212e-1(2.16e-3)−	1.2644e-1(1.45e-3)
LSMOP3			7.0884e-1(4.00e-2)−	4.4176e-1(1.30e-1)≈	7.0241e-1(9.37e-2)−	6.9024e-1(4.55e-2)−	4.1242e-1(4.68e-2)
LSMOP4			2.7326e-1(4.55e-3)−	1.8222e-1(9.13e-3)−	6.0172e-1(1.44e-1)−	1.5548e-1(4.40e-3)≈	1.5585e-1(2.04e-3)
LSMOP5			5.7900e-1(6.32e-2)−	3.2983e-1(1.23e-1)−	1.1584e+0(4.06e-1)−	3.8667e-1(3.92e-2)−	2.6932e-1(1.65e-2)
LSMOP6			1.2119e+0(2.44e-1)+	1.1094e+0(1.22e-1)+	1.8486e+0(1.39e+0)+	2.0992e+0(1.79e-1)+	1.3820e+4(3.91e+3)
LSMOP7			9.9083e-1(1.53e-1)+	1.0033e+0(2.11e-1)+	1.2275e+1(9.00e+0)−	9.1631e-1(2.81e-2)+	1.3542e+0(3.00e-1)
LSMOP8			5.1604e-1(1.67e-2)−	2.9337e-1(1.02e-2)−	4.3832e-1(5.81e-3)−	3.0537e-1(6.19e-2)−	2.3022e-1(7.57e-3)
LSMOP9			1.0600e+0(2.31e-1)−	2.3568e+0(8.52e-2)−	1.3535e+0(6.62e-1)≈	6.5805e-1(1.13e-1)≈	6.1163e-1(3.55e-2)
+/ − / ≈			2/7/0	2/6/1	1/7/1	2/5/2	

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that of LMEA, respectively.

IGD results obtained by the compared algorithms

- ✓ Effective for large-scale MaOPs

Outline

- Background
- Test problem for large-scale multiobjective optimization
- Real-world large-scale multiobjective optimization problems
- Solving large-scale many-objective optimization problems
- ➢ Accelerating large-scale multiobjective optimization
- Future Challenges

Accelerating large-scale multiobjective optimization

Existing MOEAs are inefficient in terms of function evaluation consumption and computation time.

- Reformulate the LSMOP into SOP
  - ✓ Weight variable association
  - ✓ Subproblem construction
  - ✓ Objective space reduction
- Single-objective optimization
  - ✓ Optimize the weight variable
  - ✓ Collect the solution during the optimization
- Spread the population over the entire PS
  - ✓ Start from quasi-optimal solutions

Algorithm 1 The main framework of the proposed LSMOF.

Input: Z (original LSMOP), FE_{max} (total FEs), Alg (embedded MOEA), N (population size for Alg), r (number of reference solutions) , tr (threshold).

Output: P (final population).

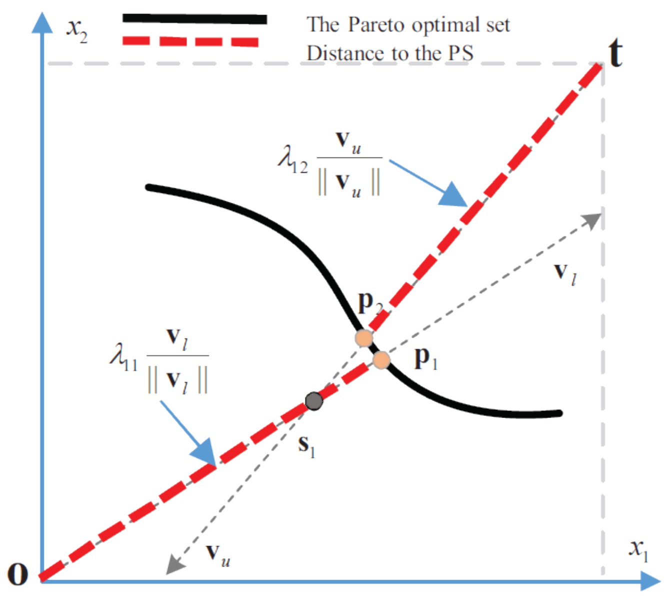
$P \leftarrow$ Initialization(N, Z)
/************First Stage************/
while $t \leq tr \times FE_{max}$ **do**
  $Z' \leftarrow$ Problem_Reformulation(P, r, Z)
  $A, \Delta t \leftarrow$ Single_Objective_Optimization(Z')
  $P \leftarrow$ Environmental_Selection$(A \bigcup P, N)$
  $t \leftarrow t + \Delta t$
end
/**********Second Stage**********/
$P \leftarrow$ Embedded_MOEA (P, N, Alg, Z)

He C, Li L, Tian Y, Zhang X, Cheng R*, Jin Y, and Yao X. Accelerating Large-scale Multi-objective Optimization via Problem Reformulation. *IEEE Transactions on Evolutionary Computation*. 2019.

Accelerating large-scale multiobjective optimization

Reformulating the LSMOP into SOP for reducing the number of decision variables and objectives.

- Reformulate the LSMOP
  - ✓ Select several reference solutions
  - ✓ Generate two reference directions for each solution in the decision space
  - ✓ Assign each direction a weight variable (denote the distance to the PS)
  - ✓ Optimize all the weight variables simultaneously (measure the quality of the solution set by an indicator)



An example of the weight variable association

$$\begin{aligned} \text{Minimize} \quad & F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{x} \in X, \end{aligned}$$

$$\begin{aligned} z_{11}(\lambda_{11}) &= F(\mathbf{o} + \lambda_{11}\frac{\mathbf{v}_l}{||\mathbf{v}_l||}l_{max}) \\ z_{12}(\lambda_{12}) &= F(\mathbf{t} - \lambda_{12}\frac{\mathbf{v}_u}{||\mathbf{v}_u||}l_{max}), \end{aligned}$$

$$Z'(\mathbf{\Lambda}) = \{z_{11}(\lambda_{11}), z_{12}(\lambda_{12}), \ldots, z_{r1}(\lambda_{r1}), z_{r2}(\lambda_{r2})\}$$

$$\begin{aligned} \text{Maximize} \quad & G(\mathbf{\Lambda}) = H(Z'(\mathbf{\Lambda})) \\ \text{subject to} \quad & \mathbf{\Lambda} \in \Re^{2r}, \end{aligned}$$

Accelerating large-scale multiobjective optimization

General performance on LSMOP using 50,000 FEs

- Convergence acceleration

Problem	M	D	NSGA-II	LS-NSGA-II	MOEA/D-DE	LS-MOEA/D-DE	SMS-EMOA	LS-SMS-EMOA	CMOPSO	LS-CMOPSO
LSMOP1	2	200	9.17E-1(2.54E-1)–	5.78E-1(5.32E-2)	3.59E-1(1.85E-2)–	2.13E-1(3.44E-2)	6.24E-1(6.25E-2)–	5.30E-1(6.69E-2)	4.38E-1(2.32E-1)+	5.75E-1(4.58E-2)
		500	2.73E+0(2.77E-1)–	6.14E-1(2.54E-2)	1.07E+0(9.70E-2)–	3.10E-1(3.44E-2)	2.09E+0(5.51E-1)–	5.98E-1(3.35E-2)	1.50E+0(1.61E-1)–	6.18E-1(2.35E-2)
		1000	4.21E+0(2.70E-1)–	6.37E-1(1.97E-2)	1.64E+0(1.17E-1)–	4.26E-1(5.07E-2)	3.72E+0(2.83E-1)–	6.22E-1(2.66E-2)	2.50E+0(1.31E-1)–	6.37E-1(1.99E-2)
	3	200	2.08E+0(2.12E-1)–	5.24E-1(1.35E-2)	1.57E+0(1.56E-1)–	5.26E-1(3.84E-2)	4.58E-1(3.02E-2)+	5.04E-1(1.13E-2)	2.12E+0(3.42E-1)–	5.20E-1(2.66E-2)
		500	5.05E+0(5.73E-1)–	5.96E-1(1.08E-2)	1.80E+0(1.59E-1)–	6.54E-1(4.31E-2)	2.94E+0(3.50E-1)–	5.84E-1(4.14E-2)	4.23E+0(5.58E-1)–	6.16E-1(1.55E-2)
		1000	6.93E+0(6.64E-1)–	6.33E-1(1.34E-2)	1.86E+0(1.97E-1)–	6.68E-1(6.41E-2)	6.29E+0(3.95E-1)–	7.09E-1(1.12E-1)	6.80E+0(5.29E-1)–	6.94E-1(2.13E-2)
LSMOP2	2	200	1.02E-1(2.93E-3)–	3.85E-2(1.08E-3)	9.64E-2(2.38E-3)–	2.71E-2(1.54E-3)	9.19E-2(2.90E-3)–	3.55E-2(2.01E-3)	9.82E-2(2.50E-3)–	3.70E-2(1.14E-3)
		500	6.20E-2(1.16E-3)–	2.32E-2(6.90E-4)	4.89E-2(1.28E-3)–	1.38E-2(1.17E-3)	5.41E-2(1.21E-3)–	1.65E-2(4.67E-4)	5.54E-2(1.42E-3)–	2.14E-2(6.83E-4)
		1000	3.70E-2(3.16E-4)–	1.81E-2(5.41E-4)	2.75E-2(9.26E-4)–	9.15E-3(1.27E-3)	3.30E-2(3.87E-4)–	9.73E-3(2.00E-4)	3.72E-2(7.32E-4)–	1.54E-2(8.72E-4)
	3	200	1.27E-1(4.79E-3)+	1.38E-1(2.76E-3)	1.05E-1(2.83E-3)–	8.51E-2(2.95E-3)	1.23E-1(1.97E-3)+	1.25E-1(5.01E-3)	1.21E-1(9.02E-4)–	1.17E-1(2.27E-3)
		500	8.25E-2(5.49E-3)+	8.71E-2(3.29E-3)	7.41E-2(8.49E-4)–	6.55E-2(9.76E-4)	7.98E-2(2.11E-3)+	8.14E-2(2.98E-3)	6.83E-2(2.81E-4)+	7.20E-2(9.77E-3)
		1000	6.72E-2(3.63E-3)+	7.05E-2(3.08E-3)	6.35E-2(2.54E-4)–	5.97E-2(4.12E-4)	6.55E-2(2.63E-3)+	6.64E-2(1.65E-3)	5.18E-2(3.66E-4)+	5.22E-2(5.09E-4)
LSMOP3	2	200	1.42E+1(2.56E+0)–	1.54E+0(1.43E-3)	5.82E+0(1.03E+0)–	1.53E+0(5.83E-3)	1.73E+1(2.63E+0)–	1.54E+0(1.12E-3)	3.85E+0(6.91E-1)–	1.52E+0(3.30E-3)
		500	1.92E+1(1.62E+0)–	1.57E+0(1.05E-3)	1.33E+1(1.29E+0)–	1.56E+0(1.41E-3)	2.21E+1(1.26E+0)–	1.57E+0(9.70E-4)	2.86E+1(1.24E+0)–	1.56E+0(2.01E-3)
		1000	2.22E+1(1.12E+0)–	1.57E+0(2.28E-4)	1.83E+1(1.22E+0)–	1.57E+0(3.30E-4)	2.35E+1(1.04E+0)–	1.57E+0(2.31E-4)	3.06E+1(1.06E+0)–	1.57E+0(8.81E-4)
	3	200	7.30E+0(1.37E+0)–	8.40E-1(2.51E-2)	7.77E+0(9.45E-1)–	8.27E-1(4.68E-2)	2.65E+0(7.63E-1)–	8.24E-1(3.15E-2)	9.46E+0(8.41E-1)–	8.60E-1(2.45E-3)
		500	1.53E+1(2.62E+0)–	8.59E-1(3.26E-3)	1.00E+1(7.92E-1)–	8.19E-1(4.79E-2)	7.81E+0(1.30E+0)–	1.60E+0(3.09E+0)	1.31E+1(8.51E-1)–	8.61E-1(1.14E-6)
		1000	1.95E+1(3.27E+0)–	8.61E-1(7.03E-5)	1.08E+1(5.73E-1)–	8.41E-1(3.55E-2)	1.63E+1(5.24E+0)–	5.97E+0(1.63E+1)	1.49E+1(7.86E-1)–	8.61E-1(1.14E-6)
LSMOP4	2	200	1.51E-1(3.96E-3)–	9.87E-2(1.69E-3)	1.59E-1(1.01E-2)–	6.99E-2(6.41E-3)	1.41E-1(2.25E-3)–	9.65E-2(1.56E-3)	1.31E-1(2.43E-3)–	9.41E-2(2.27E-3)
		500	9.71E-2(2.47E-3)–	5.05E-2(1.14E-3)	9.18E-2(1.28E-3)–	4.18E-2(2.52E-3)	7.84E-2(1.17E-3)–	4.66E-2(7.59E-4)	9.00E-2(2.21E-3)–	5.06E-2(9.20E-4)
		1000	6.26E-2(9.58E-4)–	3.20E-2(9.49E-4)	5.42E-2(9.08E-4)–	2.42E-2(1.43E-3)	4.83E-2(5.52E-4)–	2.50E-2(4.18E-4)	6.50E-2(1.08E-3)–	3.12E-2(9.27E-4)
	3	200	3.20E-1(6.48E-3)–	2.92E-1(8.37E-3)	2.87E-1(6.01E-3)–	2.31E-1(8.50E-3)	2.97E-1(1.09E-2)–	2.73E-1(1.30E-2)	3.27E-1(1.05E-2)–	2.72E-1(7.12E-3)
		500	1.93E-1(4.24E-3)+	2.13E-1(4.72E-3)	1.65E-1(1.76E-3)–	1.29E-1(3.44E-3)	1.90E-1(4.21E-3)–	1.83E-1(9.10E-3)	1.94E-1(2.27E-3)–	1.68E-1(4.74E-3)
		1000	1.29E-1(4.51E-3)+	1.41E-1(3.63E-3)	1.09E-1(1.50E-3)–	8.83E-2(2.32E-3)	1.26E-1(2.24E-3)+	1.32E-1(2.59E-3)	1.18E-1(1.42E-3)–	1.10E-1(1.88E-3)
LSMOP5	2	200	2.18E+0(4.38E-1)–	7.42E-1(1.14E-6)	6.40E-1(4.20E-2)+	7.42E-1(1.14E-6)	1.59E+0(4.72E-1)–	7.42E-1(1.14E-6)	6.33E-1(1.53E-1)+	7.42E-1(1.14E-6)
		500	8.21E+0(4.68E-1)–	7.42E-1(1.14E-6)	2.30E+0(2.69E-1)–	7.42E-1(1.14E-6)	7.33E+0(9.18E-1)–	7.42E-1(1.14E-6)	5.02E+0(3.62E-1)–	7.42E-1(1.14E-6)
		1000	1.12E+1(8.52E-1)–	7.42E-1(1.14E-6)	3.16E+0(1.86E-1)–	7.42E-1(1.14E-6)	1.10E+1(8.88E-1)–	7.42E-1(1.14E-6)	7.31E+0(5.20E-1)–	7.42E-1(1.14E-6)
	3	200	5.19E+0(5.61E-1)–	4.88E-1(5.13E-2)	2.79E+0(4.11E-1)–	4.99E-1(4.33E-2)	1.00E+0(3.95E-1)–	6.14E-1(9.91E-2)	3.35E+0(1.80E+0)–	6.30E-1(1.69E-1)
		500	1.17E+1(9.56E-1)–	5.35E-1(1.23E-2)	3.59E+0(3.91E-1)–	5.41E-1(2.47E-3)	9.42E+0(1.15E+0)–	9.51E-1(2.57E-1)	1.16E+1(1.21E+0)–	7.37E-1(2.02E-1)
		1000	1.62E+1(8.65E-1)–	5.49E-1(2.83E-2)	3.78E+0(2.09E-1)–	5.42E-1(1.60E-4)	1.75E+1(2.25E+0)–	9.02E-1(2.06E-2)	1.47E+1(1.77E+0)–	8.04E-1(1.98E-1)
LSMOP6	2	200	8.97E-1(8.91E-3)–	3.59E-1(2.37E-3)	7.59E-1(5.31E-2)–	3.32E-1(1.64E-2)	9.00E-1(8.66E-3)–	3.58E-1(4.24E-3)	9.60E-1(6.99E-1)–	3.58E-1(1.68E-3)
		500	8.09E-1(1.76E-3)–	3.22E-1(4.69E-4)	7.34E-1(8.50E-2)–	2.87E-1(3.04E-2)	8.08E-1(7.01E-4)–	3.22E-1(1.28E-3)	7.80E-1(6.42E-2)–	3.22E-1(2.63E-4)
		1000	7.75E-1(4.05E-4)–	3.14E-1(6.41E-4)	6.98E-1(1.23E-1)–	2.87E-1(2.74E-2)	7.71E-1(1.61E-2)–	3.14E-1(7.02E-4)	7.35E-1(8.16E-2)–	3.14E-1(1.70E-4)
	3	200	9.64E+1(1.55E+2)–	6.97E-1(1.63E-2)	3.05E+0(1.30E+0)–	6.76E-1(2.25E-2)	3.07E+0(1.03E+0)–	1.62E+0(9.13E-2)	5.13E+1(8.58E+1)–	8.37E-1(3.69E-1)
		500	3.76E+3(1.38E+3)–	7.42E-1(1.70E-2)	2.21E+1(1.72E+1)–	6.78E-1(4.09E-2)	8.44E+1(5.20E+1)–	2.31E+0(1.27E+0)	2.60E+3(1.05E+3)–	7.37E-1(2.11E-2)
		1000	1.24E+4(2.36E+3)–	7.45E-1(2.06E-2)	1.80E+2(8.33E+1)–	7.00E-1(1.47E-2)	1.61E+3(4.90E+2)–	2.05E+0(4.84E-1)	4.95E+3(2.10E+3)–	8.87E-1(6.58E-1)
LSMOP7	2	200	6.15E+1(8.08E+1)–	1.48E+0(2.65E-3)	4.04E+0(7.20E-1)–	1.48E+0(1.82E-3)	2.02E+1(5.37E+1)–	1.48E+0(1.71E-3)	2.52E+0(6.97E-1)–	1.47E+0(3.99E-3)
		500	1.45E+3(1.98E+3)–	1.50E+0(8.71E-4)	2.88E+1(4.97E+0)–	1.50E+0(6.11E-4)	4.74E+2(4.38E+2)–	1.50E+0(1.26E-3)	8.29E+1(1.36E+2)–	1.50E+0(1.35E-3)
		1000	8.24E+3(3.61E+3)–	1.51E+0(4.22E-4)	2.20E+2(4.85E+1)–	1.51E+0(3.19E-4)	4.15E+3(1.90E+3)–	1.51E+0(7.46E-4)	2.05E+3(5.98E+2)–	1.51E+0(7.37E-4)
	3	200	1.78E+0(8.52E-2)–	9.67E-1(2.51E-2)	1.17E+0(6.62E-2)–	8.97E-1(3.29E-2)	3.93E+1(1.85E+1)–	1.05E+0(1.71E-1)	1.89E+0(8.59E-2)–	1.04E+0(7.82E-2)
		500	1.29E+0(1.30E-2)–	8.96E-1(6.81E-3)	1.15E+0(9.17E-3)–	8.51E-1(3.19E-2)	3.98E+3(1.30E+3)–	1.03E+0(9.99E-2)	5.11E+1(2.23E+2)–	9.47E-1(7.64E-2)
		1000	1.10E+0(2.50E-3)–	8.68E-1(1.13E-2)	1.05E+0(2.96E-3)–	8.23E-1(6.78E-2)	3.17E+4(9.76E+3)–	9.75E-1(8.31E-2)	9.32E+2(3.64E+3)–	9.24E-1(8.98E-2)
LSMOP8	2	200	8.88E-1(5.54E-2)–	7.42E-1(1.14E-6)	3.79E-1(1.14E-1)+	7.40E-1(7.96E-3)	8.49E-1(6.41E-2)–	7.42E-1(1.14E-6)	6.66E-1(1.93E-1)≈	7.42E-1(1.14E-6)
		500	3.40E+0(2.81E-1)–	7.42E-1(1.14E-6)	6.34E-1(3.22E-2)+	7.42E-1(1.14E-6)	2.98E+0(3.05E-1)–	7.42E-1(1.14E-6)	2.84E+0(2.05E-1)–	7.42E-1(1.14E-6)
		1000	6.83E+0(4.47E-1)–	7.42E-1(1.14E-6)	1.26E+0(8.71E-2)–	7.42E-1(1.14E-6)	6.23E+0(3.12E-1)–	7.42E-1(1.14E-6)	4.89E+0(2.28E-1)–	7.42E-1(1.14E-6)
	3	200	5.70E-1(7.28E-2)–	3.63E-1(1.38E-2)	7.56E-1(1.02E-1)–	3.37E-1(2.79E-2)	4.42E-1(5.72E-2)+	5.34E-1(4.84E-2)	3.39E-1(4.42E-2)≈	3.56E-1(1.07E-2)
		500	9.64E-1(1.12E-2)–	3.53E-1(4.70E-2)	5.51E-1(6.05E-3)–	3.27E-1(3.14E-2)	1.74E+0(1.41E+0)–	5.40E-1(1.13E-2)	8.36E-1(9.70E-2)–	3.16E-1(3.95E-2)
		1000	9.52E-1(1.82E-2)–	3.60E-1(4.27E-2)	5.35E-1(5.24E-3)–	3.02E-1(4.71E-2)	2.43E+0(3.19E+0)–	5.35E-1(2.17E-2)	9.59E-1(2.61E-4)–	3.01E-1(2.93E-2)
LSMOP9	2	200	1.78E+0(4.84E-2)–	8.10E-1(1.14E-6)	4.44E-1(1.06E-2)+	8.10E-1(1.14E-6)	1.76E+0(2.74E-2)–	8.10E-1(2.25E-3)	1.54E+0(1.91E-1)–	8.10E-1(1.14E-6)
		500	1.38E+0(4.94E-2)–	8.10E-1(6.01E-4)	4.93E-1(2.47E-2)+	8.09E-1(8.96E-4)	1.32E+0(3.73E-2)–	8.09E-1(4.53E-4)	1.23E+0(7.34E-3)–	8.09E-1(8.64E-4)
		1000	4.80E+0(6.96E-1)–	8.08E-1(1.49E-3)	9.43E-1(1.22E-1)–	8.09E-1(1.88E-3)	4.02E+0(6.33E-1)–	8.08E-1(1.08E-3)	1.22E+0(8.51E-2)–	8.07E-1(1.29E-3)
	3	200	3.66E+0(4.05E-1)–	1.54E+0(4.56E-6)	1.29E+0(3.37E-1)≈	1.15E+0(1.46E-3)	3.60E+0(7.43E-2)–	1.37E+0(5.29E-2)	2.54E+0(1.78E-1)–	1.15E+0(4.00E-4)
		500	9.17E+0(1.32E+0)–	1.54E+0(4.56E-6)	5.25E+0(6.43E-1)–	1.16E+0(7.52E-3)	7.20E+0(8.65E-1)–	1.43E+0(1.42E-1)	3.23E+0(7.16E-1)–	1.15E+0(2.89E-4)
		1000	2.04E+1(1.53E+0)–	1.38E+0(1.97E-1)	1.33E+1(1.27E+0)–	1.16E+0(1.14E-2)	2.34E+1(2.39E+0)–	1.17E+0(6.76E-2)	2.59E+1(2.44E+0)–	1.15E+0(7.36E-4)
+/ − / ≈			5/49/0	—	5/48/1	—	6/48/0	—	4/48/2	—

IGD results obtained by the original MOEAs and their accelerated versions

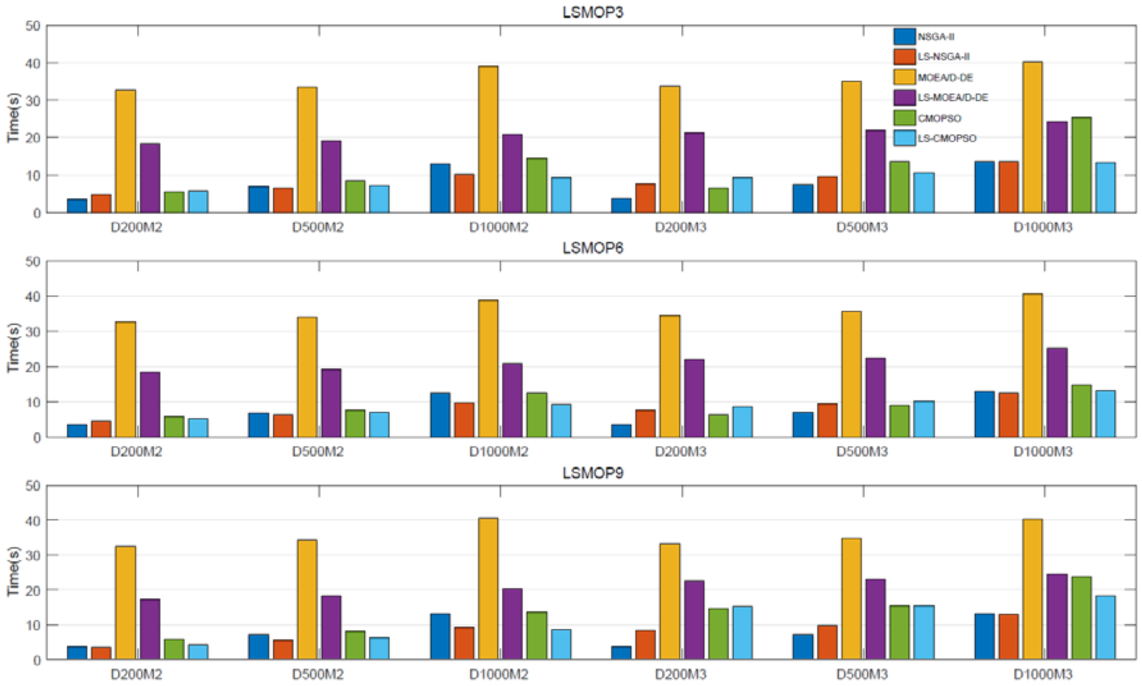
Problem	M	D	MOEA/DVA	WOF-NSGA-II	LS-NSGA-II
LSMOP1	2	200	8.66E+0(8.04E-1)–	6.30E-1(9.36E-2)–	5.78E-1(5.32E-2)
		500	1.91E+1(1.00E+0)–	6.58E-1(6.11E-2)–	6.14E-1(2.54E-2)
		1000	2.39E+1(7.84E-1)–	6.79E-1(4.22E-2)–	6.37E-1(1.97E-2)
	3	200	6.26E+0(4.62E-1)–	6.95E-1(1.32E-1)–	5.24E-1(1.35E-2)
		500	9.42E+0(2.89E-1)–	7.09E-1(8.36E-2)–	5.96E-1(1.08E-2)
		1000	1.08E+1(3.22E-1)–	8.01E-1(7.05E-2)–	6.33E-1(1.34E-2)
LSMOP2	2	200	1.51E-1(6.75E-4)–	7.46E-2(4.63E-4)–	3.85E-2(1.08E-3)
		500	7.27E-2(2.30E-4)–	3.30E-2(3.91E-4)–	2.32E-2(6.90E-4)
		1000	4.04E-2(3.87E-4)–	1.92E-2(3.40E-4)–	1.81E-2(5.41E-4)
	3	200	1.23E-1(2.61E-3)+	1.36E-1(3.84E-3)≈	1.38E-1(2.76E-3)
		500	7.89E-2(2.63E-3)+	8.54E-2(3.82E-3)≈	8.71E-2(3.29E-3)
		1000	6.48E-2(2.46E-3)+	7.00E-2(4.28E-3)≈	7.05E-2(3.08E-3)
LSMOP3	2	200	1.71E+1(1.30E+0)–	1.50E+0(6.88E-2)≈	1.54E+0(1.43E-3)
		500	2.87E+1(8.26E-1)–	1.57E+0(1.47E-3)–	1.57E+0(1.05E-3)
		1000	3.36E+1(6.07E-1)–	1.58E+0(1.61E-3)–	1.57E+0(2.28E-4)
	3	200	2.30E+1(3.53E+0)–	8.61E-1(3.38E-4)–	8.40E-1(2.51E-2)
		500	3.60E+1(2.95E+0)–	8.61E-1(1.30E-4)–	8.59E-1(3.26E-3)
		1000	4.02E+1(2.09E+0)–	8.61E-1(7.28E-4)≈	8.61E-1(7.03E-5)
LSMOP4	2	200	6.56E-1(9.76E-3)–	1.33E-1(1.51E-2)–	9.87E-2(1.69E-3)
		500	5.44E-1(1.90E-3)–	8.74E-2(6.83E-3)–	5.05E-2(1.14E-3)
		1000	4.61E-1(6.97E-4)–	5.99E-2(5.57E-3)–	3.20E-2(9.49E-4)
	3	200	3.26E-1(2.31E-3)–	3.15E-1(9.10E-3)–	2.92E-1(8.37E-3)
		500	1.94E-1(5.71E-4)+	2.14E-1(6.87E-3)≈	2.13E-1(4.72E-3)
		1000	1.20E-1(1.96E-4)+	1.39E-1(5.80E-3)≈	1.41E-1(3.63E-3)
LSMOP5	2	200	1.42E+1(6.21E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
		500	2.09E+1(5.02E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
		1000	2.41E+1(3.40E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
	3	200	1.17E+1(9.27E-1)–	5.41E-1(1.02E-3)–	4.88E-1(5.13E-2)
		500	1.70E+1(6.15E-1)–	5.41E-1(4.66E-5)–	5.35E-1(1.23E-2)
		1000	1.91E+1(5.97E-1)–	5.41E-1(7.27E-5)≈	5.49E-1(2.83E-2)
LSMOP6	2	200	7.36E+2(6.12E+2)–	6.42E-1(7.36E-2)–	3.59E-1(2.37E-3)
		500	2.24E+3(2.14E+3)–	7.33E-1(1.76E-1)–	3.22E-1(4.69E-4)
		1000	2.99E+3(2.33E+3)–	6.82E-1(9.03E-4)–	3.14E-1(6.41E-4)
	3	200	1.77E+4(3.58E+3)–	1.22E+0(3.15E-3)–	6.97E-1(1.63E-2)
		500	3.05E+4(6.34E+3)–	1.29E+0(2.01E-3)–	7.42E-1(1.70E-2)
		1000	3.68E+4(7.07E+3)–	1.31E+0(1.31E-3)–	7.45E-1(2.06E-2)
LSMOP7	2	200	5.58E+4(6.03E+3)–	1.48E+0(2.34E-3)–	1.48E+0(2.65E-3)
		500	1.06E+5(5.12E+3)–	1.51E+0(1.18E-3)–	1.50E+0(8.71E-4)
		1000	1.33E+5(4.14E+3)–	1.51E+0(1.19E-3)–	1.51E+0(4.22E-4)
	3	200	1.80E+0(3.92E-2)–	9.78E-1(4.70E-2)≈	9.67E-1(2.51E-2)
		500	1.27E+0(9.73E-3)–	9.48E-1(1.26E-1)–	8.96E-1(6.81E-3)
		1000	1.10E+0(2.56E-3)–	9.23E-1(1.38E-1)–	8.68E-1(1.13E-2)
LSMOP8	2	200	1.40E+1(8.86E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
		500	2.11E+1(4.21E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
		1000	2.39E+1(4.73E-1)–	7.42E-1(1.14E-6)≈	7.42E-1(1.14E-6)
	3	200	6.69E-1(1.07E-2)–	3.65E-1(4.56E-3)–	3.63E-1(1.38E-2)
		500	6.51E-1(6.13E-3)–	3.55E-1(1.59E-2)–	3.53E-1(4.70E-2)
		1000	6.49E-1(4.56E-3)–	3.56E-1(9.05E-3)+	3.60E-1(4.27E-2)
LSMOP9	2	200	2.26E+1(1.92E+0)–	8.10E-1(1.14E-6)≈	8.10E-1(1.14E-6)
		500	4.32E+1(1.36E+0)–	8.10E-1(3.21E-4)≈	8.10E-1(6.01E-4)
		1000	5.24E+1(1.03E+0)–	8.09E-1(4.10E-4)–	8.08E-1(1.49E-3)
	3	200	6.70E+1(5.47E+0)–	7.74E-1(3.80E-1)+	1.54E+0(4.56E-6)
		500	1.15E+2(5.42E+0)–	8.21E-1(4.13E-1)+	1.54E+0(4.56E-6)
		1000	1.37E+2(3.51E+0)–	1.08E+0(4.00E-1)+	1.38E+0(1.97E-1)
+/ − / ≈			5/49/0	4/32/18	

Comparison with the-state-of-the-art algorithms

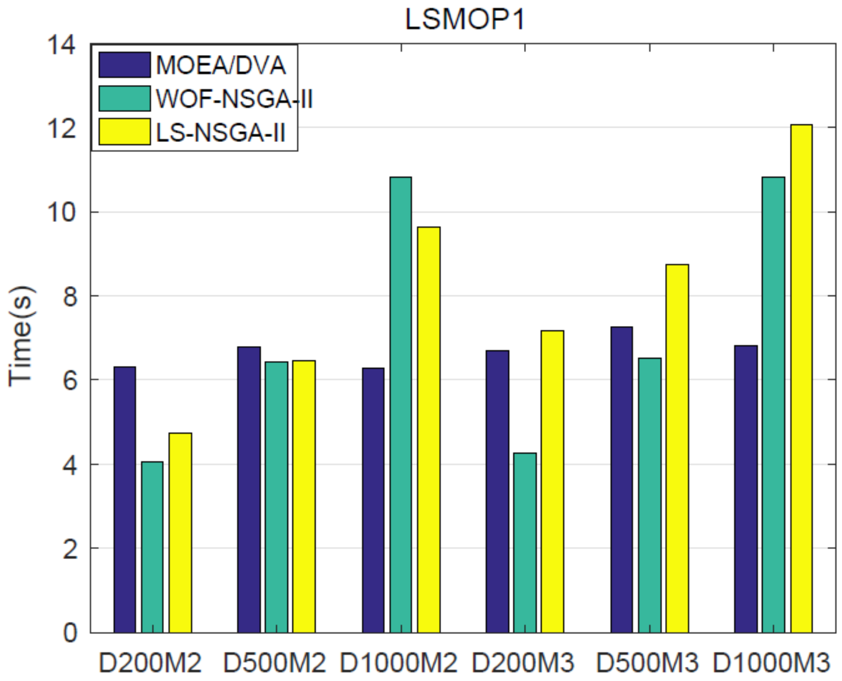
Accelerating large-scale multiobjective optimization

General performance on LSMOP using 50,000 FEs

- Computation time acceleration



The computation time used by the original MOEAs and their accelerated versions



Comparison with the-state-of-the-art algorithms

- ✓ Accelerate the convergence rate
- ✓ Accelerate the computation time

Accelerating large-scale multiobjective optimization

More results on DTLZ and WFG using 50,000 FEs

Problem	M	D	NSGA-II	LS-NSGA-II	MOEA/D-DE	LS-MOEA/D-DE	SMS-EMOA	LS-SMS-EMOA	CMOPSO	LS-CMOPSO
DTLZ1	2	200	4.97E+2(2.94E+1)–	2.40E-3(3.18E-4)	6.34E+2(2.72E+2)–	2.61E-3(1.25E-3)	4.88E+2(2.69E+1)–	1.91E-3(2.42E-4)	1.25E+3(1.12E+2)–	1.84E-3(9.18E-6)
		500	2.03E+3(6.13E+1)–	2.50E-3(2.66E-4)	1.88E+3(7.67E+2)–	2.44E-3(1.15E-3)	1.90E+3(7.49E+1)–	2.24E-3(1.05E-3)	3.68E+3(1.48E+2)–	1.84E-3(1.17E-5)
		1000	6.44E+3(3.14E+2)–	2.51E-3(3.17E-4)	4.55E+3(1.25E+3)–	2.50E-2(5.52E-2)	5.70E+3(2.17E+2)–	2.17E-3(7.14E-4)	8.81E+3(2.92E+2)–	1.84E-3(1.19E-5)
	3	200	8.22E+2(8.97E+1)–	3.00E-2(1.78E-7)	3.54E+2(2.32E+2)–	2.97E-2(2.48E-3)	4.80E+2(3.27E+1)–	5.14E-2(1.49E-2)	2.32E+3(2.36E+2)–	1.72E-1(7.83E-2)
		500	4.54E+3(3.01E+2)–	3.05E-2(2.24E-3)	1.59E+3(5.46E+2)–	3.59E-2(2.01E-2)	2.05E+3(7.96E+1)–	5.02E-2(9.42E-3)	6.05E+3(5.64E+2)–	1.71E-1(7.72E-2)
		1000	1.54E+4(6.08E+2)–	3.10E-2(3.08E-3)	2.43E+3(1.38E+3)–	6.52E-2(4.72E-2)	7.75E+3(2.36E+2)–	6.83E-2(2.65E-2)	1.23E+4(8.44E+2)–	1.82E-1(6.99E-2)
DTLZ2	2	200	2.00E-2(3.56E-8)–	1.00E-2(1.78E-8)	6.56E-2(6.58E-3)–	2.38E-2(9.42E-3)	1.23E-2(8.61E-4)–	6.70E-3(6.37E-4)	1.28E-2(9.83E-4)–	7.70E-3(1.34E-3)
		500	7.00E-1(9.01E-2)–	1.00E-2(1.78E-8)	8.14E-1(1.69E-1)–	3.99E-2(2.05E-2)	5.00E-1(5.64E-2)–	6.02E-3(3.21E-4)	2.10E-1(2.54E-2)–	6.59E-3(7.16E-4)
		1000	8.52E+0(5.77E-1)–	9.50E-3(2.24E-3)	3.35E+0(5.30E-1)–	6.46E-2(5.10E-2)	8.47E+0(5.89E-1)–	5.85E-3(2.67E-4)	3.92E+0(3.55E-1)–	6.37E-3(5.81E-4)
	3	200	1.49E-1(1.35E-2)–	1.37E-1(1.98E-1)	5.35E-1(7.63E-2)–	1.05E-1(1.67E-2)	9.82E-2(5.61E-3)–	9.37E-2(4.45E-2)	2.23E-1(1.95E-2)–	2.00E-1(2.44E-1)
		500	2.27E+0(2.53E-1)–	1.41E+0(3.43E+0)	2.04E+0(3.66E-1)–	1.33E-1(5.04E-2)	1.01E+0(1.30E-1)+	1.32E+0(3.04E+0)	2.91E+0(3.33E-1)–	9.52E-1(2.07E+0)
		1000	1.23E+1(9.30E-1)–	2.38E+0(7.12E+0)	4.72E+0(9.96E-1)–	1.66E-1(7.59E-2)	1.15E+1(8.47E-1)–	1.11E+0(4.60E+0)	1.30E+1(1.27E+0)–	1.97E+0(5.77E+0)
DTLZ3	2	200	1.37E+3(6.19E+1)–	9.00E-3(3.08E-3)	1.63E+3(8.88E+2)–	4.72E-3(1.34E-3)	1.36E+3(8.62E+1)–	6.49E-3(2.15E-3)	3.20E+3(4.39E+2)–	4.10E-3(3.00E-5)
		500	5.47E+3(1.52E+2)–	6.50E-3(4.89E-3)	5.42E+3(2.11E+3)–	1.27E-2(2.12E-2)	5.25E+3(1.94E+2)–	7.40E-3(2.40E-3)	9.75E+3(4.65E+2)–	4.11E-3(3.18E-5)
		1000	1.70E+4(4.98E+2)–	8.00E-3(4.10E-3)	1.21E+4(2.85E+3)–	7.66E-3(1.17E-2)	1.58E+4(4.67E+2)–	9.47E-3(5.58E-3)	2.33E+4(8.40E+2)–	4.12E-3(3.06E-5)
	3	200	1.86E+3(1.29E+2)–	7.20E-2(4.10E-3)	1.44E+3(8.84E+2)–	7.10E-2(2.61E-3)	1.47E+3(1.01E+2)–	1.49E-1(4.66E-2)	7.01E+3(1.15E+3)–	3.03E-1(2.54E-1)
		500	1.03E+4(5.77E+2)–	7.25E-2(5.50E-3)	3.56E+3(1.98E+3)–	1.02E-1(6.89E-2)	5.85E+3(2.92E+2)–	1.63E-1(3.97E-2)	2.04E+4(1.23E+3)–	2.05E-1(2.01E-1)
		1000	4.05E+4(2.10E+3)–	7.30E-2(8.65E-3)	8.32E+3(4.65E+3)–	1.65E-1(1.43E-1)	1.95E+4(6.81E+2)–	1.93E-1(5.30E-2)	4.11E+4(3.82E+3)–	3.10E-1(2.11E-1)
DTLZ4	2	200	9.20E-2(2.22E-1)–	6.50E-3(4.89E-3)	3.53E-1(1.69E-1)+	7.42E-1(1.04E-3)	1.23E-1(2.68E-1)–	5.91E-3(2.19E-4)	5.24E-1(3.42E-1)–	1.18E-1(2.69E-1)
		500	8.66E-1(1.51E-1)–	8.50E-3(3.66E-3)	7.46E-1(1.98E-1)–	7.43E-1(1.86E-3)	6.36E-1(1.41E-1)–	5.82E-3(2.20E-4)	4.78E-1(2.36E-1)≈	4.48E-1(3.69E-1)
		1000	1.03E+1(8.23E-1)–	8.50E-3(3.66E-3)	1.81E+0(3.87E-1)–	7.42E-1(1.14E-6)	9.68E+0(8.22E-1)–	5.86E-3(1.78E-4)	3.88E+0(1.94E+0)–	5.95E-1(3.01E-1)
	3	200	2.66E-1(2.13E-1)+	1.49E+0(1.80E+0)	7.37E-1(1.73E-1)+	9.27E-1(9.03E-2)	2.74E-1(2.93E-1)≈	5.14E-1(5.73E-1)	4.93E-1(1.04E-1)+	9.80E-1(6.71E-1)
		500	2.93E+0(8.11E-1)≈	3.66E+0(3.83E+0)	1.22E+0(1.18E-1)–	9.11E-1(1.18E-1)	1.37E+0(2.53E-1)+	3.54E+0(4.35E+0)	5.43E+0(6.88E-1)–	3.15E+0(2.70E+0)
		1000	1.40E+1(1.69E+0)–	7.69E+0(1.01E+1)	2.69E+0(3.35E-1)–	9.46E-1(3.32E-2)	1.38E+1(1.57E+0)–	1.01E+1(1.20E+1)	1.77E+1(5.67E+0)–	6.57E+0(9.05E+0)
DTLZ5	2	200	2.00E-2(3.56E-8)–	1.00E-2(1.78E-8)	6.69E-2(9.39E-3)–	2.21E-2(5.47E-3)	1.19E-2(1.01E-3)–	6.79E-3(7.03E-4)	1.24E-2(8.01E-4)–	7.35E-3(1.16E-3)
		500	6.39E-1(6.48E-2)–	1.00E-2(1.78E-8)	8.35E-1(1.43E-1)–	3.99E-2(2.13E-2)	5.14E-1(5.55E-2)–	5.95E-3(2.27E-4)	2.06E-1(1.99E-2)–	6.91E-3(9.19E-4)
		1000	8.79E+0(7.51E-1)–	8.50E-3(3.66E-3)	2.93E+0(4.33E-1)–	7.74E-2(4.41E-2)	8.21E+0(5.87E-1)–	5.85E-3(2.81E-4)	3.88E+0(3.03E-1)–	6.23E-3(4.38E-4)
	3	200	8.50E-2(1.00E-2)–	1.00E-2(1.78E-8)	3.94E-1(8.95E-2)–	2.89E-2(8.76E-3)	4.01E-2(4.51E-3)–	1.98E-2(4.66E-3)	2.82E-1(4.33E-2)–	1.78E-2(3.09E-3)
		500	2.83E+0(2.15E-1)–	1.00E-2(1.78E-8)	2.07E+0(4.64E-1)–	4.62E-2(3.42E-2)	1.13E+0(1.27E-1)–	2.03E-2(5.14E-3)	7.06E+0(1.04E+0)–	1.63E-2(3.48E-3)
		1000	1.40E+1(5.83E-1)–	9.50E-3(2.24E-3)	4.22E+0(7.25E-1)–	4.11E-2(1.41E-2)	1.26E+1(9.20E-1)–	1.96E-2(4.42E-3)	2.35E+1(2.21E+0)–	1.53E-2(3.64E-3)
DTLZ6	2	200	5.38E+1(3.91E+0)–	1.00E-2(1.78E-8)	2.33E+0(1.93E+0)–	3.97E-3(7.26E-8)	5.43E+1(4.09E+0)–	5.78E-3(4.65E-4)	4.72E+1(5.26E+0)–	4.12E-3(3.38E-5)
		500	2.58E+2(9.40E+0)–	1.00E-2(1.78E-8)	7.73E+1(8.44E+0)–	3.97E-3(1.15E-7)	2.55E+2(6.52E+0)–	6.33E-3(9.98E-4)	2.05E+2(9.04E+0)–	4.13E-3(3.99E-5)
		1000	6.43E+2(1.06E+1)–	1.00E-2(1.78E-8)	2.83E+2(1.28E+1)–	3.97E-3(3.58E-7)	6.13E+2(6.78E+0)–	7.22E-3(1.66E-3)	4.73E+2(1.62E+1)–	4.12E-3(2.87E-5)
	3	200	9.67E+1(3.23E+0)–	1.00E-2(1.78E-8)	5.71E+0(4.28E+0)–	1.22E-2(5.67E-5)	8.52E+1(4.03E+0)–	2.48E-2(1.00E-2)	1.05E+2(5.87E+0)–	3.99E-3(4.93E-5)
		500	3.52E+2(3.64E+0)–	1.00E-2(1.78E-8)	1.09E+2(1.69E+1)–	1.22E-2(5.87E-5)	3.45E+2(3.57E+0)–	2.39E-2(7.70E-3)	2.82E+2(1.28E+1)–	4.09E-2(1.65E-1)
		1000	7.92E+2(5.25E+0)–	1.00E-2(1.78E-8)	3.46E+2(2.57E+1)–	1.21E-2(1.53E-4)	7.98E+2(5.38E+0)–	3.60E-2(1.97E-2)	5.85E+2(2.82E+1)–	2.86E-2(1.09E-1)
DTLZ7	2	200	5.65E-2(1.60E-2)+	4.40E-1(1.14E-6)	2.71E+0(4.19E-1)–	4.56E-1(8.07E-3)	2.12E-2(4.81E-3)+	4.53E-2(1.36E-1)	1.65E-2(1.94E-3)+	4.21E-1(9.79E-2)
		500	1.12E+0(1.05E-1)–	4.41E-1(2.24E-3)	4.70E+0(2.64E-1)–	4.56E-1(1.33E-2)	9.15E-1(7.09E-2)–	2.28E-2(9.89E-2)	3.54E-1(7.46E-2)–	1.33E-1(2.08E-1)
		1000	2.42E+0(1.13E-1)–	4.43E-1(1.13E-2)	5.83E+0(1.36E-1)–	4.58E-1(1.99E-2)	2.44E+0(1.32E-1)–	4.51E-2(1.36E-1)	1.31E+0(1.33E-1)–	1.11E-1(1.97E-1)
	3	200	5.30E-1(5.74E-2)+	7.78E-1(1.01E-1)	4.78E+0(4.88E-1)–	8.61E-1(2.08E-2)	3.07E-1(5.54E-2)+	7.98E-1(2.20E-1)	3.50E-1(6.78E-2)+	8.03E-1(1.03E-1)
		500	1.72E+0(1.67E-1)–	1.03E+0(3.22E-1)	7.98E+0(2.53E-1)–	8.76E-1(3.18E-2)	2.03E+0(2.18E-1)–	2.07E-2(9.26E-2)	1.55E+0(1.68E-1)–	2.91E-1(4.07E-1)
		1000	3.10E+0(2.39E-1)–	1.23E+0(3.24E-1)	9.40E+0(3.01E-1)–	8.65E-1(2.96E-2)	4.23E+0(1.68E-1)–	4.71E-2(1.33E-1)	4.14E+0(3.14E-1)–	1.23E-1(3.01E-1)
+/ − / ≈			3/39/0	—	2/40/0	—	4/37/1	—	3/38/1	—

Problem	M	D	NSGA-II	LS-NSGA-II	MOEA/D-DE	LS-MOEA/D-DE	SMS-EMOA	LS-SMS-EMOA	CMOPSO	LS-CMOPSO
WFG1	2	200	8.79E-1(1.61E-2)+	1.27E+0(1.25E-2)	1.27E+0(3.60E-3)–	1.26E+0(1.15E-2)	8.56E-1(3.42E-2)+	1.20E+0(4.41E-2)	1.27E+0(4.22E-3)+	1.27E+0(2.34E-2)
		500	1.12E+0(1.00E-2)+	1.23E+0(1.25E-2)	1.27E+0(2.38E-3)–	1.26E+0(1.09E-2)	1.11E+0(2.81E-2)+	1.31E+0(3.21E-2)	1.28E+0(3.51E-3)≈	1.28E+0(1.86E-2)
		1000	1.21E+0(4.62E-3)+	1.26E+0(1.46E-2)	1.28E+0(1.95E-3)–	1.27E+0(5.73E-3)	1.20E+0(1.48E-2)+	1.33E+0(3.06E-2)	1.28E+0(3.13E-3)≈	1.28E+0(1.23E-2)
	3	200	1.36E+0(2.11E-2)+	1.48E+0(5.45E-2)	1.57E+0(2.00E-2)+	1.62E+0(6.75E-2)	1.40E+0(1.41E-2)+	1.49E+0(3.13E-2)	1.54E+0(1.65E-2)+	1.57E+0(1.70E-2)
		500	1.46E+0(1.03E-2)+	1.49E+0(1.35E-2)	1.57E+0(3.39E-2)+	1.61E+0(5.37E-2)	1.48E+0(7.90E-3)+	1.51E+0(2.06E-2)	1.53E+0(1.83E-2)+	1.56E+0(1.93E-2)
		1000	1.49E+0(1.09E-2)≈	1.49E+0(1.20E-2)	1.57E+0(1.77E-2)≈	1.61E+0(6.77E-2)	1.50E+0(7.52E-3)+	1.53E+0(1.72E-2)	1.54E+0(1.93E-2)+	1.57E+0(1.97E-2)
WFG2	2	200	1.43E-1(1.23E-2)–	7.66E-2(6.90E-2)	1.85E-1(1.18E-2)≈	1.56E-1(5.71E-2)	1.46E-1(4.00E-2)–	2.90E-2(1.74E-2)	1.55E-1(1.38E-2)–	4.41E-2(2.79E-2)
		500	2.03E-1(9.82E-3)–	4.73E-2(3.01E-2)	2.47E-1(1.09E-2)–	1.49E-1(3.70E-2)	2.05E-1(3.38E-2)–	5.16E-2(3.86E-2)	2.54E-1(9.34E-3)–	6.25E-2(4.25E-2)
		1000	2.89E-1(4.51E-2)–	8.14E-2(4.62E-2)	2.86E-1(1.65E-2)–	1.59E-1(5.36E-2)	2.68E-1(4.22E-2)–	6.62E-2(3.85E-2)	3.18E-1(8.37E-3)–	8.83E-2(5.70E-2)
	3	200	3.19E-1(4.09E-2)–	2.16E-1(3.13E-2)	6.37E-1(4.95E-2)–	4.40E-1(4.11E-2)	4.13E-1(1.10E-1)≈	3.96E-1(1.23E-1)	2.84E-1(1.28E-2)–	1.74E-1(1.61E-2)
		500	4.09E-1(2.07E-2)–	2.28E-1(4.31E-2)	6.12E-1(6.06E-2)–	4.68E-1(5.26E-2)	5.02E-1(9.47E-2)–	3.58E-1(1.24E-1)	4.07E-1(1.29E-2)–	1.88E-1(3.28E-2)
		1000	5.10E-1(2.37E-2)–	2.15E-1(2.66E-2)	6.37E-1(6.79E-2)–	4.55E-1(4.85E-2)	6.00E-1(8.40E-2)–	3.58E-1(1.25E-1)	4.76E-1(1.17E-2)–	1.83E-1(2.61E-2)
WFG3	2	200	1.42E-1(1.64E-2)–	9.36E-2(4.06E-2)	2.43E-1(1.24E-2)–	1.09E-1(3.47E-2)	1.32E-1(8.63E-3)–	4.79E-2(1.34E-2)	1.37E-1(1.01E-2)–	7.49E-2(2.03E-2)
		500	2.23E-1(1.40E-2)–	7.85E-2(2.82E-2)	3.02E-1(1.06E-2)–	9.39E-2(1.92E-2)	1.93E-1(1.49E-2)–	5.94E-2(2.57E-2)	2.45E-1(8.09E-3)–	8.90E-2(3.39E-2)
		1000	2.98E-1(1.14E-2)–	9.77E-2(3.82E-2)	3.21E-1(1.14E-2)–	1.02E-1(2.34E-2)	2.53E-1(9.45E-3)–	8.37E-2(3.00E-2)	3.29E-1(8.24E-3)–	1.01E-1(3.04E-2)
	3	200	3.56E-1(2.19E-2)–	9.11E-2(1.78E-2)	5.03E-1(2.17E-2)–	2.80E-1(4.11E-2)	3.13E-1(1.84E-2)–	1.25E-1(5.73E-2)	6.37E-1(2.45E-2)–	2.20E-1(2.90E-2)
		500	5.11E-1(2.14E-2)–	9.50E-2(1.29E-2)	5.13E-1(1.87E-2)–	2.63E-1(3.86E-2)	4.75E-1(1.49E-2)–	1.30E-1(6.09E-2)	7.45E-1(2.39E-2)–	2.19E-1(2.57E-2)
		1000	6.28E-1(2.11E-2)–	9.41E-2(1.52E-2)	5.24E-1(2.26E-2)–	2.59E-1(4.00E-2)	6.05E-1(1.72E-2)–	1.55E-1(7.33E-2)	7.64E-1(2.13E-2)–	2.18E-1(3.34E-2)
WFG4	2	200	4.74E-2(3.01E-3)+	9.36E-2(6.98E-3)	1.64E-1(1.13E-2)–	1.26E-1(1.10E-2)	3.75E-2(2.68E-3)≈	3.74E-2(4.88E-3)	1.30E-1(8.09E-3)–	1.09E-1(7.57E-3)
		500	8.46E-2(4.20E-3)–	6.78E-2(7.89E-3)	1.78E-1(9.33E-3)–	1.28E-1(8.48E-3)	6.41E-2(4.47E-3)–	5.02E-2(6.85E-3)	1.72E-1(5.35E-3)–	1.18E-1(7.31E-3)
		1000	1.33E-1(4.98E-3)–	7.76E-2(4.28E-3)	1.84E-1(5.61E-3)–	1.23E-1(8.94E-3)	1.01E-1(4.73E-3)–	6.04E-2(7.99E-3)	2.02E-1(3.48E-3)–	1.20E-1(7.42E-3)
	3	200	2.99E-1(1.12E-2)–	2.57E-1(1.12E-2)	4.08E-1(7.67E-3)–	3.75E-1(1.38E-2)	3.15E-1(1.64E-2)–	2.94E-1(1.37E-2)	3.72E-1(1.45E-2)–	2.77E-1(9.24E-3)
		500	3.45E-1(1.23E-2)–	2.65E-1(1.66E-2)	4.13E-1(8.91E-3)–	3.75E-1(1.12E-2)	3.56E-1(1.03E-2)–	2.86E-1(1.62E-2)	4.15E-1(1.16E-2)–	2.79E-1(8.67E-3)
		1000	3.87E-1(9.76E-3)–	2.61E-1(1.43E-2)	4.12E-1(1.09E-2)–	3.68E-1(1.19E-2)	4.11E-1(1.06E-2)–	2.82E-1(1.55E-2)	4.29E-1(8.97E-3)–	2.82E-1(1.14E-2)
WFG5	2	200	7.70E-2(1.79E-3)–	7.22E-2(1.62E-2)	6.94E-2(1.92E-3)+	7.14E-2(2.30E-3)	6.88E-2(1.05E-3)≈	7.04E-2(4.47E-3)	8.85E-2(5.07E-3)–	7.46E-2(4.52E-3)
		500	1.44E-1(6.26E-3)–	7.08E-2(1.04E-2)	6.86E-2(2.73E-3)+	7.01E-2(3.92E-3)	1.11E-1(4.53E-3)–	7.34E-2(9.79E-3)	1.11E-1(7.17E-3)–	7.58E-2(6.15E-3)
		1000	2.23E-1(9.55E-3)–	7.73E-2(1.96E-2)	6.78E-2(2.63E-3)+	7.14E-2(2.35E-3)	1.88E-1(6.98E-3)–	7.06E-2(5.37E-3)	1.28E-1(8.87E-3)–	7.56E-2(5.22E-3)
	3	200	3.19E-1(1.25E-2)–	2.77E-1(1.15E-2)	3.19E-1(5.84E-3)–	3.13E-1(3.65E-3)	3.43E-1(1.86E-2)–	3.21E-1(2.61E-2)	3.51E-1(2.44E-2)–	2.87E-1(2.34E-2)
		500	4.16E-1(1.34E-2)–	2.82E-1(2.81E-2)	3.23E-1(7.22E-3)–	3.14E-1(4.95E-3)	4.57E-1(1.78E-2)–	3.33E-1(1.93E-2)	3.91E-1(2.85E-2)–	2.93E-1(2.39E-2)
		1000	5.04E-1(1.08E-2)–	2.81E-1(1.70E-2)	3.18E-1(5.80E-3)–	3.13E-1(3.38E-3)	5.46E-1(1.27E-2)–	3.40E-1(3.48E-2)	3.98E-1(4.06E-2)–	2.95E-1(2.10E-2)
WFG6	2	200	5.33E-2(2.34E-3)–	2.54E-2(4.05E-3)	3.48E-2(4.52E-2)–	1.36E-2(1.22E-4)	3.83E-2(2.20E-3)–	2.73E-2(4.07E-3)	6.21E-2(4.27E-3)–	2.34E-2(3.13E-3)
		500	1.36E-1(8.13E-3)–	2.48E-2(3.33E-3)	4.01E-2(7.48E-2)–	1.50E-2(3.50E-4)	8.76E-2(3.76E-3)–	2.46E-2(2.81E-3)	1.96E-1(1.49E-2)–	2.50E-2(5.31E-3)
		1000	2.51E-1(8.08E-3)–	2.52E-2(2.56E-3)	1.44E-2(5.03E-4)≈	1.40E-2(2.34E-4)	1.84E-1(8.10E-3)–	2.49E-2(3.31E-3)	3.31E-1(1.41E-2)–	2.42E-2(3.35E-3)
	3	200	3.26E-1(1.05E-2)≈	3.29E-1(2.14E-2)	3.27E-1(4.91E-3)+	3.51E-1(4.06E-2)	3.40E-1(1.83E-2)≈	3.53E-1(2.24E-2)	5.05E-1(2.51E-2)–	2.70E-1(5.14E-2)
		500	4.43E-1(8.59E-3)–	3.39E-1(2.33E-2)	3.30E-1(5.71E-3)≈	3.37E-1(1.18E-2)	4.39E-1(1.40E-2)–	3.61E-1(3.06E-2)	7.06E-1(2.06E-2)–	2.65E-1(5.35E-2)
		1000	5.81E-1(1.60E-2)–	3.37E-1(2.58E-2)	3.27E-1(7.97E-3)+	3.44E-1(1.62E-2)	6.00E-1(1.50E-2)–	3.68E-1(3.05E-2)	7.71E-1(2.26E-2)–	2.41E-1(2.62E-2)
WFG7	2	200	3.80E-2(3.13E-3)+	7.62E-2(2.31E-2)	1.59E-1(1.53E-2)–	7.61E-2(3.04E-2)	2.15E-2(9.04E-4)+	3.10E-2(2.70E-3)	5.78E-2(4.08E-3)–	5.01E-2(6.13E-3)
		500	1.32E-1(1.04E-2)–	6.93E-2(1.29E-2)	2.47E-1(1.18E-2)–	7.76E-2(2.07E-2)	7.33E-2(8.23E-3)–	4.76E-2(1.01E-2)	1.67E-1(8.06E-3)–	6.70E-2(1.21E-2)
		1000	2.33E-1(1.54E-2)–	7.82E-2(1.88E-2)	2.81E-1(9.59E-3)–	8.19E-2(2.34E-2)	1.79E-1(2.61E-2)–	5.06E-2(1.61E-2)	2.57E-1(7.48E-3)–	7.53E-2(1.86E-2)
	3	200	4.98E-1(1.60E-2)≈	4.73E-1(8.07E-2)	4.81E-1(9.67E-3)–	4.22E-1(2.45E-2)	4.45E-1(4.04E-2)–	3.66E-1(4.31E-2)	4.22E-1(1.61E-2)–	3.31E-1(4.67E-2)
		500	5.81E-1(1.83E-2)–	4.29E-1(8.30E-2)	4.92E-1(1.18E-2)–	4.25E-1(2.72E-2)	5.75E-1(2.32E-2)–	3.90E-1(5.49E-2)	5.37E-1(1.45E-2)–	3.62E-1(5.43E-2)
		1000	6.47E-1(1.19E-2)–	4.45E-1(6.38E-2)	5.00E-1(8.60E-3)–	4.33E-1(1.67E-2)	6.58E-1(1.63E-2)–	4.00E-1(6.86E-2)	5.75E-1(1.48E-2)–	3.66E-1(5.43E-2)
WFG8	2	200	1.13E-1(7.11E-3)+	1.70E-1(5.66E-2)	2.40E-1(1.52E-2)–	1.47E-1(9.62E-2)	1.00E-1(7.13E-3)+	1.41E-1(1.30E-2)	1.60E-1(6.43E-3)–	1.01E-1(5.18E-2)
		500	1.65E-1(4.72E-3)≈	1.70E-1(1.43E-2)	2.18E-1(2.05E-2)–	1.23E-1(5.55E-2)	1.46E-1(5.83E-3)+	1.68E-1(1.16E-2)	2.34E-1(5.20E-3)–	6.11E-2(2.28E-2)
		1000	2.21E-1(5.62E-3)–	1.47E-1(5.18E-2)	2.17E-1(2.03E-2)–	1.04E-1(7.46E-2)	1.92E-1(6.31E-3)–	1.78E-1(1.28E-2)	2.91E-1(6.38E-3)–	7.43E-2(3.68E-2)
	3	200	3.43E-1(1.27E-2)+	4.82E-1(4.82E-2)	5.11E-1(1.80E-2)≈	5.08E-1(1.14E-1)	3.66E-1(1.63E-2)+	4.07E-1(2.44E-2)	4.69E-1(1.67E-2)+	4.98E-1(2.56E-2)
		500	3.93E-1(1.20E-2)+	4.27E-1(3.79E-2)	4.98E-1(1.86E-2)≈	4.75E-1(8.48E-2)	4.26E-1(1.49E-2)+	4.56E-1(3.44E-2)	5.55E-1(1.22E-2)–	5.16E-1(2.63E-2)
		1000	4.59E-1(9.96E-3)≈	4.80E-1(5.09E-2)	4.87E-1(2.15E-2)≈	4.82E-1(8.67E-2)	5.08E-1(1.51E-2)–	4.72E-1(4.90E-2)	5.86E-1(1.32E-2)–	5.15E-1(2.21E-2)
WFG9	2	200	8.14E-2(6.48E-3)–	2.04E-2(4.58E-3)	3.54E-2(3.15E-2)–	2.01E-2(1.67E-3)	3.71E-2(3.79E-3)–	2.38E-2(4.19E-3)	7.34E-2(4.65E-3)–	2.16E-2(4.21E-3)
		500	2.02E-1(1.17E-2)–	1.92E-2(3.53E-3)	2.81E-2(3.14E-2)–	2.47E-2(3.39E-3)	1.08E-1(8.98E-3)–	2.16E-2(3.12E-3)	1.58E-1(8.09E-3)–	1.88E-2(4.63E-3)
		1000	3.11E-1(1.82E-2)–	1.89E-2(3.75E-3)	4.81E-2(4.75E-2)≈	2.27E-2(1.45E-3)	2.13E-1(1.47E-2)–	2.19E-2(2.34E-3)	2.30E-1(1.25E-2)–	1.87E-2(4.07E-3)
	3	200	4.74E-1(5.20E-2)–	3.58E-1(7.84E-2)	3.10E-1(1.88E-2)+	3.46E-1(3.06E-2)	3.98E-1(2.15E-2)–	3.49E-1(4.09E-2)	4.09E-1(1.79E-2)–	2.56E-1(2.20E-2)
		500	7.01E-1(2.21E-2)–	3.48E-1(8.15E-2)	3.31E-1(3.63E-2)≈	3.32E-1(2.32E-2)	5.58E-1(2.37E-2)–	3.55E-1(5.32E-2)	5.62E-1(2.37E-2)–	2.43E-1(2.05E-2)
		1000	8.30E-1(2.84E-2)–	2.96E-1(2.99E-2)	3.32E-1(3.93E-2)≈	3.45E-1(2.56E-2)	7.17E-1(2.31E-2)–	3.65E-1(3.37E-2)	6.29E-1(3.48E-2)–	2.40E-1(2.35E-2)
+/ − / ≈			10/39/5	—	8/36/10	—	11/39/4	—	5/48/2	—

The IGD results obtained by the original MOEAs and their accelerated versions on DTLZ and WFG test suites

Outline

- Background
- Test problem for large-scale multiobjective optimization
- Real-world large-scale multiobjective optimization problems
- Solving large-scale many-objective optimization problems
- Accelerating large-scale multiobjective optimization
- ➢ Future Challenges

Future Challenges

The challenges in large-scale multiobjective optimization:

- More effective and efficient variable analysis methods
- Constraint handling
- From thousands to million or even billion scales
- From multiobjective to many-objective
- From cheap to expensive
- From machine learning to deep learning
- From continuous to discrete
- More acceleration strategies
- More real-world LSMOPs
- …

Thank you!
(Q&A)